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Numerical Verification of Amplitude Reduction of a Rotor Supported by a Superconducting Magnetic Bearing Utilizing Internal Resonance

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Abstract

Superconducting magnetic bearings (SMBs) have some advantages compared with mechanical bearings. However, whirling amplitude of a rotor supported by SMB tends to be large near the critical rotational speed because of their low damping. Furthermore, complicated phenomena of dynamics can be generated due to nonlinearity of the levitation force. Therefore it is necessary to reduce the amplitude by considering the effect of the nonlinearity on dynamics during passing through the critical speed in applications. In this paper, we investigated resonant amplitude reduction of a rotor supported by two movable superconducting bulks (SCs). For this purpose, we first introduced an analytical model consisting of a rotating permanent magnet and the unfixed SCs that can move around the rotor and are connected with springs. We evaluated electromagnetic force by the advanced mirror image method and derived governing equations of the rotor at an arbitrary rotational speed. From the nondimensionalized equations of motion, we predicted that internal resonance between whirling of the rotor and swinging of the SCs can occur if the ratio among the natural frequencies of those motions is an integer ratio. We also performed numerical calculation of equations of motion. The numerical results show that internal resonance can occur at around the critical speed and that as a result reduction of the resonant whirling amplitude can be achieved by the swinging SCs at around the critical speed.

Keywords : Internal Resonance, Rotor Dynamics, Superconducting Bulk, SMB, Nonlinear Analysis, Nonlinear Coupling, Amplitude Reduction

1. Introduction

High-Tc superconducting magnetic levitation has a feature of noncontact stable levitation. Some applications of this systems are superconducting magnetic bearings (SMBs) [1]-[4]. Compared to mechanical bearings, SMBs have various merits such as low damping, high energy efficiency and utilization under special environment due to maintenance-free property because they are able to support a magnet without physical contact. However, whirling amplitude of a rotor supported by SMB tends to be large near the critical rotational speed because of their low damping. Therefore it is necessary to reduce the resonant amplitude in systems utilizing SMBs. Several solutions for this subject include removing unbalance, adding damping and utilizing internal resonance [5]-[7] between multiple oscillation modes coupled nonlinearly.

Concerning the latest solution utilizing internal resonance, R. Kawana, et al. investigated motion of a rigid rectangular-shaped body connected with three springs and having a rotor with unbalance. They performed nonlinear analysis of the rotor with internal resonance caused by geometrical nonlinearity. In addition, they numerically and experimentally clarified that internal resonance can reduce the amplitude of the rotor [8]. R. Sakaguchi, et al. investigated nonlinear dynamics of a superconducting magnetic levitation system that consists of two permanent magnets on a rigid bar and two superconducting bulks under the bar. They numerically and experimentally confirmed that the amplitude of horizontal oscillation at its natural frequency is reduced because internal resonance is caused by magnetic nonlinearity transferring the kinetic energy from longitudinal oscillation to lateral one [9]. We considered whirling amplitude of a rotor can be reduced from R. Kawana's research and R. Sakaguchi's one utilizing internal resonance caused not by geometrical nonlinearity but by magnetic nonlinearity.

Therefore, in this study, by numerical analysis, we investigated resonant amplitude reduction of a rotor supported by SMBs utilizing internal resonance.

2. Analytical model and governing equations

Figure 1 shows an analytical model seen from side and top. A permanent magnet (PM) is supported by a couple of superconducting bulks (SCs). Each SC is connected with one spring in order to receive restoring force. Figure 2 shows coordinates related to the PM. The SCs can move keeping a constant distance R from the initial position of the PM. We define the angle of the moving SCs as θ. We regard the restoring force due to the springs as linear with respect to θ. The PM rotates at angular velocity ω and is assumed to move in the horizontal direction x and y. Suppose that we transform to a new coordinate system O-$x'y'$, which has the same origin as O-xy and is obtained by rotating the coordinate axes of O-xy through the angle θ about O. Therefore, the relationship of the two previous coordinates are described as follows:

$$x' = x\cos\theta + y\sin\theta \tag{1}$$

$$y' = -x\sin\theta + y\cos\theta \tag{2}$$

The SC in the negative y' area is represented as SC1 and the opposite SC as SC2. We utilized the advanced mirror image method [10] in order to evaluate the electromagnetic force. In this method, we consider each of the SCs as sum of mirror image (MI1) of the magnet at initial position (Mag1) and mirror image (MI2) of the magnet at present position (Mag2). MI1's and Mag1's magnetization vectors are oriented in the same direction, while MI2's and Mag2's one are in the opposite direction. Thus, the moved PM is affected by these mirror images in SCs.

