Simple and Effective Dynamic Model Identification Procedure for Magnetically Suspended Flexible Rotor Systems

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Abstract

Advanced controllers for magnetically supported flexible rotors require an accurate model of the system dynamics. Standard identification procedures can generate accurate models, but in general, do not provide the specific value of physical parameters, needed in some applications such as inertial centering. Moreover, some of these identification techniques are not trivial for implementation and may require special signal-processing skills, nonlinear optimization, etc. In the robotics community, a well-established and widely recognized identification procedure can be found for fully-actuated industrial robots. This paper suggests a procedure that makes this identification technique applicable, also to magnetically supported flexible rotors, which are under-actuated systems. The original maximum-likelihood based procedure is modified by introducing both statistics and weights to the covariance matrix of input forces. The procedure is exemplified through numerical simulations, considering an 8-DoF flexible rotor system. The results show that although the system is under-actuated, the modified identification procedure provides satisfactory results, in the sense that all rotor parameters are accurately identified, including the parameters of mass imbalance.

Key words : Model identification, Flexible systems, Rotor dynamics, Active Magnetic Bearings, Weighted least squares, Maximum likelihood

1. Introduction

Flexible rotors, suspended by active magnetic bearings (AMBs) require advanced control algorithms, which in general, demand an accurate dynamical model of the system. Numerical methods such as finite elements are effective for analyzing natural frequencies and mode shapes but not as much effective for accurate prediction of the rotors behavior in the time domain, which is essential for advanced model-based controller design. Model identification based on experiments is the most effective way to obtain an accurate dynamic model for control. The process of identification is based on system excitation by known input forces and measurements of the systems output response. Modal testing is a technique that requires special equipment and it usually cannot be implemented when the rotor is assembled on its supports. In rotating systems, it is preferred to acquire the identification data during operation in order to excite gyroscopic effects which may be dominant in the system dynamic response. Thus, utilizing the AMBs as force exciters is clearly desirable.

Common control approaches for AMB rotor systems, such as, H_∞, μ-synthesis and H_2 (Balini et al, 2011, 2012, Becker et al, 2015, Schittenhelmet et al, 2015, Tang et al, 2015a), depend heavily on the accuracy of the model. As a result, most identification procedures for AMB-rotor systems aim at finding a model which represent the system dynamics accurately, rather than representing its true physical parameters. Balini et al (2010), have utilized a predictor-based-subspace identification algorithm, also known as $\mathrm{PBSID_{OPT}}$, to obtain an initial guess for a nonlinear optimization problem which minimizes the error between the measured and the model frequency response, with respect to a vector of state space model parameters. Katja et al (2010) investigated the suitability of different frequency response functions for identification by a multi-sine excitation. Sun et al (2012) proposed a nonlinear nonparametric identification procedure for AMB systems based on Support Vector Regression. More recent work by Sun et al (2014) presented a frequency-based identification method for the transfer function matrix of an AMB system with a flexible rotor, based on systematic identification of sub-models, which are eventually combined together. Identification of modal parameters has been shown by Zheng et al (2015) and Tang et al (2016), where different optimal controllers where designed for AMB-rotor systems to pass the first bending critical speed.

In many control design problems, physical parameters are important for defining a control objective or constraints. For example, adaptive geometric and inertial centering approaches require knowledge of physical model parameters such as mass, moments of inertia, etc. (Lum et al.,1996, Lum et al., 1998, Levy and Arogeti, 2016). As a result, many of the identification techniques mentioned above are not suitable for control methods, which require knowledge of physical parameters. Furthermore, the above identification methods may be difficult for implementation due to their complexity and the required special signal processing and nonlinear optimization skills. The aim of this paper is to suggest a simple and systematic identification procedure for flexible rotor systems supported by AMBs. Inspired by (Swevers et al, 2007), where a step-by-step model identification procedure for a fully actuated robotic system was described, a modified procedure is suggested here that generalizes the applicability of that method also to under actuated systems. In particular, its application to a flexible rotor system supported by AMBs is investigated.

