"Whirl Imposer"--- Proposal for a novel passive magnetic rotor bearing system

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***Abstract*—Passive magnetic bearing systems are enticing in applications where low stiffness is acceptable, but where cost, life-time, power consumption and reliability are critical.**

This article investigates a new proposal for stabilization of permanent magnet (i.e. passive) bearing systems based on rotor precession. There will be a minimal and a maximal rotational speed for stable contact-free levitation.

The feasibility of this new stabilization scheme is investigated with the help of realistic numerical data, a demonstration system is currently in planning.

I. Introduction

Passive magnetic bearing systems [1], i.e. systems with complete contact-free levitation, but without any active control, are very appealing for certain cost-critical applications where low stiffness may be tolerated, but where simplicity, reliability and low power consumption are important.

Earnshaw's theorem [2] states that for a static system perfect (i.e., in all degrees of freedom) passive magnetic levitation is impossible. That is to say, at least one degree of freedom has to be controlled with other methods [3]-[6].

However levitation using solely permanent magnets can be achieved under certain additional conditions. Gyroscopic effects, eddy currents or superconduction can stabilize contact-free levitation. The "Levitron" toy is a proof. It is known that precession can provide a stabilizing mechanism [7].

The design proposed here consists of two passive radial bearings, for which the axial (thrust) stiffness is negative by definition. Then, negative axial stiffness compensation is added at the cost of lowering the radial stiffness. The essential part Whirl Imposer, which induces positive axial stiffness and precession, is introduced as well. A proposal will now be presented for achieving overall stable levitation in a certain range of rotational speed.

II. Concept

A. Rotating system with radial bearings

According to the Earnshaw's theorem

$$2k_r + k_z = 0 \tag{1}$$

where k_r… radial stiffness
k_z…axial stiffness

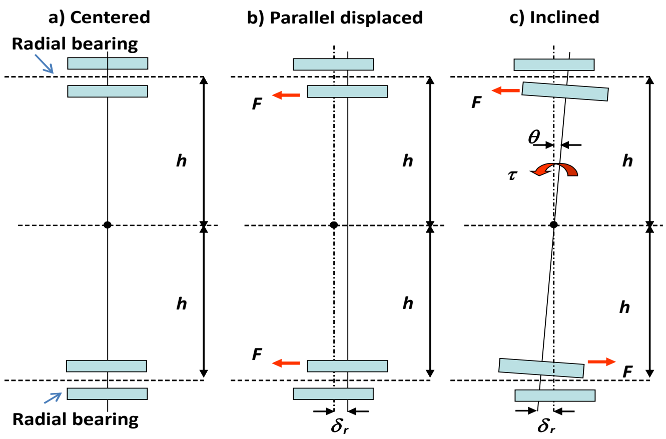

Figure. 1 Radial and conical stiffness of radial PM bearings.

Radial bearings with a positive radial stiffness therefore have a negative axial stiffness:

$$k_z = -2k_r \tag{2}$$

In order to be simple, we will now directly present a realistic numerical example of a potential test system. Consider the situation shown in Fig. 1a:

Both upper and lower bearing is composed of two identical axially magnetized permanent magnet rings in an attractive mode with stiffnesses

$$k_r = 12N/mm, k_z = -24N/mm \tag{3}$$

Let the vertical distance between radial bearings be

$$2h = 2\times 150mm = 300mm \tag{4}$$

The total stiffness of both bearings together for the cylindrical mode will then be

$$k_{rtot} = 24N/mm, k_{ztot} = -48N/mm \tag{5}$$

If the shaft is radially displaced (i.e. parallel to the vertical axis) by a distance δr = 1 mm (Fig. 1b), then the restoring force will be F = 24 N.

