

# Theoretical basics and closed loop control design of stray-flux-based measurement systems for magnetic bearings

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## Abstract

In this paper the theoretical basics of a measurement system are given in detail which uses magnetic stray fluxes for sensing the rotor position in magnetic bearings. Reluctance models for the most spread types of magnetic bearings are evolved to create the mathematic equations in the form of matrices. Based on this a description of the dependence of the magnetic stray flux that occurs and the air gap length respectively the rotor position by analytical and numerical calculations can be done. It will be shown that this behavior of stray flux depends on where it is measured. Additionally the influence of the coil current and its transient behavior on this measurement system will be discussed. Furthermore the application in an eight pole heteropolar magnetic bearing will be presented. Therefore a signal flow diagram is given that clarifies in which way the measured HALL-signals should be processed to generate a position signal as input for a PID-controller that provides a stable levitated rotor in the bearing center. Last but not least the courses of the stray flux based position signals are presented compared with the signals of an auxiliary capacitive measurement system and approaches are proposed for a further improvement of this alternative measurement system using typically neglected magnetic stray fluxes.

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**Keywords** : Magnetic Bearing, Position Sensing, Measurement System, Stray Flux, Reluctance Model, Current Compensation, Alternative Measurement System

## 1. Introduction

Magnetic bearings are naturally unstable systems. To ensure a proper levitation of the rotor in the bearing its position must be known at every time to tune the coil currents in a way that the rotor keeps in the aspired position (Schweitzer et al. 1993). The therefore needed measurement systems, typically eddy current or capacitive sensors, cause additional costs. To reduce the costs for magnetic levitated rotors in a way of avoiding conventional measurement systems sensorless bearing are in discussion since some time (Gruber and Stöckler. 2015) and (Mizuno et al. 1996). An alternative approach is the application of flux sensors which are inexpensive and can be mounted into the bearing. That brings the additional advantage of avoiding dislocation effects when the flux sensors are placed in the bearing plane. But the measurement of the rotor position using flux sensors is a rarely used method yet. The integration of flux sensors in to the air gap is associated with some problems. Most gaps are too small for using standard field sensors. A good possibility is the integration of very thin magnetic sensitive elements (Bahr et al. 2013). Another option is given by application of additional magnetic circuits which contain flux sensors like HALL-elements (Schramm and Hofmann 2007). A new and alternative opportunity is a measurement system which is based on the often neglected stray flux which also depends on the rotor position presented and discussed in the papers of Rudolph and Werner (2013 and 2014). Low cost standard Hall sensors are integrated in the space between adjacent pairs of poles. The magnetic stray flux is also a function of the air

gap length and thus of the rotor position. In previous considerations it has been shown that the direction of the flux is an important fact for positioning flux sensors with only one magnetic sensitive direction. But there is a further correlation between the stray flux and the coil current which has to be considered when the position of the rotor is derived from the stray flux.

## 2. Basics

### 2.1 Simplified model of stray flux paths in magnetic bearings

To use magnetic stray flux as input value for a closed loop control of the rotor position there is a need to have an accurate idea of the magnetic behaviors in the magnetic bearing. Therefore it is useful to develop a model of the magnetic bearing with respect to the most neglected stray fluxes. Also dependencies of current and rotor position, which are nonlinear as expected, should be part of this model. In a first step a simplified analytical model should be the starting point for further considerations. There are two types of interesting stray fluxes which can be used to identify the rotor position. Intrapolar stray fluxes between contrary polarized magnetic poles of one bearing axis represented by yellow colored magnetic resistors in Fig. 1 and interpolar stray fluxes between concordant polarized magnetic poles of adjacent bearing axes represented by red colored magnetic resistors in Fig. 1.

