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Fractional Order Control of Rotor Suspension by Active Magnetic Bearings

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Abstract

This paper demonstrates the design, analysis, and implementation of fractional order PID control (FOPID) for the control of rotor vibration in a centrifugal compressor test rig by active magnetic bearings (AMBs). This is the first time that the FOPID controller is designed for and implemented on an AMB system. In order to achieve the desired design specifications, the controller parameters are tuned by using evolutionary algorithms. Furthermore, a comparison of the performance and efficiency of the FOPID, PID, and an optimal controller is given based on simulation and experimental results.

Key words : Fractional Order Control, AMB Control, Rotor Suspension, PID Control, Centrifugal Compressor, Optimal Control

1. Introduction

Active magnetic bearings (AMBs) employ electromagnets to support machine components without mechanical contact. The magnetic forces are adjusted by feedback controllers to suspend the machine components within the magnetic field and to control the system dynamics during machine operation. Magnetic bearings offer many advantages for various applications. High-speed machines can operate smoothly because there is no friction during rotation. The maintenance cost and mechanical wear are low due to non-contact operation. A real time control of the AMB system helps to keep the rotor close to the center and to reduce vibrations during the operation.

However, controller design for AMB systems is a challenging task because of the nonlinear nature of the plant dynamics, the very small degree of natural damping, the strict positioning specifications often required by the application, and the unstable open loop system dynamics. In most cases, PID is the chosen controller due to its simplicity and intuitiveness in the tuning of the controller parameters. However, sometimes a conventional PID controller cannot meet the industry performance standards for AMB systems, such as the American Petroleum Institute (API) and the International Organization for Standardization (ISO). In these cases, more complex controllers, such as LQG, H_∞, and μ-synthesis, are used to meet the desired specifications. The tradeoff between the simplicity of the controller structure and the good performance is always one of the goals that control engineers must achieve.

Recently, fractional order calculus theory, which is the generalized version of integer order calculus, has been adopted for many applications due to its accuracy in modelling the dynamics of systems and its simplicity in model structure in representing high order processes. The fractional order control is one of the fields that many researchers and practicing engineers are interested in due to the fact that the response of a system with fractional order controller is not restricted to a sum of exponential functions, therefore a wide range of responses neglected by integer order calculus would be approached. One of the most popular fractional order controllers is the generalized PID controller, which is also called fractional order PID (FOPID) controller. FOPID has two extra parameters, the non-integer order of the integral and derivative terms, in comparison with a conventional integer order PID (IOPID) controller. It has been revealed that FOPID control can improve the performance and robustness over conventional PID control in many applications while keeping the control structure simple. This suggests that FOPID control has a good potential to reduce the gap between the simplicity of the controller structure and high closed-loop performance aspects as mentioned above.

In this paper, fractional order PID control for AMB systems is proposed. The feasibility of FOPID for AMB systems is investigated in the context of the control of rotor suspension by magnetic bearings.Tuning methods are developed based on the evolutionary algorithms for searching the optimal values of the controller parameters. The resulting FOPID controllers are then tested and compared with an integer order PID controller, as well as with an optimal controller. The comparison is based on various stability performance and robustness specifications, as well as the controller dimension as implemented. Lastly, to validate the proposed method, experimental testing is carried out on a single-stage centrifugal compressor test rig equipped with magnetic bearings, which was built and commissioned in the Rotating Machinery and Control Laboratory (ROMAC) at the University of Virginia.

The remainder of the paper is organized as follow. First, the fundamental of the fractional order calculus and the fractional order PID control is explained. Then, an overview of the centrifugal compressor test rig to be used for our experimental study is given. After that the process of the fractional order control design of rotor suspension is described. It is then followed by the validation of the proposed method by the simulation and experimental results. Finally, the paper ends with a conclusion of the study.