If the shaft is inclined so that the upper end is displaced to the right and the lower end to the left by a distance δr = 3 mm (Fig. 1c), which corresponds to an angle

$$tg\theta = \frac{\delta_r}{h} = \frac{3}{150} = 0.02 \Rightarrow \theta = 0.02rad \tag{6}$$

the restoring force at shaft ends will be

$$F_\theta = k_r \times \delta_r = 12N/mm \times 3mm = 36N \quad (7)$$

Forces at both shaft ends produce a restoring torque

$$\tau = F_\theta \times 2h = 36N \times 300mm = 10.8Nm \quad (8)$$

Thus the conical stiffness will be

$$k_\theta = \frac{\tau}{\theta} = \frac{10.8}{0.02} = 540Nm/rad \quad (9)$$

B. *Principle of negative axial stiffness compensation*

Consider now what happens, when a compensation for the negative axial stiffness will be introduced as shown in (Fig. 2a).

The negative axial stiffness compensation system consists of two coaxial axially magnetized permanent magnet cylinders in an attractive mode. The system is located exactly at the middle between radial bearings. The bearing-rotor system's properties with respect to parallel displacement are chosen to be the same as above (Fig. 2b).

The radial stiffness will be increased, say, by a factor 5 (arbitrary chosen), i.e. from

$k_r = 12N/mm$ to $k_r' = 60N/mm$

The new axial stiffness will be

$$k_z' = -120N/mm$$

Total stiffnesses of both radial bearings together

$$k_r' = 120N/mm, k_z' = -240N/mm$$

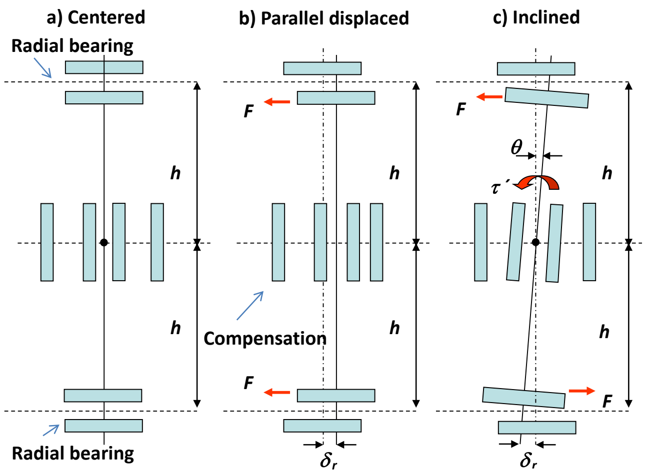

Figure 2. Bearing behavior with negative stiffness compensation.

In order to obtain the same properties as above, the compensating system has to possess the following stiffnesses

$k_{cr}' = -96$ N/mm , $k_{cz}' = 192N/mm$

(Note that, this structure introduces a negative radial and positive axial stiffness). When all corresponding stiffnesses are added together

$$\begin{aligned} k'_{rtot} &= 2 \times 60 - 96 = 24N/mm, \\ k'_{ztot} &= -2 \times 120 + 192 = -48N/mm \end{aligned} \quad (10)$$

which is the same as in the first case.

However, concerning the conical stiffness, with the same inclination, i.e. also with the same displacement distance δr = 3 mm as above (Fig. 2c), the forces at both shaft ends will be

$$F'_\theta = k'_r \times \delta_r = 60N/mm \times 3mm = 180N \quad (11)$$

These forces produce a stabilizing torque

$$\tau' = F'_\theta \times 2h = 180N \times 300mm = 54Nm \quad (12)$$

Hence the conical stiffness will be

$$k'_\theta = \frac{\tau'}{\theta} = \frac{54}{0.02} = 2700Nm/rad \quad (13)$$

which is five times more than without the compensation.

By increasing radial bearing stiffness of the radial bearings x-times, then with the negative axial stiffness compensation the resulting conical stiffness will also be increased by the same factor, on condition that the radial and axial stiffness of the system remain the same.

Note that when the rotor is working in conical mode, there will be much increased restoring force and stabilizing torque, due to the increased radial stiffness of the outer radial bearings. This is why precession (i.e. the conical mode of the rotor) could contribute to stabilizing.