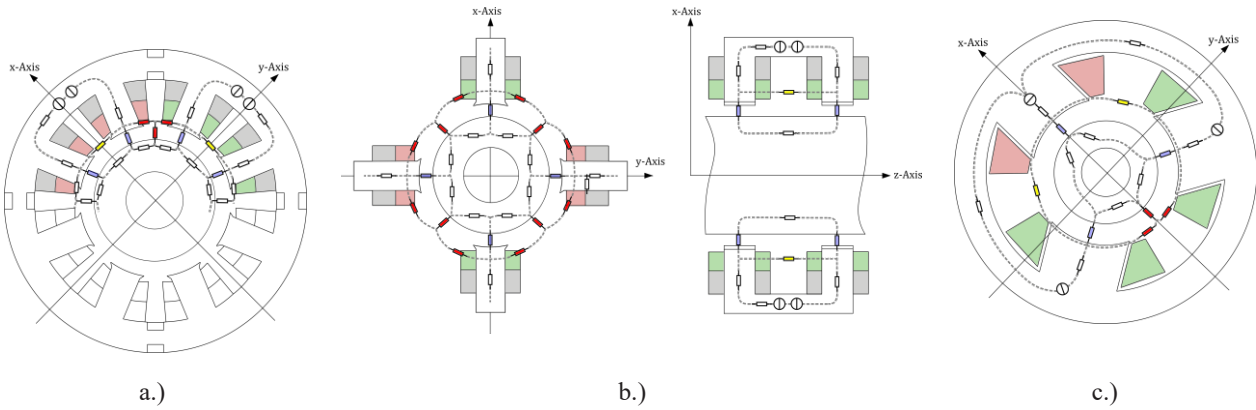


Fig. 1 **a.)** Reluctance model of a heteropolar magnetic bearing with intrapolar and interpolar stray flux (two of four magnetic circuits are shown). **b.)** Reluctance model of a homopolar magnetic bearing with interpolar stray flux in the bearing plane and interpolar stray flux in the orthogonal x-z-plane. **c.)** Reluctance model of a unipolar magnetic bearing (arrangement of magnetic poles is not constant).

Due to that the polarization of adjacent magnetic poles of x- and y-axis normally are the same the interpolar stray flux superpose in this region so that they share the same area indeed the according magnetic circuits can be considered independent of each other. The direction of the magnetic flux in this case is predominantly radial. The intrapolar stray flux has a tangential direction (Rudolph and Werner, 2013). With a given geometry and the assumption that all paths of magnetic fluxes are well known (length, cross section, e.g.) one magnetic circuit of hetro- and homopolar bearing can be described by the model in Fig. 3. Based on this the following equation can be obtained,

$$\Theta_{(n)} = R_{(n)} \cdot \Phi_{(n)} \quad (1)$$

where  $n$  is the number of the magnetic circuit. For example a heteropolar magnetic bearing with eight poles has a pair of poles per each positive and negative axis. In such a case four pairs of poles have to be considered ( $n = 4$ ). The reluctance matrix  $R$  and the vectors  $\Theta$  and  $\Phi$  arise as following.

$$R_{(n)} = \left\{ \begin{array}{cccc} R_{mFe1} + R_{m\sigma1} & R_{m\sigma1} & 0 & 0 \\ -R_{m\sigma1} & 2 \cdot R_{m\delta} & -R_{m\delta} & -R_{m\delta} \\ 0 & -R_{m\delta} & R_{m\delta} + R_{m\sigma21} + R_{m\sigma22} + R_{mFe22} & 0 \\ 0 & -R_{m\delta} & 0 & R_{m\delta} + R_{m\sigma21} + R_{m\sigma22} + R_{mFe22} \end{array} \right\} \quad (2)$$

$$\Phi_{(n)} = \left\{ \begin{array}{c} \Phi_{M1} \\ \Phi_{M2} \\ \Phi_{M3} \\ \Phi_{M4} \end{array} \right\} \text{ with } \begin{array}{l} \Phi_{M1} = -\Phi_{Fe1} \\ \Phi_{M2} = -\Phi_{\sigma2} - \Phi_{\delta} \\ \Phi_{M3} = -\Phi_{\sigma2} \\ \Phi_{M4} = -\Phi_{\sigma2} \end{array} \quad (3)$$

$$\Theta_{(n)} = \left\{ \begin{array}{c} \Theta \\ 0 \\ 0 \\ 0 \end{array} \right\} \text{ with } \Theta = \Theta_1 + \Theta_2 \quad (4)$$