Electromagnetic forces $F_{x'1}$ and $F_{y'1}$ due to the mirror images in the SC1 act on the PM, and $F_{x'1}$ due to the PM acts on the frozen mirror image (MI1). $F_{y'12}$ due to the PM act on the moving mirror image. $F_{x'2}$ and $F_{y'2}$ for SC2 are also given in the same way. Using that method, the equations of electromagnetic forces are derived as follows:

$$F_{x'1} = -\frac{3M^2}{4\pi\mu_0}\frac{x'}{\{x'^2 + (y' + 2R)^2\}^{\frac{5}{2}}} \tag{3}$$

$$F_{y'1} = F_{y'11} + F_{y'12} \tag{4}$$

$$F_{y'11} = -\frac{3M^2}{4\pi\mu_0}\left[\frac{y' + 2R}{\{x'^2 + (y' + 2R)^2\}^{\frac{5}{2}}} - \frac{1}{16(y' + R)^4}\right] \tag{5}$$

$$F_{y'12} = -\frac{3M^2}{4\pi\mu_0}\frac{1}{16(y' + R)^4} \tag{6}$$

$$F_{x'2} = -\frac{3M^2}{4\pi\mu_0}\frac{x'}{\{x'^2 + (y' - 2R)^2\}^{\frac{5}{2}}} \tag{7}$$

$$F_{y'2} = F_{y'21} + F_{y'22} \tag{8}$$

$$F_{y'21} = -\frac{3M^2}{4\pi\mu_0}\left[\frac{y' - 2R}{\{x'^2 + (y' - 2R)^2\}^{\frac{5}{2}}} - \frac{1}{16(y' - R)^4}\right] \tag{9}$$

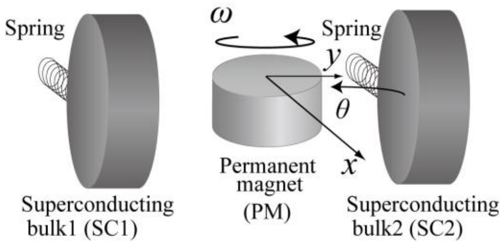
$$F_{y'22} = -\frac{3M^2}{4\pi\mu_0}\frac{1}{16(y' - R)^4} \tag{10}$$

Here M is magnetic moment of the PM with dipole approximation and μ_0 is the permeability of vacuum. Using Eq.(1)-(10), the equations of motion of the PM and the SCs are derived as follows:

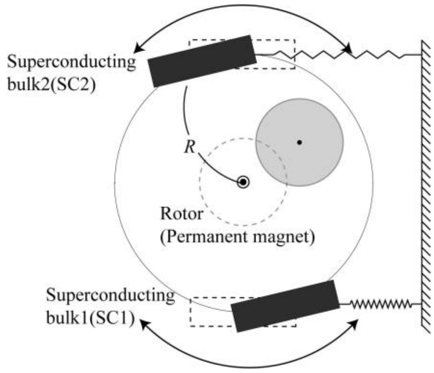
$$m\ddot{x} + c\dot{x} = (F_{x'1} + F_{x'2})\cos\theta - (F_{y'1} + F_{y'2})\sin\theta + me\omega^2\cos\omega t \tag{11}$$

$$m\ddot{y} + c\dot{y} = (F_{x'1} + F_{x'2})\sin\theta + (F_{y'1} + F_{y'2})\cos\theta + me\omega^2\sin\omega t \tag{12}$$

$$I\ddot{\theta} + c_\theta\dot{\theta} = -2k_\theta\theta + 2R(-F_{x'1} + F_{x'2}) - x'(F_{y'12} + F_{y'22}) \tag{13}$$



(a) Side view.



(b) Top view.

Fig. 1 Analytical model.

