

# Smooth voltage controller and observer for a three-pole active magnetic bearing system

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## Abstract

In this study, a simple and smooth voltage controller for a 3-pole active magnetic bearing (AMB) system is proposed. Based on the voltage-controlled AMB system, an observer is designed for the rotor displacement estimation. The observer will be based on the input voltages and coil currents that drive the magnetic poles. Such an observer can be utilized in the sensorless control of the AMB system. Due to the strong nonlinearity in the dynamics of 3-pole AMB system, the theory of nonlinear high-gain observer is adopted to design the observer. The proposed smooth voltage controller and nonlinear observer are verified numerically and experimentally.

**Keywords** : Three-pole, Active magnetic bearing, Smooth voltage control, Nonlinear observer, High-gain

## 1. Introduction

Cost reduction is an important research direction in the development of the active magnetic bearings (AMB) system. It is the expensive cost that makes AMB not as common as the conventional bearing in industry. Eliminating or reducing the hardware requirements of an AMB can significantly decrease its overall cost. This is due to the fact that a large portion of an AMB's cost comes from the hardware. One typical example is using zero or low bias currents to lower down the heat dissipation so that the requirement of sophisticated cooling systems is unnecessary (P. Tsiotras, et al., 2003, 2005). On the other hand, the 3-pole AMB has been proposed for this particular purpose (C.-T. Hsu, et al., 2002, 2003; S.-L. Chen, et al., 2002, 2005). As shown in (C.-T. Hsu, et al., 2002, 2003; S.-L. Chen, et al., 2002, 2005), for the 3-pole AMB system, less power amplifiers are needed, and less copper and iron losses are generated. Also, it can provide more space for heat dissipation, coil winding, and sensor installation. As a result, the cost requirement for power amplifiers and cooling system can be reduced.

To further reduce the overall cost, simple and smooth controller is preferred since less computational load and memory are required. This will make real-time implementation easier and reduce the hardware requirement. Furthermore, position sensors are in general more expensive than electrical sensors. It has been found that electrical signals may be used to estimate the mechanical information, leading to the so-called self-sensing technique (T. Mizuno, et al., 1998; Beltran-Carbajal, et al., 2013; Xiaodong Sun, et al., 2013). Thus, sensor cost can be reduced. There have been a large number of self-sensing studies in the literature (see, e.g., (Eric H. Maslen, 2006; Darbandi, S.M., et al., 2014) and the references therein). Several methods have been proposed, including linear observer based, linear observer with parameter identification, switching ripple (from switching amplifier) based, and interrogation signal based.

The objectives of this study are to propose a smooth voltage controller and to design an observer for the 3-pole AMB system. Due to the strong nonlinearity of the 3-pole AMB system, the stable levitation controller is in general complicated. Although a simple and smooth current controller for the 3-pole AMB system was proposed in (David Meeker, et al., 2006; Chen, Shyh-Leh, 2011), voltage controller is in general more cost-effective. Based on the voltage-controlled system, the estimation of the rotor position will be achieved by designing an observer. A Luenberger type observer with Kalman filter has been proposed in (Darbandi, S.M., et al., 2014) based on the linearized 3-pole AMB system. However, the results are valid only when coil currents and rotor displacements are relatively small. The

overall stability and observability cannot be guaranteed by linear controller and observer. In view of the strong nonlinearity of the system, the method of nonlinear high-gain observer will be applied here (Farza, M., et al., 2014). It will be shown that the 3-pole AMB system is open loop observable and hence a nonlinear high-gain observer can be designed.

The contributions of the paper include:

- (i) A nondimensional model for the voltage-controlled 3-pole AMB system is derived.
- (ii) A smooth voltage controller for stable levitation of the 3-pole AMB system is proposed and verified experimentally.
- (iii) The 3-pole AMB system is shown to be open loop observable.
- (iv) A nonlinear high-gain observer for the 3-pole AMB system is proposed and verified experimentally. It can be used for self-sensing control in the future.

Note that to emphasize on the design of smooth voltage controller and nonlinear high-gain observer, the system under study only consists of a two-degree-of-freedom (2-DOF) disk-like rotor without motor. In other words, the rotation motion is not considered here.