The unipolar magnetic bearing consists of three magnetic poles whose polarization depends on the direction of the force which is needed to keep on the rotor levitated. The reluctance model shown in Fig. 2 is unraveled for better understanding. In practice the left and right ends have to be considered as connected. The lever arrangement stands for the correlation of the three air gaps to each other in the unipolar magnetic bearing. Based on this the equations for the unipolar bearing are given by:

$$\Theta_{uni} = R_{uni} \cdot \Phi_{uni} \quad (5)$$

$$R_{uni} = \left\{ \begin{array}{cccccc} R_{mFe1} + 2 \cdot R_{mFe2} + R_{m\sigma1} & -R_{mFe2} & 0 & -R_{m\sigma1} & 0 & 0 \\ -R_{mFe2} & R_{mFe1} + 2 \cdot R_{mFe2} + R_{m\sigma1} & 0 & 0 & -R_{m\sigma1} & 0 \\ 0 & 0 & R_{m\delta1} + R_{m\sigma21} + R_{m\sigma22} + R_{m\delta1} & -R_{m\delta1} & 0 & 0 \\ -R_{m\sigma1} & 0 & -R_{m\delta1} & R_{m\delta1} + R_{m\delta2} + R_{m\sigma1} + R_{mFe4} & -R_{m\delta2} & 0 \\ 0 & -R_{m\sigma1} & 0 & -R_{m\delta2} & R_{m\delta1} + R_{m\delta2} + R_{m\sigma1} + R_{mFe5} & -R_{m\delta3} \\ 0 & 0 & 0 & 0 & -R_{m\delta3} & R_{m\delta3} + R_{m\sigma21} + R_{m\sigma22} + R_{mFe6} \end{array} \right\} \quad (6)$$

$$\Phi_{uni} = \left\{ \begin{array}{c} -\Phi_{Fe1} \\ \Phi_{Fe3} \\ -\Phi_{\sigma3} \\ -\Phi_{\delta1} + \Phi_{\sigma3} \\ \Phi_{\delta3} + \Phi_{\sigma4} \\ \Phi_{\sigma4} \end{array} \right\} \quad (7)$$

$$\Theta_{uni} = \left\{ \begin{array}{c} \Theta \\ 0 \\ 0 \\ 0 \end{array} \right\} \text{ with } \Theta = \Theta_1 + \Theta_2 \quad (8)$$

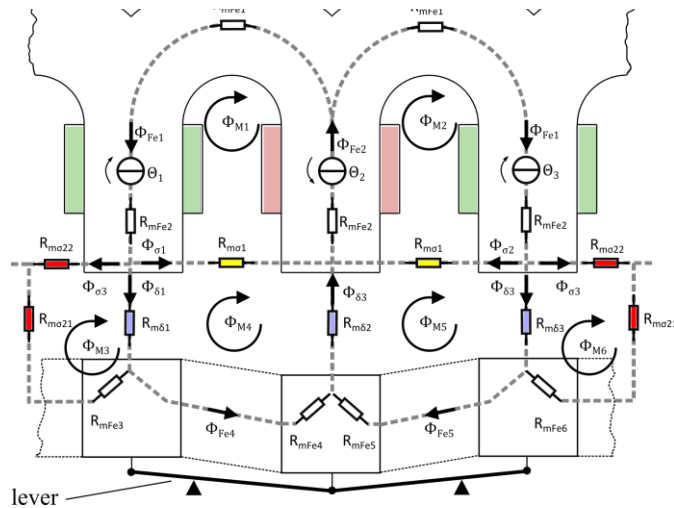


Fig. 2 Entire reluctance model of a unipolar magnetic bearing with magnetic resistors, sources, fluxes and imaginary lever arrangement.