2. Fractional Order Calculus and Control

2.1. Fractional Order Calculus Definition and Its Applications

Even though it does not sound familiar to many, fractional order calculus was developed about 300 years ago, around the same time when integer order calculus was invented. Fractional calculus is a generalization of integer order differentiation and integration to non-integer orders. There are two main definitions of fractional calculus that have been widely used, namely Riemann-Liouville (RL) and Caputo definitions (Das, 2011). These two definitions are derived from the concept of the Cauchy n^{th} integration of function $f(t)$ (Folland, 2002). When n is a positive integer number, n^{th} integration of function $f(t)$ is given by

$$f^{(-n)}(t) = \frac{1}{(n-1)!}\int_0^t (t-\tau)^{n-1} f(\tau)d\tau, \tag{1}$$

and if n is any positive real number, the formula is generalized to

$$f^{(-n)}(t) = \frac{1}{\Gamma(n)}\int_0^t (t-\tau)^{n-1} f(\tau)d\tau, \tag{2}$$

where the gamma function $\Gamma(n) = \int_0^\infty e^{-t}t^{n-1}dt$ is the generalization of the factorial function (Oldham and Spanier, 1974). By combining the concept of integer order derivative and the Cauchy n^{th} integration, the α^{th} order derivative of a function $f(t)$ with respect to t defined by Riemann-Liouville (RL), also called the Left Hand Definition (LHD), is given as

$${}_a\mathcal{D}_t^\alpha f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau\right], \quad (m-1) \le \alpha \le m, \tag{3}$$

where m is an integer. On the other hand, the α^{th} order derivative of a function $f(t)$ defined by Caputo, also called the Right Hand Definition (RHD), is given as follows (Das, 2011),

$${}_a\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)}\int_a^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau, \quad (m-1) \le \alpha \le m, \text{where } m \text{ is an integer} \tag{4}$$

Most natural phenomena can be modeled and explained more accurately by a fractional order differential equation. Consequently, many researchers and practicing engineers have been attempting to incorporate fractional calculus into their applications.

2.2. Fractional Order PID Control

Over the years, engineers and industrial practitioners have been aspired to substitute the conventional PID controller with a more powerful one. However, the PID controller remains the most popular due to its simplicity and the clear physical interpretation of its controller parameters. Recently, there has been an extension of the conventional PID controller by substituting the orders of the derivative and integral components to any arbitrary real numbers instead of fixing those orders to one. The fractional order PID (FOPID) controller was first introduced by Podlubny in 1998 (Podlubny, 1998). The transfer function of an FOPID controller takes the form of

$$C_{\text{FOPID}}(s) = K_{\text{P}} + \frac{K_{\text{I}}}{s^\lambda} + K_{\text{D}}s^\mu, \tag{5}$$

where λ is the order of the integral, μ is the order of the derivative, and K_{P}, K_{I}, and K_{D} are the controller gains similar to the conventional PID controller.

Similar to a conventional PID controller, an FOPID controller behaves like a bandstop filter that passes most frequencies unaltered, but attenuates those in a specific range to very low levels. Generally, the integral part in the conventional PID control helps to eliminate the steady state error due to its infinite gain at zero frequency but it has 90 degree phase lag. On the other hand, the derivative part provides 90 degree phase lead, but has a large gain at high frequencies, which is susceptible to noise. By tuning the derivative and the integral orders in an FOPID controller, one can adjust the sharpness of the filter independently as illustrated in Fig. 1.