This paper is organized as follows. After the introduction, the mathematical model of the 3-pole AMB system is described and a smooth voltage controller is proposed in Section 2. In Section 3, a nonlinear high-gain observer for the voltage-controlled 3-pole AMB system is proposed. Numerical and experimental results are presented in Section 4. Finally, conclusions are drawn in Section 5.

## 2. System modeling and smooth voltage control

### 2.1 The voltage-controlled 3-pole AMB system

The system under study is a voltage-controlled 3-pole AMB system, as shown in Fig. 1. The stator of the AMB is of 3-pole type arranged in a symmetrical Y-shape, which is optimal in the sense that the power loss and number of power amplifiers can be minimized (S.-L. Chen, et al., 2002). The disk-like rotor is made of laminated sheets of silicon steels. The rotor motion is constrained in the axial direction by a thrust ball bearing. The rotor can only move in the radial direction. Hence, the overall system is of two degrees of freedom. The coil current for the lower pole is denoted by  $i_1$ . The coils for the upper two poles are wound in a differential way. In other words, the two poles share the same current  $i_2$ , but with opposite winding directions. The two independent coil currents,  $i_1$  and  $i_2$ , are generated by two input voltages,  $v_1$  and  $v_2$ . The gravity is assumed to be in the negative  $y$ -direction. This system has been investigated previously in (C.-T. Hsu, et al., 2002, 2003; S.-L. Chen, et al., 2002, 2005). The mathematical model is briefly reviewed here. For more details, please refer to the above-mentioned references.

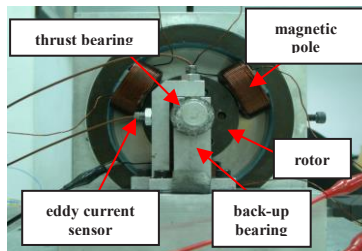


Fig. 1 The voltage-controlled 3-pole AMB system

Several assumptions are made regarding the mathematical model. The first assumption is that the magnetic field is linear in the sense that the magnetic flux density  $B$  is linearly proportional to the magnetic field intensity  $H$ . Another assumption is no flux leakage. Also, the fringing effect is negligible. Finally, the magnetic reluctance exists only in the air gap. Based on the assumptions, a set of dynamic equations for the system can be derived. Please refer to, e.g., (Chen, Shyh-Leh, et al., 2010), for details.

The system variables and parameters are described in order. The rotor displacements are represented by  $x_r$  and  $y_r$ . The most important system parameter is the nominal air gap denoted by  $l_0$ . Below is a list of the other system parameters:  $\mu$  is the magnetic permeability of the air,  $A$  is pole face area,  $N$  is the number of coil turns for each pole,  $R_1$  and  $R_2$  are the coil resistances, and  $m$  is the rotor mass.

Next, a nondimensional model will be derived to simplify the design of the smooth voltage controller and nonlinear observer. Another advantage of adopting the nondimensional model is that the selection of control parameters will be easier. Let the set of nondimensional state variables be

$$x_1 = \frac{x_r}{l_0}, x_2 = \frac{x'_r}{\sqrt{gl_0}}, x_3 = \frac{y_r}{l_0}, x_4 = \frac{y'_r}{\sqrt{gl_0}}, x_5 = \frac{\Phi_1}{\Phi_0}, x_6 = \frac{\Phi_2}{\Phi_0} \quad (1)$$

where  $(\bullet)' = \frac{d(\bullet)}{dt}$ ,  $\Phi_0 = \sqrt{2g/c_0}$  and  $c_0 = 4\mu AN^2/3m$ . The variables  $\Phi_1$  and  $\Phi_2$  are proportional to the magnetic flux through the magnetic poles and are defined by (C.-T. Hsu, et al., 2002; Chen, Shyh-Leh, et al., 2010)

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \frac{-1}{L} \begin{bmatrix} 2l_0 - y_r & \sqrt{3}x_r \\ x_r & \sqrt{3}(2l_0 + y_r) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (2)$$

where  $L = 4l_0^2 - (x_r^2 + y_r^2)$ . Note that  $L$  is always positive in the operation range because that the rotor displacement is always smaller than the nominal air gap, i.e.  $(x_r^2 + y_r^2) \leq l_0^2$ . The first 4 system states represent the mechanical states and the last two the electrical states.