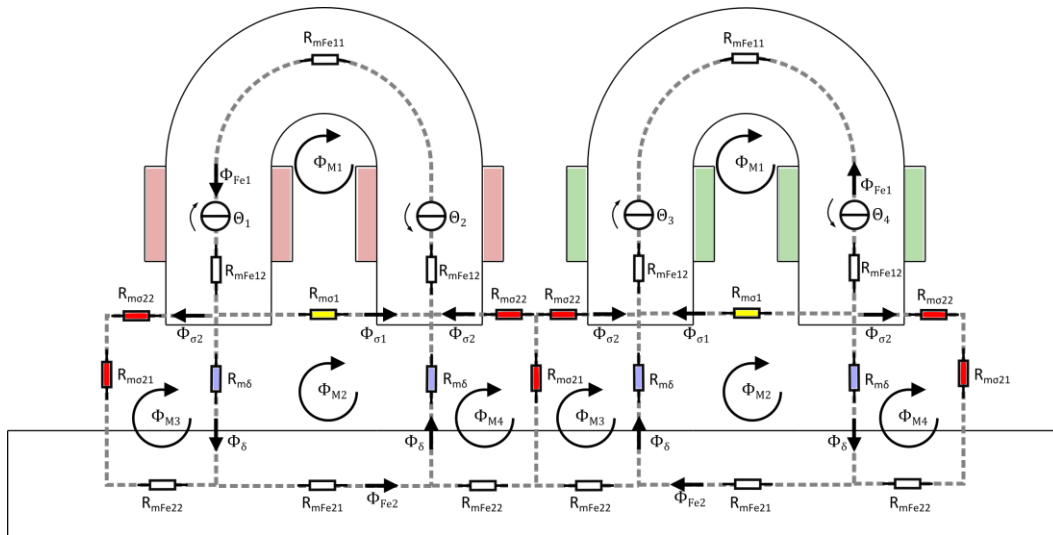


Fig. 3 Entire magnetic model of two magnetic circuit of hetero- or homopolar magnetic bearing including all relevant magnetic resistors, sources and magnetic fluxes.

## 2.2 Behavior of magnetic stray fluxes depending on air gap length

To get an overview a separate consideration of inter- and intrapolar stray flux is useful. There are two states of the system which leads to a simplification of the equations. Assuming the values of the magnetic resistors and the ampere turns  $\theta$  are known an analytical solution is possible. In the first case the air gap is infinitesimal small. The magnetic circuit can be considered as closed iron. The magnetic resistor of the intrapolar stray flux  $R_{m\sigma1}$  is parallel connected to  $R_{mFe21}$ . Thus the minimal value of  $\Phi_{\sigma1}$  is given. Interpolar stray fluxes does not occur because there is no magnetic potential difference between yoke and back iron in this area. If the air gap length becomes infinite the magnetic resistors of the yoke  $R_{mFe21}$  and  $R_{mFe22}$  as well as the air gap resistor  $R_{m\delta}$  are no longer needed. The magnetic circuit now is closed via  $R_{m\sigma1}$ . It can be shown that the intrapolar stray flux in that case has its biggest value. Interpolar stray flux will as well as in the first considered case not occur. In practice the value of the intrapolar stray flux will increase like shown in Fig. 4 with the air gap length from its smallest to its biggest value given by the mechanical, magnetic and electrical conditions of the bearing. However the interpolar stray flux starts with nearly zero when the air gap is very small. It increases to its maximum and decreases to zero again (Fig. 4). This maximum and the point of inflection depending on the bearing properties. If the interpolar stray fluxes are used for position sensing this fact has to be considered carefully

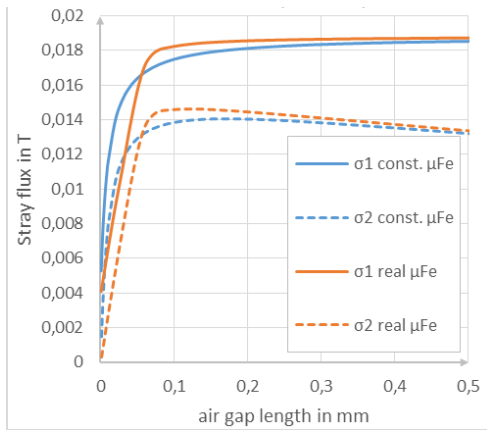


Fig. 4 Inter- and intrapolar stray flux ( $\sigma_2$ ,  $\sigma_1$ ) vs. air gap length with constant  $\mu_{Fe}$  and saturation at 100A ampere turns.