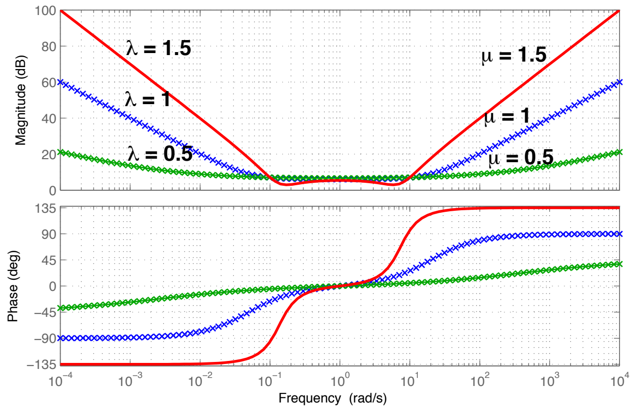

Fig. 1 Frequency domain effects of fractional orders λ and μ

2.3. Fractional Order PID Tuning Methods

With the additional flexibility of the fractional derivative and integral orders of the FOPID introduced in the Section 2.2, controller parameter tuning is another important factor to pay attention to. There have been tremendous effort spent on tuning methods for the FOPID controller in the past years. FOPID tuning methods can be categorized into three major approaches, analytical, rule-based, and numerical tuning methods (Valerio and da Costa, 2010). The analytical and rule-based methods are widely used in many studies. These methods mainly concern the phase margin, gain margin, gain crossover frequency, and dominant poles. The studies of analytical tuning for FOPID can be found in (Caponetto et al., 2004, Zhao et al., 2005, Maione and Lino, 2007). The available rule-based methods can also be extended to the auto-tuning method by incorporating an additional test such as relay feedback test into the loop (Vinagre et al., 2007, Monje et al., 2008). One of the drawbacks of these two methods is the assumption that a plant is of minimum phase and open loop stable. Because of this limitation, this study will focus only on numerical tuning methods due to the fact that AMB systems are open loop unstable. The aim of numerical tuning methods is to optimize the specified objective functions with respect to the five adjustable parameters, K_{P}, K_{I}, K_{D}, λ, and μ.

Many optimization algorithms for control design have been studied to examine their effectiveness for different purposes. Evolutionary Algorithms (EAs) are one of the most efficient and robust optimization methods. EAs are influenced by the principles of natural selection proposed by Charles Darwin. The idea of "the survival of the fittest" is the key concept behind all evolutionary algorithms (Fleming and Purshouse, 2002). These algorithms are also able to cope with systems that are highly nonlinear, discontinuous, and time-varying. The reason that EAs have become a popular alternative optimization algorithm is that the *evolution* process enhances the global optimum search whereas conventional optimization algorithms are based on a local gradient search. One of the most popular EAs used in FOPID control design is the Genetic Algorithm (GA) (see Chang and Lee, 2008, Chang and Chen, 2009). Other popular evolutionary algorithms used in FOPID control design include particle swarm optimization (PSO) (Bingul and Karahan, 2011, Cao and Cao, 2006), differential evolution (DE) (Biswas et al., 2009, Chang, 2009), and various modifications of the mentioned methods. In this study, GA, PSO, and DE algorithms are used to achieve optimal values of the controller parameters. It is also convenient that all of the mentioned evolutionary algorithms are easily implemented in software. Because of the limited space in this paper, we suggest readers to consult with the references mentioned above for the details of optimization processes and steps.

2.4. Implementation of Fractional Order PID Controller

A feasible way to implement a fractional order operator is to use a finite dimensional integer order transfer function, which requires approximation. The fractional order operator can be approximated in both continuous time and discrete time. This work will focus on the continuous time approximation because the result of approximation in the continuous time is more suitable for further analysis.

One of the well-known approximation methods was proposed by Oustaloup (Oustaloup, 1991). Oustaloup's approximation of fractional order α in the specified frequency range $[\omega_l, \omega_h]$ is given as

$$s^{\alpha} = K \prod_{k=1}^{N} \frac{s + \omega_k'}{s + \omega_k}, \qquad 0 < \alpha < 1, \tag{6}$$

where N is the number of poles and zeros which is chosen beforehand and a good approximation strongly depends on this number. Then, the zeros, poles, and gain are determined as

$$\omega_k' = \omega_l \omega_u^{(2k-1-\alpha)/N}, \quad \omega_k = \omega_l \omega_u^{(2k-1+\alpha)/N}, \quad \omega_u = \sqrt{\omega_h/\omega_l}, \quad K = \omega_h^{\alpha}.$$

For the case $\alpha < 0$, the right hand side of equation (6) will be inverted. But if $|\alpha| > 1$, the approximation becomes unsatisfactory. Accordingly, it is usual to split the fractional power of s into the following form

$$s^{\alpha} = s^{n} s^{\gamma},$$

where n is an integer number, $\alpha = n + \gamma$, and $\gamma \in (0,1)$. In this manner, only the s^{γ} term needs to be approximated.