In addition, a set of nondimensional system parameters, and nondimensional input voltages and measured outputs (coil currents) are defined by

$$u_1 = \frac{v_1}{v_0}, u_2 = \frac{v_2}{v_0}, y_1 = \frac{i_1}{i_0}, y_2 = \frac{i_2}{i_0}, r_1 = \frac{R_1}{R_0}, r_2 = \frac{R_2}{R_0}, \tau = \sqrt{g/l_0}t \quad (3)$$

where  $v_0 = 2R_2l_0\Phi_0/\sqrt{3}$  and  $i_0 = \sqrt{8gl_0^2/3c_0}$  are the bias voltages and currents that are required to suspend the rotor weight at the bearing center,  $R_0 = 3mc_0\sqrt{g/l_0^3}$ , and  $\tau$  is the nondimensional time. Then, the nondimensional state space model of the system is given by

$$\dot{x} = f(x) + Gu, \quad y = h(x) \quad (4)$$

where  $\dot{x} = \frac{dx}{d\tau}$  and

$$f(x) = \begin{bmatrix} x_2 \\ 2x_5x_6 \\ x_4 \\ x_6^2 - x_5^2 - 1 \\ 3r_1[-(2+x_3)x_5 + x_1x_6] \\ r_2[x_1x_5 - (2-x_3)x_6] \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -2\sqrt{3}r_2 & 0 \\ 0 & -2r_2 \end{bmatrix}, \quad h(x) = \begin{bmatrix} -\frac{\sqrt{3}}{2}(2+x_3)x_5 + \frac{\sqrt{3}}{2}x_1x_6 \\ \frac{1}{2}x_1x_5 - \frac{1}{2}(2-x_3)x_6 \end{bmatrix}$$

## 2.2 Smooth voltage controller

Stable levitation controller is necessary for the AMB system since it is inherently unstable (G. Schweitzer, et al., 2002). The stabilizing feedback controller is also required in order to verify the proposed observer experimentally. For the feasibility of real-time implementation, a simple and smooth control law is preferred. In view of the smooth current controller proposed in (Chen, Shyh-Leh, 2011), a smooth voltage controller is proposed here. It is to incorporate the smooth current controller with the backstepping procedure. To this aim, we decompose the state equation (4) into two parts: the mechanical dynamics and electrical dynamics as

$$\dot{x}_a = f_a(x); \quad \dot{x}_b = f_b(x) + G_b u \quad (5)$$

where  $x_a = [x_1 \ x_2 \ x_3 \ x_4]^T$  represents the mechanical states,  $x_b = [x_5 \ x_6]^T$  represents the electrical states, and  $f_a(x)$ ,  $f_b(x)$ ,  $G_b$  can be easily recognized from  $f(x)$ ,  $G$ .

Consider the first in equation (5). The first step in the backstepping procedure is to take  $x_b$  as the virtual control input, resulting in the current-controlled system mode. Thus, the smooth current controller proposed in (Chen, Shyh-Leh, 2011) can be applied here. By the results in (Chen, Shyh-Leh, 2011),  $x_b$  can be designed as

$$x_5 = -k_{11}\zeta_1 - k_{12}x_1 - k_{13}x_2 \equiv \psi_1, \quad x_6 = 1 - k_{21}\zeta_2 - k_{22}x_3 - k_{23}x_4 \equiv \psi_2 \quad (6)$$

where  $\dot{\zeta}_1 = x_1$ ,  $\dot{\zeta}_2 = x_3$ , and  $k_{ij}$ 's are the PID feedback gains. The gains are related to the stiffness and damping coefficients of the closed-loop system, and can be designed by assigning proper damping ratios and natural frequencies. Alternatively, one can also design the feedback gains by the standard pole placement method.