2. Methodology

The identification procedure in (Swevers et al, 2007) is based on the linear maximum-likelihood estimator (MLE) with periodic band-limited excitation trajectories and measurements of the corresponding joint angular positions and motor torques (of a robotic system). Here, it is assumed that the AMB models are known, the forces are measured (e.g., by Hall effect sensors or by an accurate model of the actuator) and the rotors degrees of freedoms (as described in the model) are fully measured (e.g., by proximity sensors). The periodic AMB excitation forces are designed as a finite Fourier series where all sine and cosine functions have a common fundamental frequency such that the discrete Fourier transform (DFT) does not introduce any leakage errors. This allows averaging data over multiple fundamental periods and removing all frequency components which are not included in the input excitation, by setting their spectrum to zero. The time derivatives are then calculated by means of inverse DFTs and as a result, the AMB forces (u), positions (q), velocities ($\dot{q}$) and accelerations ($\ddot{q}$) are derived with a significant noise reduction and without phase distortions, which are generally caused by standard numerical differentiation and filtering techniques. The systems is then represented in the continuous form, as,

$$\Phi\left(q, \dot{q}, \ddot{q}\right)\theta = u \tag{1}$$

where $\Phi\left(q, \dot{q}, \ddot{q}\right)$ is the observation matrix, θ is a vector containing a minimal set of model parameters to be estimated, and the averaged AMB force measurements are represented by u. Then, the weighted least squares estimator (WLSE) is formulated for M measurement points, in the following way,

$$\min_{\theta}\left(F\theta - b\right)^{T}\Sigma^{-1}\left(F\theta - b\right) \tag{2}$$

such that F is a matrix containing all of the samples of the measured observation matrices $\Phi\left(q\left(k\right), \dot{q}\left(k\right), \ddot{q}\left(k\right)\right)$ and b is a vector containing all force samples $u\left(k\right)$, where in both cases $k = 1..M$. Here, Σ is the weighting matrix which allows the estimator to discriminate between accurate and inaccurate input data (if chosen properly). The batch processing solution of the widely known WLSE problem is given by,

$$\theta_{WLS} = \left(F^{T}\Sigma^{-1}F\right)F^{T}\Sigma^{-1}b \tag{3}$$

Now, by assuming that the observation matrix Φ is almost free of noise, as a result of the above mentioned signal processing procedure, the WLSE may be generalized to a simplified maximum likelihood estimator. This estimator provides unbiased estimates with minimal uncertainty if the weighting matrix Σ is properly assigned with the covariance of the input AMB excitation forces at each sample $k \in \{1..M\}$. However, since an AMB rotor system is usually an under-actuated system (as opposed to most industrial robotic systems), it is impossible to impose external excitation at all given rotor points. In these points, where the external forces are identically zero, the covariance is also theoretically zero, which leads to a singular (non-invertible) covariance matrix Σ. To solve this, we propose to assign a relatively high covariance weights to the inputs of zero force, compared with the calculated covariance weights of the periodic AMB force inputs. For example,

$$\Sigma = diag\left(\Sigma_{AMB}, \Sigma_{free}\right) \tag{4}$$

where $\Sigma_{AMB} = E\left\{\left(u - \tilde{u}\right)\left(u - \tilde{u}\right)^{T}\right\}$ and $\Sigma_{free} = k\Sigma_{AMB}$, such that $k \gg 1$. This will cause the estimator to relay more on data that is causally related to the AMB forces rather than to the zero force inputs. The problem is that k cannot be assigned with an infinite value, and furthermore, due to the inverse of Σ in the MLSE solution, it is not recommended to assign k with terms that are too large, due to numerical considerations. In order to assess the minimal value of k required for achieving satisfactory results, an alternative solution is proposed, as follows. Instead of a 1-step batch processing, the estimation is made recursively by a sequential set of calculations, starting with the set of eqnarrays where all input forces are available,

$$\hat{\theta}_1 = \left(F_1^T \Sigma_1^{-1} F_1\right) F_1^T \Sigma_1^{-1} u_{AMB} \tag{5}$$

and followed by a new set of eqnarrays where the zero force inputs are replaced with estimated inputs of internal forces, depending on the measurements and the estimated parameters from previous steps, i.e.,

$$\hat{\theta} i = \left(F_i^T \Sigma_i^{-1} F_i\right) F_i^T \Sigma_i^{-1} \hat{u}_i \tag{6}$$