It has been proven that this approximation method is accurate enough for the implementation purpose (Oustaloup et al., 2000). Modified versions of Oustaloup's approximation can be found in (Xue et al., 2006).

3. System Description and Modelling

3.1. Overview of the Test Rig

For the purpose of investigating the capability of AMBs in high-speed compressor applications, the single-stage centrifugal compressor equipped with AMBs was built and commissioned in the Rotating Machinery and Control Laboratory (ROMAC) at the University of Virginia, as shown in Figs. 2 and 3. Specifically, this test rig is used as a platform to demonstrate flow instabilities caused by surge in a centrifugal compressor. The rotor is levitated by two radial AMBs for smooth rotation without mechanical contact. The rotor is supported axially by the thrust AMB, which is also used to modulate the impeller tip clearance for the purpose of surge control. The designed maximum operational speed is 23,000 rpm, which requires a power supply of 52 kW. Within the operating speed range (maximum at 23,000 rpm), the rotor is considered to be a rigid rotor since the first bending mode is at 40,792 rpm. The rotor has a length of 0.517 m and is 27 kg in mass. AMBs used for radial suspension are 12 pole E-core design. The 12 poles are separated into four quadrants. The width of the primary and secondary poles are 27.94 mm and 13.97 mm, respectively. Each pole has 51 turns of 17 AWG wire. The stators of the radial AMBs are laminated in order to reduce the eddy current effect. The designed maximum load capacity per quadrant is 1,414 N and the nominal air gap is 0.5 mm. The corresponding values of negative stiffness

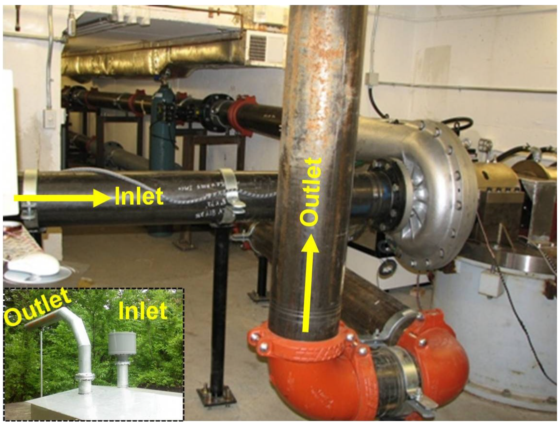

Fig. 2 Centrifugal compressor test rig.

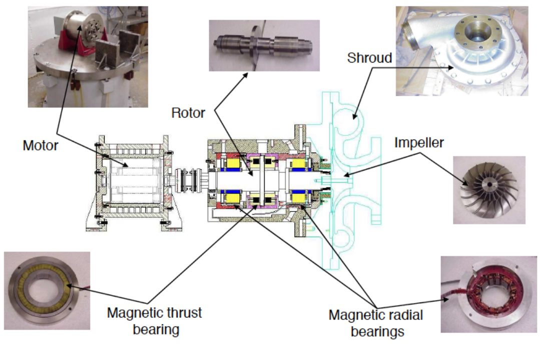

Fig. 3 Centrifugal compressor components.

K_X and current gain K_i were obtained experimentally and summarized in Table 1. The instrumentation properties are summarized in Table 2.

Radial AMB	I_b (A)	K_X (N/m)	K_i (N/A)
Motor side	3	1.27×10^6	199.34
Compressor side	4	2.26×10^6	265.86

Table 1 Radial AMB properties

AMB	Motor side	Compressor side	Thrust
Amplifier gain (A/V)	1.5	1.5	1.5
Amplifier bandwidth (rad/s)	5026.5	5026.5	5026.5
Sensor gain (V/m)	3.937×10^4	3.937×10^4	3.937×10^4
Sensor bandwidth (rad/s)	1.26×10^4	1.26×10^4	1.26×10^4
Maximum slew rate (N/s)	2.2×10^6	2.2×10^6	1.9×10^6

Table 2 Instrumentation properties

In this paper, the fractional order controller design is focused only the rotor lateral dynamics.