The next step of backstepping procedure is to design the actual control input  $u$  so that equation (6) can be fulfilled. To the end, the error state is defined:  $e = x_b - \psi$ . Then, the error dynamics can be obtained from equation (5) as

$$\dot{e} = f_b(x) + G_b u - \dot{\psi} \quad (7)$$

Note that  $G_b$  is nonsingular. Thus, the actual control input can be designed to cancel out the nonlinearity as

$$u = G_b^{-1}(-f_b(x) + \dot{\psi} + w) \quad (8)$$

where  $w$  is the new control input. A simple PI control law can be utilized to design the new input  $w$  as

$$w = \begin{bmatrix} -k_{31}\zeta_3 - k_{32}e_1 \\ -k_{41}\zeta_4 - k_{42}e_2 \end{bmatrix}, \quad \dot{\zeta}_3 = e_1, \quad \dot{\zeta}_4 = e_2 \quad (9)$$

The gains should be designed so that  $e$  can be converged faster than the mechanical dynamics. Also, since we are dealing with the nondimensional model, the feedback gains  $k_{ij}$ 's should be taken within order one to avoid leading to too large gains practically.

Finally, the overall feedback control algorithm is

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \frac{r_1}{r_2} [-(2+x_3)x_5 + x_1x_6] + \frac{1}{2\sqrt{3}r_2} [k_{31}\zeta_3 + k_{32}e_1 - \dot{\psi}_1] \\ \frac{1}{2} [x_1x_5 - (2-x_3)x_6] + \frac{1}{2r_2} [k_{41}\zeta_4 + k_{42}e_2 - \dot{\psi}_2] \end{bmatrix} \quad (10)$$

where note that from equation (6), the term  $\dot{\psi}$  can be expressed in terms of the original system states as

$$\dot{\psi}_1 = -k_{11}x_1 - k_{12}x_2 - 2k_{13}x_5x_6, \quad \dot{\psi}_2 = -k_{21}x_3 - k_{22}x_4 - k_{23}(x_6^2 - x_5^2 - 1)$$

Similar to the smooth current control, the smooth voltage control law (10) is also polynomial functions of degree two in the states, and the closed-loop system is also a quadratic nonlinear system.

### 2.3 Stability analysis

The closed-loop system of the 3-pole AMB system (4) with the smooth voltage controller given by (10) can be expressed in an extended state equation as

$$\dot{z} = A_l z + g(z) \quad (11)$$

where the state vector is defined by

$$z = [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6 \ z_7 \ z_8 \ z_9 \ z_{10}]^T = [\zeta_1 \ x_1 \ x_2 \ \zeta_2 \ x_3 \ x_4 \ \zeta_3 \ e_1 \ \zeta_4 \ e_2]^T \quad (12)$$

where  $\zeta_i$  are the new states due to the integral action. The errors in the backstepping procedure  $e_1$  and  $e_2$  are taken to be the states instead of  $x_5$  and  $x_6$  as in (4) to simplify the analysis. Note that the closed-loop state equation

(11) is a quadratic nonlinear system. The  $10 \times 10$  linearized matrix can be expressed in a block diagonal form as

$$A_l = \begin{bmatrix} A_1 & 0_{3 \times 3} & E & 0_{3 \times 2} \\ 0_{3 \times 3} & A_2 & 0_{3 \times 2} & E \\ 0_{2 \times 3} & 0_{2 \times 3} & A_3 & 0_{2 \times 2} \\ 0_{2 \times 3} & 0_{2 \times 3} & 0_{2 \times 2} & A_4 \end{bmatrix} \quad (13)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2k_{11} & -2k_{12} & -2k_{13} \end{bmatrix}; A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2k_{21} & -2k_{22} & -2k_{23} \end{bmatrix}; A_3 = \begin{bmatrix} 0 & 1 \\ -2k_{31} & -2k_{32} \end{bmatrix}; A_4 = \begin{bmatrix} 0 & 1 \\ -2k_{41} & -2k_{42} \end{bmatrix}; E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 2 \end{bmatrix}$$

The quadratic nonlinear function in (11) is

$$g(z) = [0 \quad 0 \quad 2\eta_a(z)\eta_b(z) \quad 0 \quad 0 \quad \eta_b^2(z) - \eta_a^2(z) \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (14)$$

where

$$\eta_a(z) = -k_{11}z_1 - k_{12}z_2 - k_{13}z_3 + z_8 \quad \eta_b(z) = -k_{21}z_4 - k_{22}z_5 - k_{23}z_6 + z_{10}$$

are linear functions in the states. Since the matrix  $A_i$  is block diagonal, the equilibrium at the origin will be exponentially stable if and only if all of the feedback gains  $k_{ij}$  are positive and  $2k_{12}k_{13} > k_{11}$ ;  $2k_{22}k_{23} > k_{21}$  by Routh-Hurwitz's criterion.