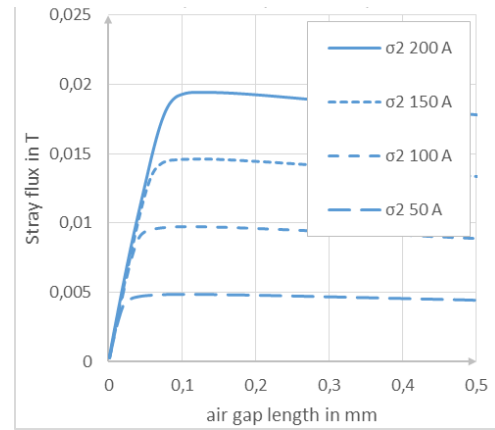


Fig. 5 Interpolar stray flux  $\sigma_2$  vs. air gap length at varying ampere turns ( $\mu_{Fe}$  not constant).

to ensure a biunique characteristic of stray flux and air gap length. In addition to the dependence of stray flux and air gap length the ampere turns have a significant influence shown in Fig. 5. With an increasing value of the coil current the magnetic fluxes in the bearing increases too. This leads to more distinct stray fluxes. To use these as indicator of the rotor position it is necessary to measure the coil currents to achieve a current compensation of the position signal.

### 2.3 FEM analysis of the simplified reluctance model

In a next step the analytically found basics with several assumed simplifications such as predefined paths of magnetic flux with constant and known cross sections should be verified by numerical calculations with realistic allocation of

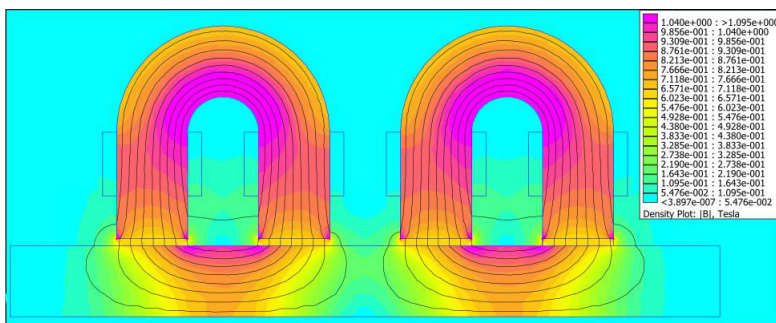


Fig. 6 Field pattern of the simplified model at 50A ampere turns and an air gap length of one millimeter.

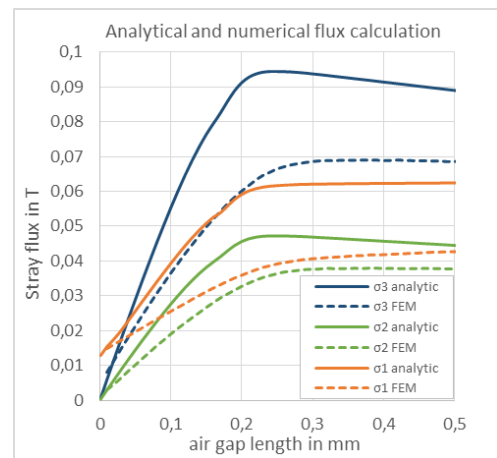


Fig. 7 Analytical and numerical flux calculations depending on the air gap length at 500A ampere turns with respect to a nonlinear behavior of iron in both calculations.

magnetic fluxes. With the help of a two dimensional model with the same properties like the simplified model shown in Fig. 6 the distribution of stray flux depending on the air gap length has been calculated. It becomes clear that the resulting curves displayed in Fig. 7 arise in principle in the same manner as the analytical curves. Of course there are differences in maximum values which are lower and rising is less steep. That is caused by the now realistic distribution of stray fluxes. The interpolar stray flux  $\sigma_3$  is the result of the superposition of the two stray fluxes  $\sigma_2$  between the two magnetic circuits. Based on the considerations of the simplified model  $\sigma_3$  is twice as large as  $\sigma_2$  as expected under the premise that the ampere turns in both magnetic circuit are equal.

## 2.4 Transient behavior of the stray flux

So far quasi static behavior of stray flux has been discussed. For usage as a measurement system in magnetic bearing an investigation in dynamic properties is necessary. In case of a very slow movement of the rotor in the bearing the changing of the magnetic main flux is very low. Due to that voltage induction in the bearing coils is negligible. However caused by shock or other fast movements of the rotor the induction law

$$-U_{\text{ind}} = \frac{d\Phi}{dt} \quad (9)$$

becomes more important. Induced voltage in the bearing coils will cause an additional current which leads to a smaller or bigger magnetic flux. Since a current measurement is needed anyway for signal compensation this effect does not occur. As opposed to that the delay of the magnetic flux in relation of steeply rising currents has a significant impact. This leads to a current over compensation which causes an increased apparent rotor deviation. In the considerations of Rudolph and Werner, 2014 theoretical background is discussed in detail and possibilities to solve this problem are presented.