3.2. Lateral Rotor Dynamics

The rotor lateral dynamics can be represented as the block diagram shown in Fig. 4, where the rotor model is derived by the finite element analysis approach. The values of negative stiffness K_X and current gain K_i are summarized as mentioned in Table 1. The complete radial AMB system combines the rotor-AMB model with the power amplifiers, sensors, and time delay models as shown in Fig. 5. The control output voltage v_C is the input to the system and the sensor measurement voltage v_S is the output of the system. A time delay is also added to complete the model in order to represent the sampling and computational delays that occur in the digital controller.

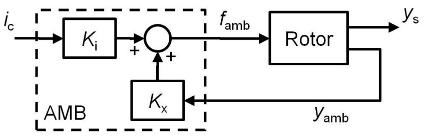

Fig. 4 Rotor-AMB system block diagram

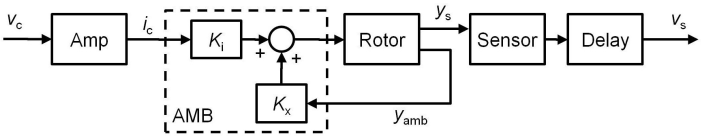

Fig. 5 Radial AMB system block diagram

4. Fractional Order Control of Rotor Suspension

4.1. Control Objectives

The control objectives mainly pertain to rotor vibration and stability margin (sensitivity function peak) stated by ISO14839 specifications (ISO, 2004 and ISO, 2006). Furthermore, the step response performance objective is also important to include in order to provide smooth rotation during the transition between rotational speeds and an ability to reject disturbances. The performances considered in the step test are rise time, settling time, and overshoot. Thus, the objective functions used for the control of the rotor lateral dynamics are listed below.

- Stability of closed-loop system (J_1): Closed-loop stability will be determined by the number of poles that have a positive real part. The optimization goal of this objective is zero.
- Stability margin (J_2): A peak magnitude of the sensitivity function will be used to determine a stability margin.
- Vibration level (J_3): A maximum magnitude of forced responses among three cases (translate mode, conical mode, and overhung cantilevered). This objective will be used to determine the maximum vibration.
- Integral square error (ISE) of a unit step response (J_4): Instead of specifying transient response performance separately, the performance index ISE will be used in order to reduce the conflict between different performances.

4.2. FOPID Controller Design and Tuning

The FOPID controller for the rotor lateral dynamics takes the form of

$$C_{\mathrm{FOPID}}(s) = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s^{\lambda}} + K_{\mathrm{D}} s^{\mu}. \tag{7}$$

This controller is also coupled with a second order low-pass filter in order to limit the bandwidth as well as to make the controller implementable. Here it is assumed that the two control axes (x and y) are symmetric. Therefore, the controllers used for both control axes will be identical.

In addition, all controller parameters are tuned using the Genetic Algorithm (GA), Differential Evolution (DE) and Particle Swarm Optimization (PSO). All objective functions as previously metioned are combined as a single cost function

J during the optimization process. Therefore, it is proper to add some constant weight, w_i, for each specified objective to obtain a normalized overall objective,

$$J = \max\{w_1 J_1, w_2 J_2, w_3 J_3, w_4 J_4\}.$$

These constant weights are respectively the inverse of the desired value of each specification. For example, the desired value of peak sensitivity must be less than 3 according to ISO specification, thus the initial value of w_2 is 1/3.

Note that the main difference of this study from the existing works in the field is that the approximation of the fractional order operators, s^λ and s^μ, occur during the tuning process. Studies in the past generally have addressed the approximation step after the optimization is completed, which can degrade the performance of the FOPID controller due to errors from the approximation. Therefore, in this study, the approximation is included in the optimization process and uses the approximated FOPID controller to evaluate all objective functions. Oustaloup's method is used for the fractional order approximation as explained in Section 2.4 and the chosen number of poles and zeros used for the approximation is 2.

