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Characteristic quantities of active magnetic bearings

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Abstract
In the design of a magnetically levitated system the question arises whether the closed loop can be stabilized by a certain active magnetic bearing (AMB). In this paper we introduce several characteristic quantities which evaluate the static and dynamic capacity of an AMB system. A controller design is not necessary in this novel stability estimation. The calculation of the proposed characteristic quantities can be used to estimate how well an AMB is suited for the stabilization task. Furthermore, the derived dimensionless quantities allow easy comparison of different AMB systems.

Key words : active magnetic bearing, AMB, stability, design, characterisitc quantity, performance, dimensionless quantitiy

1. Introduction

AMBs accomplish levitation and vibration control without mechanical contact (Chiba et al., 2005). They have to keep the vibration amplitude, temperature in the coil and electronics, and the coil current in acceptable ranges. In this paper we investigate 1-axis-controlled AMBs which are characterized by few parameters: the mass m of the levitated body or rotor, the destabilizing stiffness k_x^*, the force constant k_i, the coil parameters (resistance R and inductance L) and the maximum voltage u_{max}. We do not include rotordynamics (gyroscopic effects) or disturbances in our considerations. In this paper we follow a more basic approach and investigate certain fundamental physical limits of an AMB system.

1.1. Comparison with ISO 14839

ISO 14839-2 evaluates the vibrations of rotors supported by AMBs. This standard is applicable to rotating machines with a nominal power greater than 15 kW, which excludes small-scale rotors such as turbomolecular pumps. In ISO 14839-2 one criterion is that the maximum radial displacement of the rotor should be smaller than 0.4 times the clearance of the auxiliary bearing for long-term operation. The vibration magnitude and the resonance severity are regulated by ISO 10814. Compared to ISO 14839-2, we do not examine the AMB during operation but derive characteristic quantities of the AMB system from its parameters.

ISO 14839-3 evaluates the stability margin of an AMB at nominal speed. The maximum peak of the sensitivity function, which is gained from measurements, should be below 12 dB. In this study we do not consider any control algorithm for the derivation of the characteristic quantities. While ISO 14839 describes how to evaluate already operational AMB systems our investigation is based on calculations assuming linear relationships.

2. System model

2.1. Dynamic behavior

We investigate an unstable, one degree of freedom (1-DOF) system with a negative stiffness k_x^* ($k_x^* < 0\,\mathrm{N/m}$, $k_x = |k_x^*|$) as shown in Fig. 1. The voltage u is typically generated by a switching amplifier (with a fixed switching frequency usually in the order of tens of kilohertz), and the duty cycle is the controlled variable. Due to the high switching frequency we can use the average voltage over one switching period as input voltage u. The position of the body and the current is typically

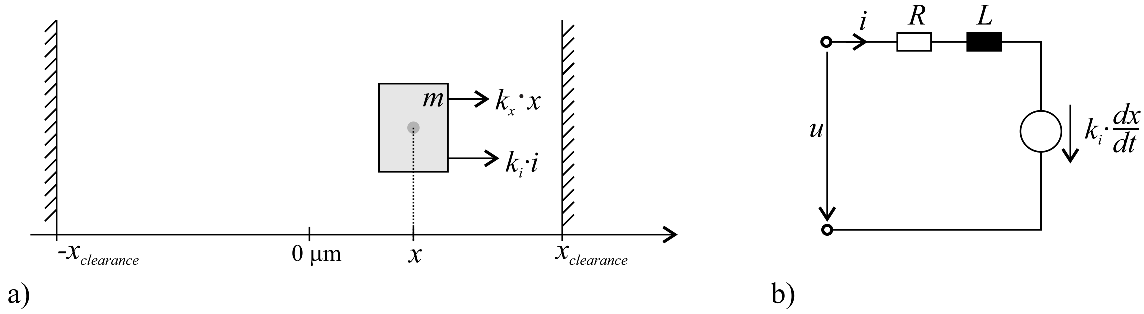

Fig. 1 a) Mechanical and b) electrical model of 1-DOF AMB. A positive current i generates a force in positive x-direction.