where, $\hat{u}_i = \hat{u}_i \left(q, \dot{q}, \ddot{q}, \hat{\theta}_{i-1}, \hat{\theta}_{i-2}, \ldots, \hat{\theta}_1\right)$. In this case, the AMB force inputs are utilized for estimation of several parameters, which are then utilized for propagation of the original AMB forces in order to excite other observation matrices, according to the dynamic model of the rotor (i.e., according to the internal mechanical relation of rotors mass elements). By comparing these two approaches, it is possible to determine if there is a sufficiently small $k \gg 1$, which would lead to satisfactory results in the first approach that is clearly much easy to apply.

3. Numerical Results

An 8 degrees of freedom (DoF) flexible rotor model supported by two AMBs, as illustrated in Fig. 1, was utilized for the numerical results, which are presented in the following. In this model, three masses (m_1, m_2, m_3) are lumped by massless shaft-connections with linear elastic characteristics (k_1, k_2), and different lengths (L_1, L_2). The dynamic bending of the shaft is in two mutually perpendicular lateral planes (xy planes). The AMBs are located at the end masses (m_1, m_3) and modeled as force inputs ($F_{AMB,x1,y1,x2,y2}$). The central mass (m_2) has transverse and polar moments of inertia (I_t, I_p), meaning that it can rotate in the x and y directions (θ_x, θ_y) and excite gyroscopic effects. The central mass also has static and dynamic imbalances (parameterized by ξ, ζ and μ, η, respectively). The dynamic eqnarrays of the 8-DoF model are given by,

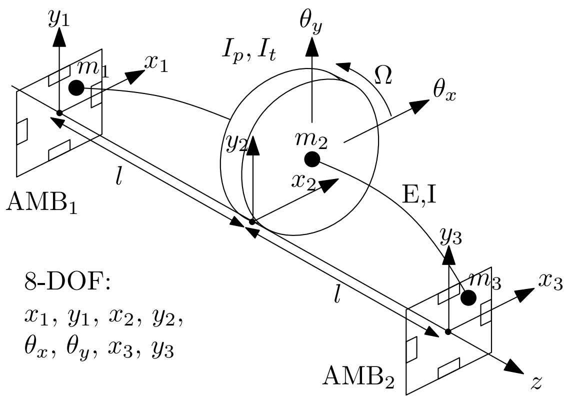

Fig. 1 8-dof model of a flexible rotor-AMB system

$$\begin{aligned}
m_1\ddot{x}_1 &= -k_1\left(x_1 - x_2 + \theta_y L_1\right) + F_{AMB,x1} \\
m_1\ddot{y}_1 &= -k_1\left(y_1 - y_2 - \theta_x L_1\right) + F_{AMB,y1} \\
m_2\ddot{x}_2 &= -k_1\left(x_2 - x_1 - \theta_y L_1\right) - k_2\left(x_2 - x_3 + \theta_y L_2\right) + m_2\Omega^2\left(\xi\cos(\Omega t) - \zeta\sin(\Omega t)\right) \\
m_2\ddot{y}_2 &= -k_1\left(y_2 - y_1 + \theta_x L_1\right) - k_2\left(y_2 - y_3 - \theta_x L_2\right) + m_2\Omega^2\left(\xi\sin(\Omega t) + \cos\zeta(\Omega t)\right) \\
I_t\ddot{\theta}_x &= -k_1 L_1\left(\theta_x L_1 + y_2 - y_1\right) - k_2 L_2\left(\theta_x L_2 + y_3 - y_2\right) - I_p\Omega\dot{\theta}_y + \left(I_t - I_p\right)\Omega^2\left(\mu\cos(\Omega t) - \eta\sin(\Omega t)\right) \\
I_t\ddot{\theta}_y &= -k_1 L_1\left(\theta_y L_1 + x_1 - x_2\right) - k_2 L_2\left(\theta_y L_2 + x_2 - x_3\right) + I_p\Omega\dot{\theta}_x + \left(I_t - I_p\right)\Omega^2\left(\mu\sin(\Omega t) + \eta\cos(\Omega t)\right) \\
m_3\ddot{x}_3 &= -k_2\left(x_3 - x_2 - \theta_y L_2\right) + F_{AMB,x3} \\
m_3\ddot{y}_3 &= -k_2\left(y_3 - y_2 + \theta_x L_2\right) + F_{AMB,y3}
\end{aligned} \tag{7}$$