5. Simulation and Experimental Results

5.1. Simulation Results

Based on the same design objectives, three optimization methods are utilized for the controller parameters tuning in order to determine the most suitable method for the FOPID control design of the AMB system. The performance of the FOPID controllers tuned by the GA, DE, and PSO methods are summarized in Table 3. It can be observed that the FOPID controller tuned by the DE algorithm gives better results in terms of the peak of the sensitivity function and maximum vibration levels. In addition, the corresponding FOPID controller has a good transient response for a smooth rotation. Therefore, this FOPID controller will be used for implementation as well as for comparison with other kinds of controllers.

For comparison, the conventional PID controller is tuned based on the same objectives and algorithms as the best case of the FOPID controller. Moreover, performances based on the LQG controller that was designed for the same system reported in (Yoon et al., 2012) are compared with both the FOPID and PID controllers in Table 4.

Specifications	GA	PSO	DE
Sensitivity function peak	2.4414	2.3947	2.2727
Peak unbalance vibration (mm)	0.0029	0.0027	0.0024
Controller output peak (V)	0.3909	0.3639	0.4176
Overshoot (%)	6.1020	5.7632	2.7300
Rise time (s)	0.0077	0.0077	0.0071
Settling time (s)	0.0205	0.0161	0.0151

Table 3 Comparison of performances of FOPID controllers tuned by different Evolutionary algorithms

Specifications	PID	FOPID	LQG
Sensitivity function peak	2.6742	2.2727	2.4794
Peak unbalance vibration (mm)	0.0037	0.0024	0.0026
Controller output peak (V)	0.4228	0.3340	0.1810
Overshoot (%)	0.172	0.033	0.178
Rise time (s)	0.003	0.003	0.005
Settling time (s)	0.023	0.042	0.016
Bandwidth (rad/s)	12757	13759	14377
Controller dimension as implemented	6	7	11

Table 4 Comparison of performances in radial AMBs

As shown in Table 4, the stability margin of all controllers fall within Zone A specification (smaller than 3) as specified by ISO (ISO, 2004). Moreover, the sensitivity function peak of the FOPID controller is smaller than the value achieved by the PID and LQG controllers. Another advantage of the FOPID controller over the LQG controller is the reduction of the controller size by 50 percent. Transient response of each controller is approximately the same. Each controller has a bandwidth within the limit for digital implementation at a 5 kHz sampling frequency.

5.2. Experimental Results

To validate the FOPID controller designed in Section 4 for the lateral rotor dynamics, two types of measurements are made. The first type of measurement is the rotor vibration magnitude for speeds ranging from 500 rpm to 16,500 rpm, in 500 rpm increments. Three separate cases are tested, the IOPID and FOPID controllers tuned by the DE algorithm and the LQG controller that was previously designed in (Yoon et al., 2012).

The results in Figs. 6 and 7 show the rotor vibration within the specified speed range of the motor side and the compressor side, respectively. The FOPID controller leads to the smallest vibration magnitude throughout the speed range among all three tested controllers and its peak magnitude is well within the limit of Zone A specified by ISO (ISO, 2004). The IOPID controller leads to the largest vibration magnitude which can be observed from the motor side measurement. The result agrees with the prediction of the maximum vibration magnitudes illustrated in Table 4.

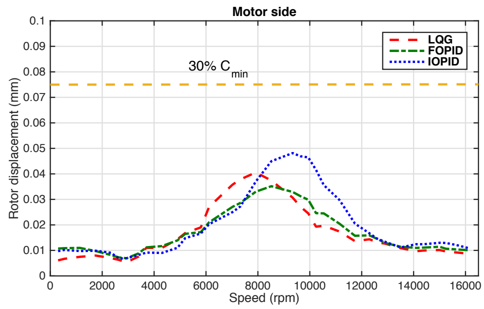

Fig. 6 Rotor displacements at the motor side under the IOPID, FOPID, and LQG controllers