measured. The instability of the levitated mass m is determined by the mechanical time constant

$$\tau_m = \sqrt{\frac{m}{k_x}}. \tag{1}$$

The lower the time constant τ_m the quicker the body moves away from the equilibrium position $x = 0\,\mu$m if the AMB is not energized ($i = 0$ A). The AMB has the task to get the mass from the position at $x \neq 0\,\mu$m to the position $x = 0\,\mu$m. The AMB coil has a resistance R and an inductance L, which gives us the electrical time constant

$$\tau_{el} = \frac{L}{R}. \tag{2}$$

The force of the AMB is equal to $k_i i$, where k_i is the force constant and i the coil's current. A further important parameter is the maximum voltage u_{max} of the AMB system. The dynamic behavior can be described by two simple differential equations

$$m\frac{d^2x(t)}{dt^2} - k_x x(t) = k_i i(t) \tag{3}$$

$$Ri(t) + L\frac{di(t)}{dt} + \underbrace{k_i\frac{dx(t)}{dt}}_{\sim 0\,\mathrm{V}} = u(t) \tag{4}$$

The following assumptions are made:

- The AMB has no bias current. However, the principle can be also easily applied to AMBs with bias current.
- Linear relationships (no magnetic saturation - linear inductance L, linear force characteristic).
- The power electronics is capable of producing a current between $i_{min} = \frac{-u_{max}}{R}$ and $i_{max} = \frac{+u_{max}}{R}$. Otherwise, the maximum voltage u_{max} should be adjusted accordingly.
- The induced voltage due to motion $k_i \frac{dx}{dt}$ is usually small and will be neglected in the further investigations.
- The weight force is not considered (usually small compared to the AMB force).
- The temperature of the AMB is constant (constant coil resistance R).
- The sensor (measurement of position x) and the actuator (force $k_i i$) are collocated.

2.2. Analytical solution

2.2.1. AMB not energized If the AMB is powerless ($i(t) = 0$ A) we can easily calculate the position of the body at the time t. The solution of (3) with the initial conditions $x(0) = x_0$ and $\frac{dx}{dt}(0) = v_0$ is

$$x(t) = x_0 \cosh\left(\frac{t}{\tau_m}\right) + v_0\tau_m \sinh\left(\frac{t}{\tau_m}\right). \tag{5}$$

2.2.2. AMB energized It is possible to solve the differential equations (3) and (4) for a constant input voltage u. The solution of the system of the differential equations given by (3) and (4) with the initial conditions $x(0) = x_0$, $\frac{dx}{dt}(0) = v_0$ and $i(0) = 0$ A is

$$x(t) = -\frac{k_i u}{k_x R} + \left(\frac{k_i u m R}{k_x c} + x_0\right)\cosh\left(\frac{t}{\tau_m}\right) + \left(v_0\tau_m - \frac{k_i u L \tau_m}{c}\right)\sinh\left(\frac{t}{\tau_m}\right) - \frac{k_i u L^2}{Rc}e^{-\frac{t}{\tau_{el}}} \tag{6}$$

In (6) we used the constant c

$$c = mR^2 - k_x L^2. \tag{7}$$

Since we can rewrite this equation as

$$c = k_x R^2 \left(\tau_m^2 - \tau_{el}^2\right) \tag{8}$$

the sign of c tells us whether the mechanical or the electrical time constant is greater.

3. Derivation of characteristic quantities

We will now calculate three different quantities which describe characteristic properties of an AMB:

- The static equilibrium position $x_{balanced}$.
- The maximum delay time $t_{delay,max}$ for a given initial position x_0.
- The maximum dynamic start position $x_{0,stable}$.

Each quantity will be calculated for the AMB parameters given in Table 1.

Table 1 Parameters of an AMB system at 20 °C coil temperature.