## 3. Closed loop control and linearization

It is clear to see that a measurement system using stray fluxes is not linear over the whole range of air gap length. A feasible solution is a differential arrangement of flux sensors such as HALL-sensors in the bearing. Four sensors are applied in the bearing. One in each positive and one in each negative axis. In a first step the stray flux air gap

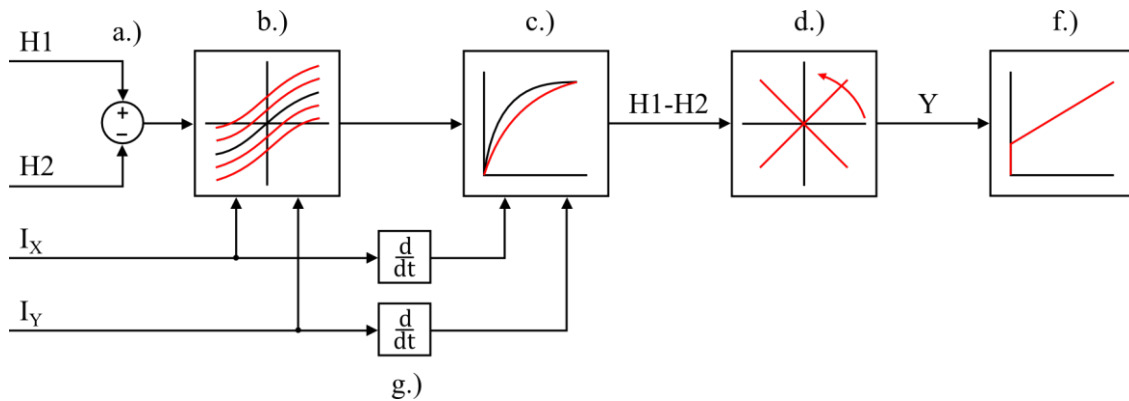


Fig. 8 Schematic of the stray flux measurement system for position sensing in magnetic bearings in one axis. **a.)** First linearization by subtraction opposing signals. **b.)** Second linearization and static current compensation by look-up-table. **c.)** Dynamic current compensation (field delay). **d.)** Rotation of sensor coordinate system. **f.)** PID-controller **g.)** Derivation of control currents.

characteristic is linearized by subtracting the opposing sensor signals. If like in this case interpolary stray flux is used for position sensing the measuring coordinate system is turned 45° adverse the bearing coordinate system and the x- and y-control currents always have to be used for current compensation at the same time. This approach implies some advantages and disadvantages which are described more detailed in the paper of Rudolph and Werner (2013). Fig. 9 displays the linearized position signal of an eight pole heteropolar magnetic bearing where the stray flux measurement system is applied. A further linearization should be done by look-up tables. Based on what has been discussed until now the stray flux measurement is ready for application in a magnetic bearing.

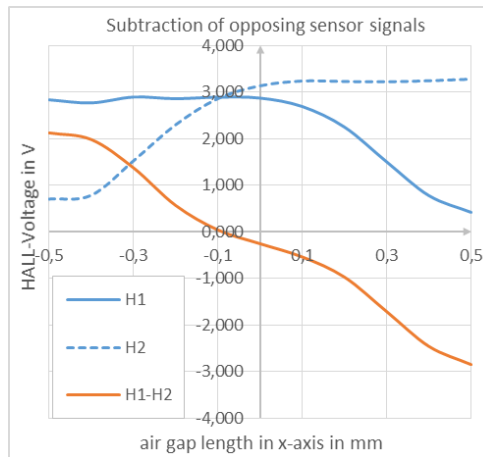


Fig. 9 Linearization of the stray flux air gap characteristic by subtraction of opposing sensor signals. Sensors are applied in an eight pole heteropolar magnetic bearing between the bearing axes ( $\sigma_3$  is measured).