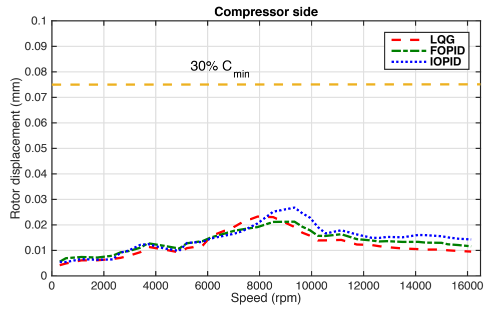

Fig. 7 Rotor displacements at the compressor side under the IOPID, FOPID, and LQG controllers

The second test is the sensitivity function frequency response measurement. Again, the three controllers used in the rotor vibration experiment are tested. For this testing, the perturbation signal of 100 mv with frequencies ranging from 0.1 Hz to 1200 Hz is added at the controller input and the sensitivity function frequency response is obtained from the relationship between the sum of perturbation and controller input signals and the perturbation signal itself. The frequency response plots for all three controllers for both the motor side and the compressor side are shown in Figs. 8 and 9, respectively. From these results, the sensitivity function peak under the FOPID controller is the smallest and falls into Zone A specification of the ISO standard (ISO, 2006), while the IOPID controller results in the largest sensitivity function peak and its magnitude falls into Zone B specification. Lastly, the trend of the sensitivity function frequency responses match the theoretical prediction in Table 4.

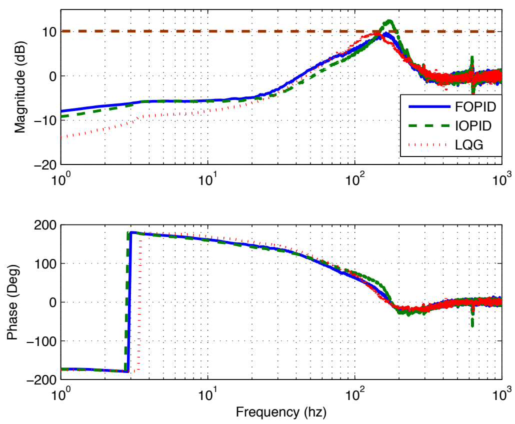

Fig. 8 Bode plots of the lateral AMB sensitivity function at the motor side

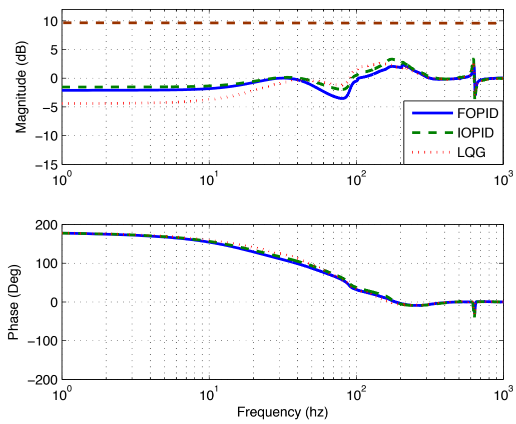

Fig. 9 Bode plots of the lateral AMB sensitivity function at the compressor side

6. Conclusions

This paper demonstrated the design and analysis of fractional order PID controllers for rotor-AMB systems. Simulation results showed that the Differential Evolution method achieves the best performance among three chosen methods for FOPID control design. For comparison, the IOPID controller was designed based on the same objectives and the optimization method. Moreover, the designed and implemented LQG controller reported in (Yoon et al., 2012) was also compared with the designed FOPID controller. The experimental results for rotor vibration with the rotor spinning at speeds ranging from 500 rpm to 16,500 rpm showed that the FOPID results in the smallest rotor vibration and the IOPID results in the largest vibration peak, while the rotor vibration under the LQG controller falls in between the results under the FOPID and the IOPID controllers. In addition, in terms of the peak value of the sensitivity function, the FOPID controller results in the smallest and it falls into Zone A specified by the ISO Standard, while LQG controller also achieves the Zone A specification with a slightly larger peak value of the sensitivity function than the FOPID controller. On the other hand, the IOPID controller can achieve only Zone B specification. These results showed the effectiveness of the FOPID controller for rotor-AMB systems. The future work will extend the concept of fractional order control to the axial rotor dynamics as well as the surge control by active magnetic bearings.

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