	Parameter	Value
m	Mass	0.3 kg
k_x	Value of destabilizing stiffness	150 N/mm
R	Resistance	550 Ω
L	Inductance	0.3 H
k_i	Force constant	116 N/A
u_{max}	Voltage	240 V
τ_{el}	Electrical time constant	0.55 ms
τ_m	Mechanical time constant	1.41 ms

3.1. Static equilibrium position

We can calculate the theoretical rotor position $x_{balanced}$ where the maximum force of the AMB ($k_i i_{max}$) is equal to the destabilizing force ($k_x x_{balanced}$). This force equilibrium is at the position

$$x_{balanced} = \frac{k_i u_{max}}{k_x R} \tag{9}$$

For the parameters given in Table 1 we get $x_{balanced} = 337\,\mu\text{m}$. In a feasible AMB system the touch-down bearing clearance $x_{clearance}$ (compare Fig. 1) must be smaller than the position of force equilibrium ($x_{clearance} < x_{balanced}$).

3.2. Maximum delay time

In the 1-DOF AMB with the nominal parameters of Table 1 the electrical system is only 2.5 times faster than the unstable mechanical system ($\tau_{el} = \frac{\tau_m}{2.5}$). Therefore, it is not clear from the outset if the AMB can hold the body in a stable, force-free position.

To analyze the dynamic behavior given by (3) and (4) we use the Matlab Simulink model shown in Fig. 2. At start of the simulation the position of the mass is 1 μm above the force free position ($x(0) = 1\,\mu\text{m}$). The initial speed is set to zero ($\frac{dx}{dt}(0) = 0\,\text{m/s}$). Between the mechanical and electrical system we use a transport delay block. Therefore, we can delay the AMB force by the time t_{delay} which is equally to use the input voltage

$$u(t) = \begin{cases} 0\,\text{V} & t <= t_{delay} \\ -u_{max} & t > t_{delay} \end{cases} \tag{10}$$

in equation (4). The negative voltage generates an AMB force in negative x-direction while the destabilizing force acts in positive x-direction at $t = 0\,\text{s}$ since $x(0) > 0\,\mu\text{m}$ (compare Fig. 1). Note that we can also use the already calculated solutions (5) for $t <= t_{delay}$ and (6) for $t > t_{delay}$ instead of the numerical solver to get the position x of the levitated body. With (5) we can calculate the position $x\left(t_{delay}\right)$ and speed $v\left(t_{delay}\right)$ which can be used as initial conditions in (6).

We want to find the maximum delay time $t_{delay,max}$ where the position of the levitated body can be brought back to equilibrium position $x = 0\,\mu\text{m}$. Since we continue to apply the maximum negative voltage $-u_{max}$ in our model the position

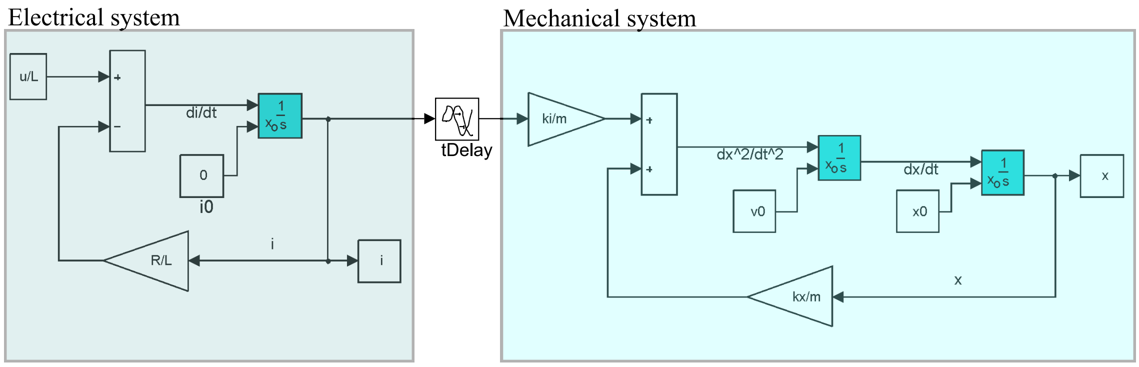

Fig. 2 Matlab Simulink model of the dynamic behavior.