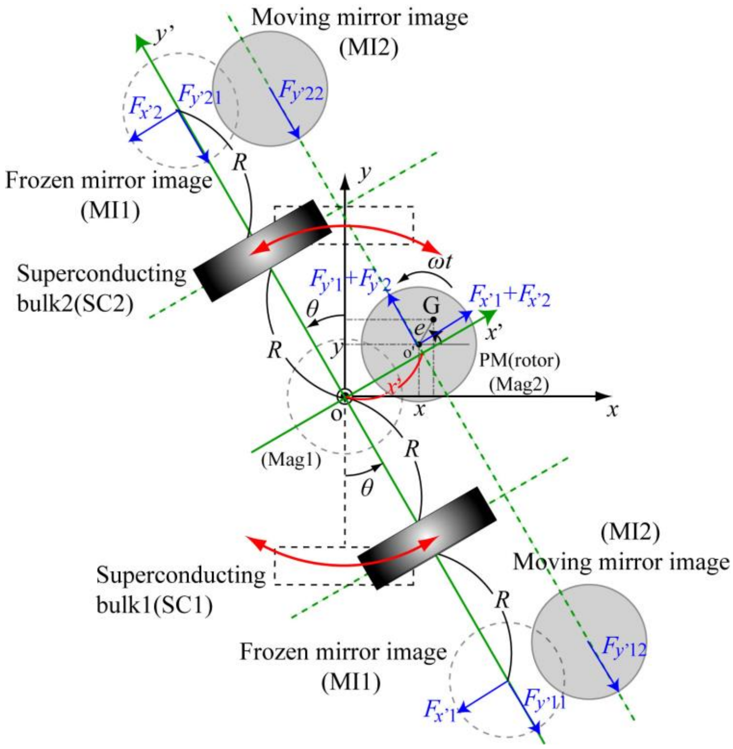


Fig. 2 Coordinates and forces exerted on the magnet.

Here m is mass of the PM, c and c_θ are damping coefficients of this system, k_θ is the product of the spring constant and the moment arm, and I is moment of inertia of this system. The dot symbol denotes derivation with respect to time t. Equations (11) and (12) show that PM's movement in the x and y directions and SC's swinging in the θ direction are nonlinearly coupled with each other. Equations (11)-(13) can be expanded into Taylor series around the origin of the coordinates (O) up to the 3rd order terms as follows:

$$m\ddot{x} + c\dot{x} = -2k_1x + 2(k_5 - k_1)y\theta - 2k_3xy^2 - 2k_5x\theta^2 + 2k_4x^3 + me\omega^2\cos\omega t \tag{14}$$

$$m\ddot{y} + c\dot{y} = -2k_5y + 2(k_5 - k_1)x\theta - 2k_3x^2y - 2k_1y\theta^2 - 2k_6y^3 + me\omega^2\sin\omega t \tag{15}$$

$$I\ddot{\theta} + c_\theta\dot{\theta} = -2k_\theta\theta + 2(k_7 - 2Rk_2)xy - 2(k_7 - 2Rk_2)x^2\theta + 2(k_7 - 2Rk_2)y^2\theta \tag{16}$$

where k_n ($n = 1,2,\cdots,7$) are coefficients of terms appearing in the expanded form of Eq. (11)-(13). Equations (14)-(16) can be nondimensionalized using the following relations:

$$x = Rx^*\,, y = Ry^*\,, t = \sqrt{\frac{m}{2k_5}}t^* = \frac{1}{\omega_y}t^* \tag{17}$$

Nondimensional equations of motion are obtained as below.

$$\ddot{x} + 2\gamma\dot{x} = -\frac{1}{4}x + \frac{3}{4}y\theta - \frac{15}{16}xy^2 - x\theta^2 + \frac{5}{32}x^3 + \epsilon\nu^2\cos\nu t \tag{18}$$

$$\ddot{y} + 2\gamma\dot{y} = -y + \frac{3}{4}x\theta - \frac{15}{16}x^2y - \frac{1}{4}y\theta^2 - \frac{35}{4}y^3 + \epsilon\nu^2\sin\nu t \tag{19}$$

$$\ddot{\theta} + 2\gamma_\theta\dot{\theta} = -k_{\theta\theta}\theta + k_{\theta xy}xy - k_{\theta xx\theta}x^2\theta + k_{\theta yy\theta}y^2\theta \tag{20}$$

where the asterisks denoting nondimensional variables are omitted for simple description. Here γ and γ_θ are the nondimensional damping coefficients of this system, ν is the nondimensional rotational speed defined by the ratio of ω to ω_y, and ϵ is the nondimensional eccentricity. Other constants such as $k_{\theta\theta}$ and $k_{\theta xy}$ are also nondimentional ones. From Eq. (18)-(20), it is found that the ratio of the natural frequencies of the rotor's oscillation in the x direction ω_x to y direction ω_y is theoretically 1 to 2.