Although the origin of the overall system (11) is exponentially stable, it is not globally exponentially stable since there exists another equilibrium point where all of the states are 0 except that  $z_4 = 2/k_{21}$ . It can be verified easily that this equilibrium is a saddle point. Thus, it will constrain the domain of attraction of the origin. To estimate the domain of attraction, let us define a Lyapunov function

$$V(z) = z^T P z; \quad P A_i + A_i^T P = -I \quad (15)$$

Then, following the procedure outlined in (H. K. Khalil, 2011), one can obtain an estimate of domain of attraction as

$$\left\{ V(z) \leq \frac{\lambda_{\min}(P)}{4\|P\|^2\|Q\|^2} \right\} \quad (16)$$

where  $\lambda_{\min}(P)$  denotes the minimum eigenvalue of  $P$ . As the nature of Lyapunov analysis, the estimated domain of attraction is conservative. However, it provides a guideline on designing the control gains  $k_{ij}$  since both matrices  $P$  and  $Q$  depend on these gains. Note that the rotor is initially at rest on the back-up bearing. We need to choose the gains so that this initial state is in the domain of attraction.

### 3. Observer design

#### 3.1 Observability analysis

Next, the observer of the 3-pole AMB system in the voltage-controlled mode, i.e., system (4), will be investigated. Analyzing the observability is the important first step in designing an observer for a system. This is especially true for the nonlinear system like 3-pole AMB system. Unlike linear systems whose observability is independent of the system input, the observability of a nonlinear system is more involved. It may depend on the inputs. Fortunately, for the high-gain observer that will be designed here, it is only required that the system with zero inputs be observable. In this case, the observability matrix is given by (Farza, M., et al., 2004)

$$Q_o(x) = \frac{\partial}{\partial x} \begin{bmatrix} L_f^0 h(x) \\ L_f^1 h(x) \\ \vdots \\ L_f^5 h(x) \end{bmatrix} \quad (17)$$

It is required to be of full rank in the domain of interest, where the Lie derivatives are defined by

$$L_f^0 h(x) = h(x), \quad L_f^i h(x) = f(x) \partial L_f^{i-1} h(x) / \partial x, \quad \forall i \geq 1 \quad (18)$$

Obviously, determining the exact observability domain is extremely difficult, if not impossible. To simplify the analysis, we consider only the upper half of  $Q_o(x)$  denoted by  $Q(x)$ . The dimension of  $Q_o(x)$  is  $12 \times 6$  and hence

$Q(x)$  is a  $6 \times 6$  square matrix. From the previous section, it is known that the equilibrium point of the system (4) is  $x^* = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T$ . At this equilibrium, it can be shown that  $\det(Q(x^*)) = -3\sqrt{3}(r_2^2 - 0.25)^2$ . In other words, the matrix  $Q(x)$  is nonsingular in the neighborhood of  $x^*$ , implying that the system is at least locally observable. Thus, the nonlinear high-gain observer can be designed for the AMB system.

### 3.2 Nonlinear high-gain observer

To design a nonlinear high gain observer for the system (4), we need to transform it into the regular form. This can be achieved by the diffeomorphic coordinate transformation (Farza, M., et al., 2004)

$$\xi = T(x) = \begin{bmatrix} L_f^0 h(x) \\ L_f h(x) \\ L_f^2 h(x) \end{bmatrix}, \quad \xi \in \mathfrak{R}^6 \quad (19)$$

In other words, the new coordinates are taken to be the measured outputs  $y$  (coil currents) and their time-derivatives. The resulting regular form is given by

$$\dot{\xi} = A\xi + \Psi(\xi) + G_\xi(\xi)u, \quad y = C\xi \quad (20)$$

where

$$A = \begin{bmatrix} 0_{4 \times 2} & I_{4 \times 4} \\ 0_{2 \times 2} & 0_{2 \times 4} \end{bmatrix}, \quad \Psi(\xi) = \begin{bmatrix} 0_{4 \times 1} \\ \alpha(\xi)_{2 \times 1} \end{bmatrix}, \quad G_\xi(\xi) = Q(x)G|_{x=T^{-1}(\xi)}, \quad C = [I_{2 \times 2} \quad 0_{2 \times 4}]$$

The explicit expression of  $\alpha(\xi)$  and  $G_\xi(\xi)$  are complicated and are omitted here.