of the mass goes further to $x \to -\infty$. The size of $t_{delay,max}$ is a measure of the stability of the AMB. Fig. 3 shows the simulation results for different delay times. We can see that the maximum delay where the rotor changes the direction of movement is $t_{delay,max} = 7.7\,\text{ms}$. If the delay time t_{delay} gets bigger than 7.7 ms the force produced by the AMB is not sufficient to pull the mass back to $x = 0\,\mu\text{m}$. If we compare $t_{delay,max}$ to a typical sampling time T_s of up-to-date DSPs ($0.05\,\text{ms} < T_s < 0.2\,\text{ms}$) we come to the conclusion that the closed loop control will succeed in stabilizing the system. This result was verified on a 4-DOF passive magnetic bearing (PMB) and 1-DOF AMB. We succeeded in stabilizing an AMB which had the parameters shown in Table 1.

If we reduce the mass from $m = 0.3\,\text{kg}$ to $m = 0.045\,\text{kg}$ we have equal time constants $\tau_{el} = \tau_m = 0.55\,\text{ms}$. In this case the maximum delay time reduces to $t_{delay,max} = 2.7\,\text{ms}$. If we additionally change the start position from $x(0) = 1\,\mu\text{m}$ to $x(0) = 10\,\mu\text{m}$ we get a maximum delay time of $t_{delay,max} = 1.5\,\text{ms}$. If we increase the initial position to $x(0) = 50\,\mu\text{m}$ the delay time reduces to $t_{delay,max} = 0.6\,\text{ms}$.

The calculation of the maximum delay time $t_{delay,max}$ has two drawbacks:

- It depends on the initial position $x(0)$.
- It is not possible to find an explicit equation for the maximum delay time.

We will therefore derive a second method to characterize the stability of an AMB in the next section where it is not necessary to assume an initial position $x(0)$. Furthermore, we can calculate this characteristic quantity using a compact analytical equation.

3.3. Maximum dynamic start position

We have seen in Fig. 3 that the solution goes to plus infinity ($\lim_{t\to\infty} x(t) = +\infty$) if the delay time is to big, i.e., the body did not return to the equilibrium position. The solution goes to minus infinity ($\lim_{t\to\infty} x(t) = -\infty$) if the force of the AMB succeeded in bringing the mass back to $x = 0\,\mu\text{m}$. As we do not change the applied voltage, the mass goes further to minus infinity. When we calculate the limit of (6) we get

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\left[-\underbrace{\frac{k_i u}{k_x R}}_{const.} + \underbrace{\left(\frac{k_i u m R}{k_x c} + x_0\right)}_{a=const.}\underbrace{\cosh\left(\frac{t}{\tau_m}\right)}_{\to\infty} + \underbrace{\left(v_0\tau_m - \frac{k_i u L \tau_m}{c}\right)}_{b=const.}\underbrace{\sinh\left(\frac{t}{\tau_m}\right)}_{\to\infty} - \underbrace{\frac{k_i u L^2}{Rc}e^{-\frac{t}{\tau_{el}}}}_{\to 0}\right] \tag{11}$$

which goes either to plus or minus infinity. Now we can use the mathematical relationship $\lim_{t\to\infty}(a\cosh(t) + b\sinh(t)) = \infty(a+b)$ and get

$$\lim_{t\to\infty} x(t) = \infty\cdot\left[\left(\frac{k_i u m R}{k_x c} + x_0\right) + \left(v_0\tau_m - \frac{k_i u L \tau_m}{c}\right)\right]. \tag{12}$$

If the expression in square brackets is negative the AMB succeeds in pulling the mass back towards the equilibrium position for the given initial conditions $x_0 > 0\,\mu\text{m}$ and v_0. For a negative initial condition $x_0 < 0\,\mu\text{m}$ a positive value of the square brackets expression implies that the body could be brought back towards the equilibrium position.