The physical values in this model were chosen such that the critical speed of the first bending mode is at 10.5 krpm (175Hz). During the simulations, the rotor is spinning at $\Omega = 8400$ rpm (140Hz) and the excitation frequencies are 20,60,100,140,180,220. The AMB excitation forces are given by,

$$F_{AMB,x2} = F_{AMB,y1} = 2F_{AMB,x1} = -2F_{AMB,y2} = \sum_{i=0}^{5} (10\sin(2\pi \cdot (20 + 40 \cdot i)t) + 10\cos(2\pi \cdot (20 + 40 \cdot i)t))$$

Note that the fundamental frequency is 20Hz and the spinning speed is also a multiplication of that frequency by a natural number. Also, the spinning speed was chosen high enough to excite gyroscopic coupling moments. The rest of the physical parameter values are given in table 1, along with the numerical results of the identification process. Both the input excitation forces and the output position measurements where corrupted by white noise. As a result, the excitation forces covariance elements of the kth sample of the ith AMB where calculated according to,

$$\sigma_i(k) = \frac{1}{n}\sum_{j=1}^{n}\left(F_{\mathrm{AMB}i,j}(k) - \bar{F}_{\mathrm{AMB}i}(k)\right)^2 \tag{8}$$

where $F_{\mathrm{AMB}_{i,j}}(k)$ is the kth sample within the jth period of the ith AMB ($AMBx1$, AMB$y1$, etc.), and $\bar{F}_{\mathrm{AMB}_i}(k)$ is the kth sample of the n period mean force, i.e., $\bar{F}_{\mathrm{AMB}i}(k) = \frac{1}{n}\Sigma_{j=1}^{n}\left(F_{\mathrm{AMB}ij}(k)\right)$. For the batch processing, the covariance matrix Σ is then calculated by, $\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, \ldots, \Sigma_m)$ where,

$$\Sigma_k = \mathrm{diag}\left(\sigma_{x1}(k), \sigma_{y1}(k), \alpha\sigma_{max}(k), \ldots, \alpha\sigma_{max}(k), \sigma_{x2}(k), \sigma_{y2}(k)\right) \tag{9}$$

and $\sigma_{max}(k) = \max(\sigma_i(k))$, $\alpha \gg 1$.

The sequential processing may be accomplished here by a two-step procedure. In the first step, the input excitation vector is given only by the AMB forces, without zero elements as in the batch processing. In the second step, the input excitation vector is defined based on the measured data and the estimated parameters from step 1. As a result, the first step is defined by,

$$\underbrace{\begin{bmatrix} \ddot{x}_1 & 0 & x_1 - x_2 & 0 & \theta_y & 0 \\ \ddot{y}_1 & 0 & y_1 - y_2 & 0 & -\theta_x & 0 \\ 0 & \ddot{x}_2 & 0 & x_3 - x_2 & 0 & -\theta_y \\ 0 & \ddot{y}_2 & 0 & y_3 - y_2 & 0 & \theta_x \end{bmatrix}}_{\text{Defines } H_1} \underbrace{\begin{bmatrix} m_1 \\ m_3 \\ k_1 \\ k_2 \\ k_1 L_1 \\ k_2 L_2 \end{bmatrix}}_{\theta_1 \triangleq} = \underbrace{\begin{bmatrix} F_{AMB,x1} \\ F_{AMB,y1} \\ F_{AMB,x3} \\ F_{AMB,y3} \end{bmatrix}}_{\text{Defines } F_1} \tag{10}$$