With the regular form (20) in hand, the high-gain observer can be constructed as

$$\dot{\hat{\xi}} = A\hat{\xi} + \Psi(\hat{\xi}) + G_\xi(\hat{\xi})u + S_\infty^{-1}C^T(y - C\hat{\xi}) \quad (21)$$

where  $S_\infty$  is a symmetric positive definite matrix satisfying

$$-\theta S_\infty - A^T S_\infty - S_\infty A + C^T C = 0 \quad (22)$$

where  $\theta$  is a positive gain to be designed. Since the matrices  $A$  and  $C$  in (20) consist of only identity matrices, the solution to equation (22) can be easily obtained as

$$S_\infty = \begin{bmatrix} \frac{1}{\theta} I_{2 \times 2} & \frac{-1}{\theta^2} I_{2 \times 2} & \frac{1}{\theta^3} I_{2 \times 2} \\ \frac{-1}{\theta^2} I_{2 \times 2} & \frac{2}{\theta^3} I_{2 \times 2} & \frac{-3}{\theta^4} I_{2 \times 2} \\ \frac{1}{\theta^3} I_{2 \times 2} & \frac{-3}{\theta^4} I_{2 \times 2} & \frac{6}{\theta^5} I_{2 \times 2} \end{bmatrix} \quad (23)$$

For the purpose of state feedback, it is preferred to have the estimate of original state  $\hat{x}$ , rather than the transformed state  $\hat{\xi}$ . Hence, the observer dynamics (21) should be transformed back to the original state to yield

$$\dot{\hat{x}} = f(\hat{x}) + Gu + Q(\hat{x})^{-1}S_\infty^{-1}C^T[y - h(\hat{x})] \quad (24)$$

The theory of high-gain observer asserts that for sufficiently large  $\theta$ , the estimation error  $\|x - \hat{x}\|$  will approach to zero asymptotically (Farza, M., et al., 2004). The gain  $\theta$  is usually taken by trial and error. Fortunately, we have the nondimensional model and hence  $\theta$  can be taken with order one. A reasonable choice will be  $1 < \theta < 5$ .

## 4. Simulation and experimental results

### 4.1 Simulation results

Numerical simulations are performed for the smooth voltage controller and observer of the 3-pole AMB system. It will be the same system studied in the next section for the experimental validation. The nominal values of the system

parameters are given by

$$m = 0.6435 \text{ kg}, l_0 = 0.95 \times 10^{-4} \text{ m}, \mu = 4\pi \times 10^{-7} \text{ H/m}, N = 300, R_1 = 1.3 \Omega, R_2 = 2.6 \Omega$$

The nondimensional control parameters for the proposed smooth voltage controller are given by

$$k_{11} = 0.3, k_{12} = 1.85, k_{13} = 1.85, k_{31} = 0.1, k_{32} = 3, k_{21} = 0.15, k_{22} = 1.675, k_{23} = 1.8, k_{41} = 0.1, k_{42} = 3$$

and the observer gain of the nonlinear high gain observer is taken to be  $\theta = 1.5$ . The initial conditions for the system state  $x(0)$  and  $\hat{x}(0)$  are taken to be  $x(0) = [0 \ 0 \ -5 \times 10^{-4} \text{ m} \ 0 \ 0 \ 0]^T, \hat{x}(0) = [0 \ 0 \ -5 \times 10^{-4} \text{ m} \ 0 \ 0 \ 400 \text{ A/m}]^T$ . The position  $-0.5 \text{ mm}$  is where the backup bearing is placed. In what follows, there is a dotted circle of radius  $0.5 \text{ mm}$  representing the boundary formed by the backup bearings. The initial condition is to simulate the situation that the rotor is at rest on the backup bearing initially. This is exactly the case that will be repeated in the experiments. Fig. 2 shows the estimation error in the rotor displacements, indicating that the estimations possess good accuracy.