In our first evaluation (section 3.2) we assumed that the start position of the levitated mass is $x(0) = 1\,\mu\text{m}$. We can avoid this arbitrary assumption by searching for the maximum position $x_{0,stable}$, where the AMB is able to bring back the levitated body to x equal zero. The speed at $t = 0\,\text{s}$ is defined to be zero ($v_0 = 0\,\text{m/s}$). As our derivation is based on the

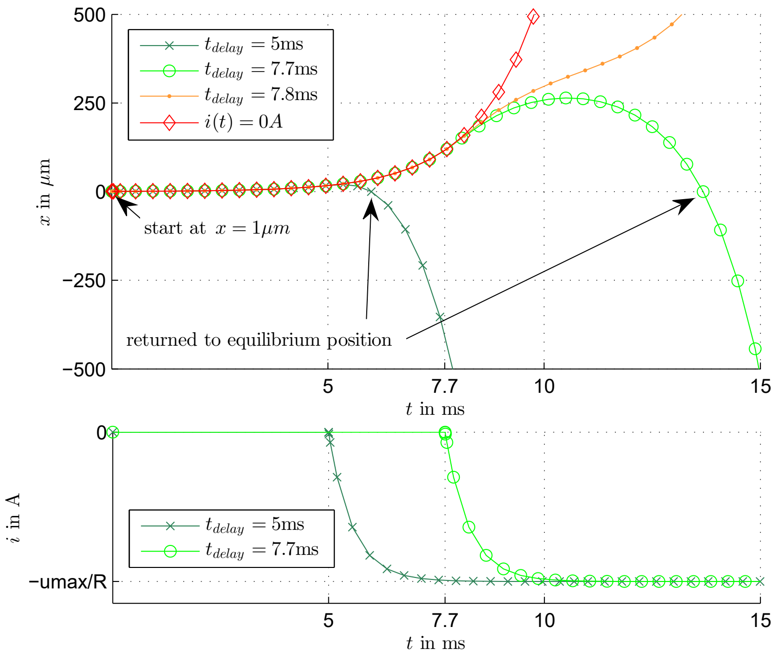

Fig. 3 Numerical simulation results for the position $x\,(x\,(0) = 1\,\mu\text{m})$ and the current i. The voltage of the AMB is delayed by the time t_{delay}. If the delay time is below 7.7 ms the levitated body can be brought back to the equilibrium position $x = 0\,\mu$m.

analytical solution (6) the start current is also zero ($i = 0$ A). We can calculate the highest possible start position by using (12)

$$x_{0,stable} = \frac{k_i u L \tau_m}{c} - \frac{k_i u m R}{k_x c}. \tag{13}$$

That means that if the voltage $u = -u_{max}$ is applied at $t = 0$ s and $0 < x\,(0) < x_{0,stable}$ the mass can pulled back to $x = 0\,\mu$m. For the parameters given in Table 1 we get $x_{0,stable} = 244\,\mu$m. The position of the mass is plotted in Fig. 4 for three slightly different starting positions $x\,(0)$:

- The first curve $x\,(0) = x_{0,stable}$ shows the theoretical result where the AMB force gets exactly the same magnitude as the magnetic force after the transient response ($k_i \frac{-u_{max}}{R} + k_x x_{balanced} = 0$ N).
- The second curve $x\,(0) = x_{0,stable} - 1\,\mu$m shows that the AMB pulls the body back towards the equilibrium position $x = 0\,\mu$m.
- The third curve $x\,(0) = x_{0,stable} + 1\,\mu$m shows that this initial condition cannot be stabilized.

In other words: the limit of stability is reached when we move the rotor to the position $x_{0,stable}$ and switch on the AMB without delay ($t_{delay} = 0$ s).

Note that the curves in Fig. 4 do not represent a typical start procedure from a mechanical stop. As it takes time to impress the AMB coil current, the body is moving even further away from the force free position ($x = 0\,\mu$m) in the time range from 0 ms till 4 ms.