and the second step is formulated as,

$$\underbrace{\begin{bmatrix} \ddot{x}_2 & 0 & 0 & 0 & -\Omega^2\cos(\Omega t) & \Omega^2\sin(\Omega t) & 0 & 0 \\ \ddot{y}_2 & 0 & 0 & 0 & -\Omega^2\sin(\Omega t) & -\Omega^2\cos(\Omega t) & 0 & 0 \\ 0 & \ddot{\theta}_x & \theta_x & \Omega\dot{\theta}_y & 0 & 0 & -\Omega^2\cos(\Omega t) & \Omega^2\sin(\Omega t) \\ 0 & \ddot{\theta}_y & \theta_y & -I_p\Omega\dot{\theta}_x & 0 & 0 & -\Omega^2\sin(\Omega t) & -\Omega^2\cos(\Omega t) \end{bmatrix}}_{\text{Defines } H_2} \underbrace{\begin{bmatrix} m_2 \\ I_t \\ k_1 L_1^2 + k_2 L_2^2 \\ I_p \\ m_2\xi \\ m_2\zeta \\ \left(I_t - I_p\right)\mu \\ \left(I_t - I_p\right)\eta \end{bmatrix}}_{\theta_2 \triangleq} \tag{11}$$

$$= \underbrace{\begin{bmatrix} -k_1\left(x_2 - x_1 - \theta_y L_1\right) - k_2\left(x_2 - x_3 + \theta_y L_2\right) \\ -k_1(y_2 - y_1 + \theta_x L_1) - k_2(y_2 - y_3 - \theta_x L_2) \\ -k_1 L_1(y_2 - y_1) - k_2 L_2(y_3 - y_2) \\ -k_1 L_1(x_1 - x_2) - k_2 L_2(x_2 - x_3) \end{bmatrix}}_{\text{Defines } F_2}$$

The covariance matrices in each step are then calculated in a similar way to the batch processing, with F_1 and F_2 as input excitation vectors. In either way, it is assumed that the inputs are not correlated, and hence the covariance matrix, Σ, is a diagonal matrix.

In order to reduce signal to noise ratio, all of the output measurements (i.e., displacement and rotation) where averaged over 260 fundamental periods. Then, all of the frequency components which are not included in the multi-sine excitation were filtered by setting their spectrum to zero. This filtering process is presented in Fig. 2. The time derivatives are then calculated by means of an inverse Fourier transform of the filtered spectrum. Finally, the physical parameters are estimated by the batch processing formula (3) and the resulting estimation-error is presented in table 1.

Four different estimators were implemented: Two 1-step MLE with $k = 10$ and $k = 100$, a sequential MLE and a Minimum Least Squares Estimator (MLSE). The results indicate that the MLE with $k = 100$ provides similar performance as the sequential MLE and both provide the best results. The MLE with $k = 10$, which relays more on zero input force data, provides less accurate results than the MLE with $k = 100$. The MLSE, which is calculated with $\Sigma = I$ and does not incorporate the statistics of the input forces, provides the worst result.

Table 1 Numerical results for 8-dof flexible rotor model supported by two AMBs

Parameter	True value	Estimation error [%]				
		1-step MLE, $k = 100$ (1)	1-step MLE, $k = 10$ (2)	Sequential MLE (3)	1-step MLSE, $\Sigma = I$ (4)	Accuracy ranking Highest...Lowest
m_1[kg]	8.3339e-01	0.120	0.177	0.150	0.963	1,3,2,4
m_2[kg]	1.1322e+01	0.219	0.298	0.223	1.245	1,3,2,4
m_3[kg]	8.3339e-01	0.268	0.350	0.287	1.266	1,3,2,4
$I_t\,[\mathrm{kgm}^2]$	2.5425e-02	0.182	0.222	0.157	0.986	3,1,2,4
k_1 [Nm]	8.8357e+05	0.119	0.178	0.139	0.996	1,3,2,4
k_2 [Nm]	8.8357e+05	0.305	0.403	0.311	1.466	1,3,2,4
$k_1L_1\,[\mathrm{Nm}^2]$	2.6507e+05	0.194	0.243	0.214	1.014	1,3,2,4
$k_2L_2\,[\mathrm{Nm}^2]$	2.6507e+05	0.259	0.362	0.259	1.430	1&3,2,4
$k_1L_1^2 + k_2L_2^2\,[\mathrm{Nm}^3]$	1.5904e+05	0.229	0.306	0.220	1.225	3,1,2,4
$I_p\,[\mathrm{kgm}^2]$	4.8275e-02	0.308	0.344	0.357	1.042	1,2,3,4
$\xi m_2\Omega^2$[N]	7.7112e+01	0.031	0.045	0.048	0.956	1,2,3,4
$\zeta m_2\Omega^2$[N]	1.3356e+02	0.352	0.430	0.346	1.372	3,1,2,4
$\mu\,(I_t - I_p)\,\Omega^2$[Nm]	-1.3145e+00	1.389	1.692	1.340	3.868	3,1,2,4
$\eta\,(I_t - I_p)\,\Omega^2$[Nm]	-2.2767e+00	0.984	0.943	0.910	1.202	3,2,1,4