C. *Whirl Imposer*

Whirl imposer consists of a rotor and a stator, which are coaxial discs. There are magnets distributed equally in circumferential direction on both the rotor and the stator. The magnets are oriented so that the forces between the stator and rotor are attractive. Thus the whirl imposer introduces positive axial stiffness. A sample is shown in Fig.3.

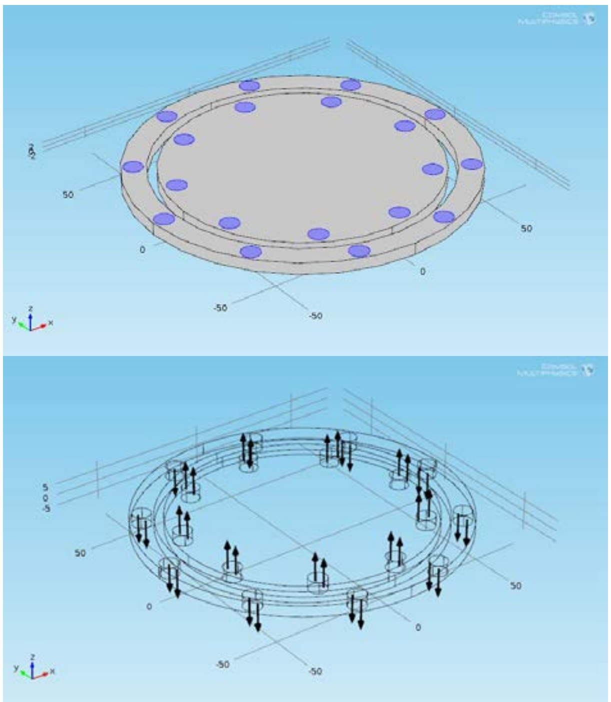

Figure 3. Whirl Imposer with $n_{R}=9$, $n_S = 10$, the magnetization direction of magnets are shown.

Moreover, due to the disc shape of the rotor, gyroscopic stiffness will be high, which does contribute to stability, similar to the gyroscopic effect in the Levitron toy [8].

In principle, precession can easily be introduced in a rotating system by means of a weak unbalance with two additional masses at appropriate rotor locations. This is called torque-free precession, which means there is no external torque applied to the rotor [9]. The forces depend strongly on the rotation speed and rotate synchronously with the rotor. This property may not be desirable for our purpose

However, when the principle of whirl imposer is used, precession will be induced as well, but the forces don't depend on the rotational speed and rotate either in the same or in the opposite sense than the rotor angular speed. In this case, the precession is induced by external forces (magnetic force on the rotor of whirl imposers, F_R). The angular speed of these forces can be made different from the rotor angular speed, depending on numbers of permanent magnet poles on stator and rotor.

The multiplication coefficient κ between the angular speed of the radial force F_R and rotor angular speed is given by the following expression:

$$\kappa = \frac{n_R - n_S}{n_R} \tag{14}$$

where

n_R …number of PMs within the rotor

n_S …number of PMs within the stator

It can be seen that this factor is a positive or negative rational number, hence both forward and backward whirls may be achieved.

Precession angular speed will be inversely proportional to the rotor angular speed in this case, which will induce an upper limit of the rotational speed in order not to obtain a too low precession speed. It is expected that a stable behaviour will only be achieved in a limited range of rotational speeds. This range of rotational speeds will depend on the properties of the whole system.

The PMs may be oriented either radially or axially. The operation of both variants is similar but not exactly the same (PMs are used more efficiently in the second option, as shown in Fig.3). In order to obtain some preliminary feeling about whirl imposer behaviour, both variants have been manufactured (but not yet the whole system).

The number combination of PMs on the whirl imposer can be chosen such as to minimize torque ripple. Essentially, the whirl imposer introduces a positive axial stiffness to the system. It can thus compensate for the residual axial negative stiffness. For a given azimuth angle the Earnshaw theorem (equation (1)) is not satisfied; however, if the torques are integrated over all azimuth angles then the theorem holds as well for the whirl imposer.