Generally, if there is an integer ratio between natural frequencies in a multi-degree-of-freedom system which has quadratic nonlinear coupling terms such as $x\theta$, $y\theta$ and xy, internal resonance between corresponding motions can simultaneously occur and the kinetic energy can be exchanged between them. We considered that by adjusting $k_{\theta\theta}$, the ratio of the natural frequency of the rotor's oscillation in the y direction ω_y to that of the SCs' swing motion ω_θ can be 2 to 1. It is expected that amplitude reduction of y can be achieved by energy transfer from y to θ caused by internal reasonance between the two motions.

3. Numerical Calculation

We performed numerical calculation of Eq. (11)-(13) by means of the Runge-Kutta method, taking nonlinear terms into consideration. We dealt with two different conditions: (a) with the SCs unfixed with optimal springs that adjust the ratio of ω_y to ω_θ to be 2 to 1 and (b) with the SCs fixed. Figure 3 shows frequency responses of y and θ obtained by decreasing the rotational speed. It is found that in (a) the amplitude of y is reduced at around the critical speed $v=1.0$ compared with the amplitude of y in (b). On the other hand, the amplitude of θ is increased in (a). Figure 4 shows time histories and FFT spectra of y and θ at v=1.0. It is found that the frequency of the main component in the y direction is 1.0, while that in the θ direction is 0.5. Therefore, this amplitude reduction of y at around the critical speed $v=1.0$ is caused by internal resonance between the rotor's whirling and the SCs' swinging, which transfers the kinetic energy from y to θ.

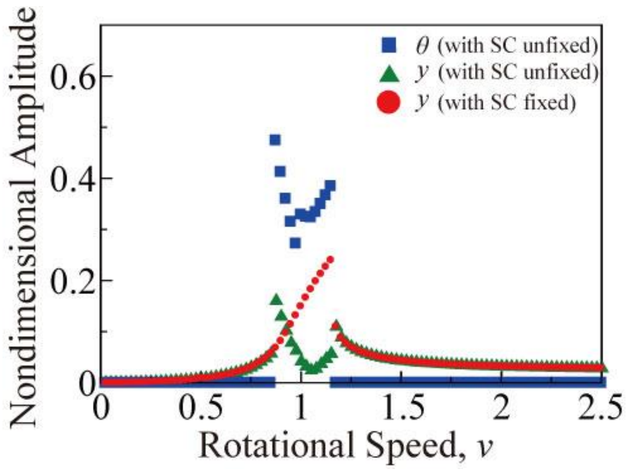


Fig. 3 Numerically obtained frequency responses of y and θ.

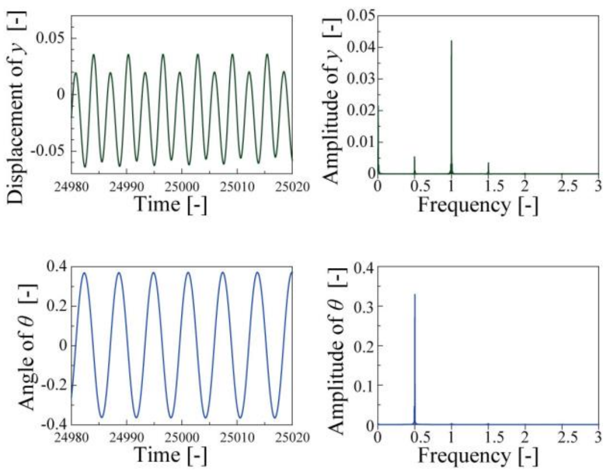


Fig. 4 Time histories and FFT spectra of y and θ at v=1.

4. Conclusion

In this study, we numerically investigated nonlinear dynamics of a rotor supported from its sides by a couple of superconducting bulks. From derived governing equations for dynamics of the system with optimal springs attached to the superconducting bulks, we predicted that internal resonance between whirling of the rotor and swinging of the SCs can occur by quadratic nonlinear terms of the electromagnetic forces. In addition, we performed numerical calculation taking nonlinear terms into consideration. Numerical results show that the whirling amplitude can be reduced by internal resonance at around the critical speed.

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