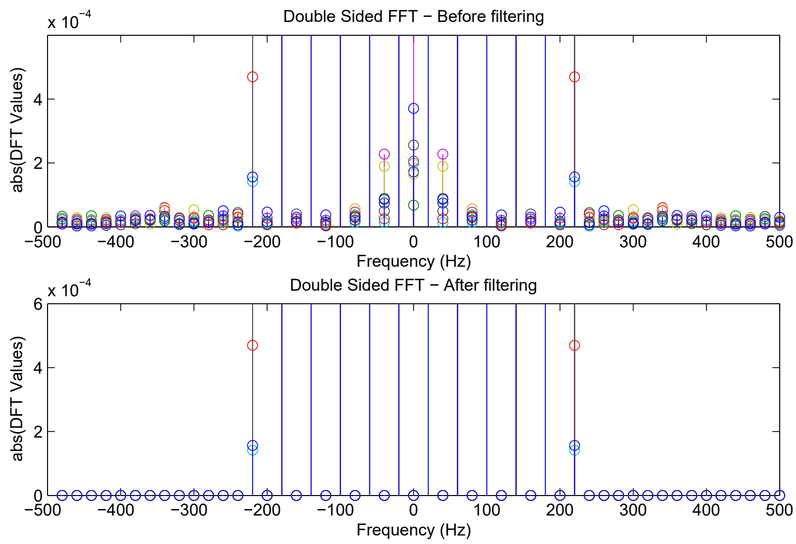

Fig. 2 A zoom on the spectrum - all of the frequency components which are not included in the multisine excitation are filtered by setting their spectrum to zero

4. Implementation Remarks

On the practical side, the suggested identification procedure requires knowledge of, 1) the AMB forces, 2) all DoFs, displacement and rotation, and 3) the spinning position of the rotor. Measuring the forces applied by the AMBs can be based on the AMB model, its known input signals (which are typically, currents) and the measured displacements at the AMB plane (Trumper et al, 1997). An alternative way can be based on magnetic flux density measurements at the AMBs (Forch et al, 1996). The requirement for measurements of all DoFs displacement and rotation variables is another challenge related to practical implementation of the suggested procedure. The number of required sensors increases with the complexity of the model, according to the chosen finite approximation of its continuous deformation. In the current example this means that, except for the AMB displacement sensors (for x_1, y_1, x_3, y_3), the displacement and rotation variables of the central mass, i.e., $x_2, y_2, \theta_x, \theta_y$, should also be measured. Nevertheless, these sensors may be used only in the identification stage, which will allow an accurate model and consequently, enhanced control performance. As to the shaft spinning position, it can be measured by means of a rotary encoder or hall effect sensors.

Once the model parameters are identified, it is recommended to validate their accuracy. This can be done by inspecting the predicted excitation force,

$$\hat{b} = F\hat{\theta}_{WLS} \tag{12}$$

or by analyzing the covariance matrix of the estimated parameter vector of WLSE which is given by,

$$C = F^T \Sigma^{-1} F \tag{13}$$

5. Conclusions

A simple and systematic identification procedure for a flexible rotor, supported by AMBs, was suggested in this paper. The theory of the presented MLE based algorithm is well-established and widely recognized in the robotics community. The original procedure, which is not suitable for under-actuated systems, was modified by introducing both weights and statistics to the covariance matrix of the input forces. An assisting stage, based on an alternative sequential estimation procedure, which does not incorporates data from zero force inputs (i.e., unactuated DoFs), was presented. The results show that although the system is under-actuated, the modified MLE-based identification procedure provides satisfactory results, as all of the rotors parameters were accurately identified, including parameters of the mass imbalance.

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