4. Normalized characteristic quantities

The quantity $x_{balanced}$, defined in (9), describes the static force characteristic of the AMB. A dimensionless, charasteristic quantity can be gained by relating $x_{balanced}$ to the touch-down bearing clearence $x_{clearance}$

$$\kappa_s = \frac{x_{balanced}}{x_{clearance}}. \tag{14}$$

The parameter κ_s should be as high as possible. As the AMB force must at least be high enough to lift the rotor from the mechanical stop the quantity κ_s must be greater than one ($\kappa_s > 1$).

A second characteristic, normalized quantity is the relation between the maximum dynamic start position $x_{0,stable}$ and

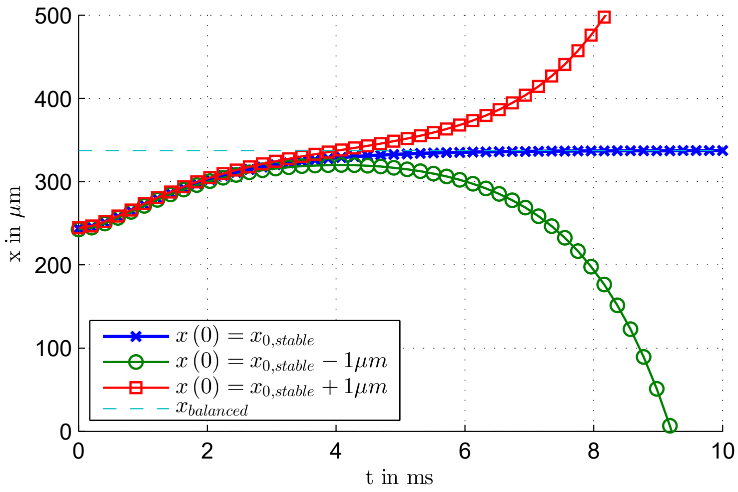

Fig. 4 Position of mass for different initial conditions x_0.

the force equilibrium position $x_{balanced}$

$$\kappa_d = \frac{x_{0,stable}}{x_{balanced}}. \tag{15}$$

The parameter κ_d characterizes the quickness of the AMB force generation as well as the instability due to the negative stiffness k_x^* and lies between zero and one ($0 < \kappa_d < 1$). The closer it is to one, the higher is the dynamic capacity of the AMB system. If we assume zero inductance ($L = 0\,\mathrm{H}$) the current can be immediately impressed and we get $x_{0,stable} = x_{balanced}$ and therefore $\kappa_d = 1$. The value of κ_d considers both relevant time constants: τ_m of the mechanical system and τ_{el} of the electrical system.

The results of the investigated AMB system are summarized in Table 4. For comparison we also calculated the characteristic quantities for a coil temperature of 100 °C. We see that the investigated AMB system is not well suited for the high coil temperature as the position of the force equilibrium ($x_{balanced}$) is very close to the position of the mechanical stop ($x_{clearance}$).

Table 2 Characteristic quantities of the investigated AMB system.

	Parameter	20 °C coil temp.	100 °C coil temp.
$t_{delay,max}$ @ $x_0 = 1\,\mu$m	Maximum delay time for the initial cond. x_0	7.7 ms	7.5 ms
$x_{balanced}$	Position of theoretical force equilibrium	337 μm	265 μm
$x_{clearance}$	Touch-down bearing clearance	250 μm	250 μm
$x_{0,stable}$	Maximum dynamic start position	244 μm	203 μm
κ_s	Static force characteristic ($\kappa_s > 1$)	1.35	1.06
κ_d	Dynamic force characteristic ($0 < \kappa_d < 1$)	0.72	0.77

5. Further Investigations

The following topics are planned to be investigated in the further research

- Influence of the number of windings on the characteristic quantities.
- Find a relationship between the presented characteristic quantities and the closed loop stability. How do these parameters correlate with the AMB transient behavior.
- Comparison of the calculated results with measurements.
- Extension of the principle for the stability analysis of bearingless motors (magnetic bearing is integrated into the motor unit).

6. Summary

We have derived several characteristic quantities which evaluate the static and dynamic performance of an AMB system. These quantities can be easily calculated from the basic AMB parameters, no controller design is necessary. The two normalized quantities are well suited for comparison of different AMB systems.

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