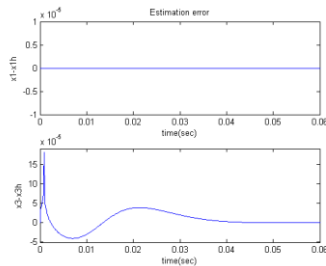


Fig. 2 Estimation error (unit: m): numerical simulation

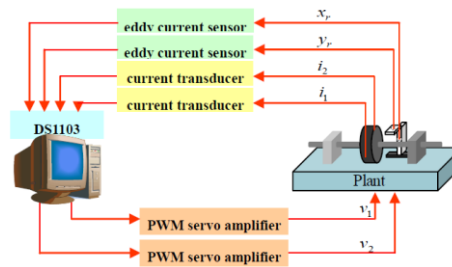
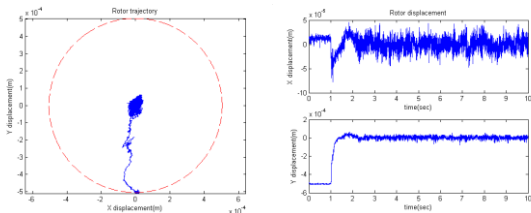


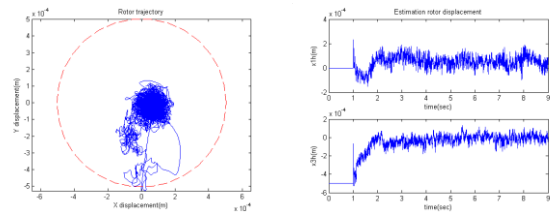
Fig. 3 Experimental set-up

## 4.2 Experimental results

The experimental set-up of the 3-pole AMB system is shown in Fig. 3. It is the same as that presented in [10]. Please refer to (Chen, Shyh-Leh, et al., 2010) for more details. For the voltage-controlled experiments, one needs current sensors and voltage power amplifiers in addition to the eddy current displacement sensors. The current sensors used here is the Hall sensors (LEM, LTS 6-NP) that can measure primary current up to  $19.2 \text{ A}$ , with  $0.7\%$  of accuracy. Finally, the same control parameters and observer gain in the numerical simulations are applied here. The rotor trajectory with the proposed smooth voltage controller is shown in Fig. 4(a), with the corresponding displacement response shown in Fig. 4(b). As one can see from the figures, the rotor can reach the steady state within  $2 \text{ sec}$ , with the steady state error of  $23 \mu\text{m}$  (root of mean square, RMS). These results verify the effectiveness of the proposed smooth voltage controller. Then, the input voltages and output coil currents are utilized for the observer to estimate the rotor displacements. The estimated rotor trajectory and displacement response are shown in Fig. 5. As one can see, the estimated results catch the major trend of the actual results. Larger discrepancy exists at the transient period. It is due to the sensor noise existing in the current transducers. The steady state RMS estimation error for X displacement is  $68 \mu\text{m}$ , and that for the Y displacement is  $45 \mu\text{m}$ , indicating the effectiveness of the proposed nonlinear observer.



(a) rotor trajectory (b) rotor displacements  
Fig. 4 Levitation by smooth voltage controller



(a) rotor trajectory (b) rotor displacements  
Fig. 5 Estimation results by nonlinear observer

## 5. Conclusions

A simple and smooth voltage controller and a nonlinear high-gain observer have been proposed for a 3-pole

voltage-controlled AMB system in this study. The smooth voltage controller is designed based on the previously proposed smooth current controller incorporated with the backstepping technique. The purpose of the observer is to use the information of input voltages and output coil currents for the estimation of the rotor displacements. Such estimate can subsequently replace the position sensors and implement the sensorless control the AMB system. First, it was shown that the strong nonlinear 3-pole AMB system is observable under the zero input condition, which guarantees the existence of a high-gain observer. Then, the system was transformed into a regular for the design of observer. Finally, a nonlinear observer was designed following the theory of high-gain observer. Both numerical and experimental results verify the effectiveness of the proposed smooth voltage controller and the observer.

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