With this presented approach shown in Fig. 8 as a very simplified example for one bearing axis, an eight pole magnetic bearing has been equipped with such a measurement system for sensing the rotor position. The major difficulties have been caused by very noisy measurement signals. The HALL-sensors are mounted in direct vicinity to the bearing coils. This naturally induces extreme disturbance. A further inappropriate factor is the current measurement. This signals are also disturbed. In consequence that they serve as input signals for the current compensation more noisy position signals are the result. In practice it is a good approach to choose a moderate and well balanced filtering, for that the signals are smooth enough while still ensuring a sufficiently dynamic behavior. In a first test the rotor could be stably held in levitation. The current compensation, coordinate rotation and the closed loop control is executed by a dSpace® system. The influence of field delay is not applied yet. This causes a low frequent oscillation of the rotor position in the bearing center. Nevertheless the system was stable even when small strokes occur.

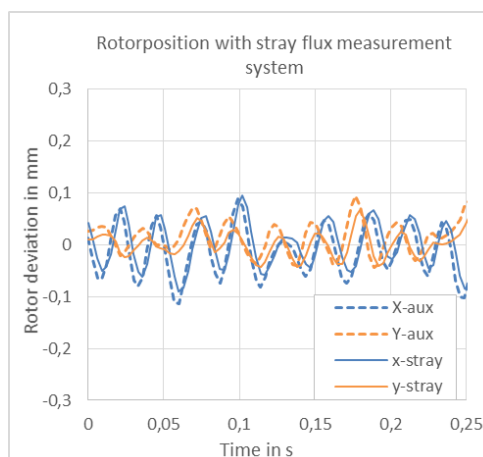


Fig. 10 Rotor position measured by auxiliary capacitive sensors and the stray flux measurement system while rotor is levitated and the closed loop controller works with stray flux sensors.

## 4. Conclusion and outlook

In this paper the theoretical basics of magnetic bearings have been presented. Based on reluctance models for the most common bearing types a simplified model is derived. With the help of analytical and numerical calculations the dependence of different forms of magnetic stray flux on the air gap length is analyzed. The example of an eight pole magnetic bearing is used to describe the application of this measurement system with respect on special issues and a number of difficulties.

In a next and last step the compensation of the influence of field delay on the stray flux based measurement system has to be embedded in the controlling unit to suppress the low frequent oscillation of the rotor in the bearing. Furthermore it should be considered whether alternative closed loop controller concepts and advanced filtering e.g. the KALMAN-filter could be used to handle the noisy signals of the flux sensors and the current measurement.

## References

- G. Schweitzer, A. Traxler, H. Bleuler, *Magnetic bearings-Basics, Properties und Application contactless electromagnetic bearings Lager* (1993), Springer ,(in German)
- F. Bahr, M. Melzer, D. Karnaushenko, et al., Flux Based Control and Monitoring of Active Magnetic Bearings Using Ultra-Thin and Flexible Bismuth Hall Sensors, 9. Workshop Magnetlagertechnik Zittau/Chemnitz, p. 96, Oct. 2013.(in German)
- M. Schramm, W. Hofmann, Novel magnetic displacement sensor for mechatronical systems, in The 33rd Annual Conference of the IEEE Industrial Electronics Society, IEEE, Nov. 2007
- J. Rudolph, R. Werner, and I. Maximow, Measurement of the rotor position of an active magnetic bearing using interpolar stray flux, in 1. Brazilian Workshop on Magnetic Bearings, 2013
- J. Rudolph, R. Werner, Effect of Eddy Currents on the Stray Flux based Measurement System for Magnetic Bearing, ISMB14, Linz, 2014
- N. Skricka, Development of sensorless active magnetic bearing, VDI progress report 8 No. 1027, Darmstadt, 2004
- Mizuno, T., Bleuler, H., Tanaka, H., Hashimoto, H., Harashima, F. and Ueyama, H. (1996), Industrial application of position sensorless active magnetic bearings. *Elect. Eng. Jpn.*, 117: 124–133. doi: 10.1002/ej.4391170511
- W. Gruber, M. Stöckler, On the self-sensing technique based on the interlink voltage of two serially connected phase coils, 2015 IEEE 11th International Conference on Power Electronics and Drive Systems, p. 645-651, ISSN: 2164-5256, Sydney, June 2015