The magnitudes of axial stiffness and radial forces depend on the number and size of magnet poles, and on the air gap between rotor and stator. These properties may be chosen for optimal overall system behaviour. The precession producing torque depends on the whirl imposer axial locations with respect to the axial centre of the rotor.

The FEM code COMSOL 4.4 has been used to simulate a realistic model of whirl imposer with n_R= 9 n_S= 10 as shown in Fig.3. Our interest is mainly with the radial force on the rotor. Here κ is expected to be −9 according to equation (14).

TABLE I. PARAMETERS OF A WHIRL IMPOSER

	Parameter	Value
d_r	Outer rotor diameter	112 mm
ds	Outer stator diameter	140 mm
dsi	Inner stator diameter	116 mm
t	Thickness of magnets	5 mm
rm	Radius of magnets	4 mm
mB	Magnetic flux density	1.22 T

Parameters of this whirl imposer are listed in Table I while some results of obtained forces on the rotor are shown in Table II. In this case, the rotor is in the nominal position $(x, y, z)=(0, 0, 0)$ and rotates in the CCW direction.

For azimuth angle 4° and its multiples, two rotor and stator magnets are located at the same radius vector. This is repeated periodically.

TABLE II. RESULTS OF FORCES ON THE ROTOR AT NOMINAL POSITION

φ [°]	F_x [N]	F_y [N]	F_R [N]	φ_F [°]
0	2.27	0.01	2.27	0
1	2.19	-0.38	2.22	-9.79
2	2.13	-0.69	2.24	-17.95
3	2.03	-1.06	2.29	-27.46
4	1.89	-1.30	2.29	-34.52

where:

φ… rotor position azimuth in degrees.

F_x… x-component of rotor force in N.

F_y… y-component of rotor force in N.

φ_F…azimuth of radial force in degrees.

Results show that the force vector magnitude remains always the same and it rotates with a higher speed than the rotor in the CW direction (κ= −9 proved here).

When the rotor is displaced off the nominal position, the force vector magnitude will vary according to rotor position azimuth angle. Then the period 4° will not hold, and turns to 40° depending only on the symmetry of the rotor magnets distribution.

For the radial displacement d_x=2mm, a simulation has also been made in COMSOL.

Forces on the rotor are listed in Table III. The azimuth of the rotor is in the range (0,20°).

TABLE III. RESULTS OF FORCES ON THE ROTOR WHEN DX=2MM

φ[°]	F_x [N]	F_y [N]	F_R [N]	φ_F [°]
0	4.031	0.11	4.032	1.563
2	3.82	-1.116	3.98	-16.29
4	2.96	-2.047	3.60	-34.67
5	2.512	-2.286	3.40	-42.30
6	2.129	-2.487	3.27	-49.44
8	1.054	-2.602	2.81	-67.95
10	0.294	-2.30	2.315	-82.72
10.5	0.051	-2.134	2.135	-88.66
11	-0.066	-2.195	2.196	-91.72

φ[°]	F_x [N]	F_y [N]	F_R [N]	φ_F [°]
11.5	-0.342	-2.013	2.041	-99.65
12	-0.353	-1.87	1.903	-100.69
14	-0.89	-1.484	1.73	-120.95
15	-1.102	-1.178	1.612	-133.09
16	-1.086	-1.00	1.475	-137.39
18	-1.266	-0.443	1.34	-160.70
20	-1.292	-0.02	1.292	-179.12

With the Curve Fitting tool in Matlab, the function of the force vector magnitude could be approximately written as:

$$F_R = 1.433\cos(0.151\varphi + 0.346) + 2.75 \quad (15)$$

while the azimuth of the radial force could be written as a function of the rotor azimuth angle:

$$\varphi_F = -8.94\varphi + 4 \quad (16)$$

Figures of F_R and φ_F are plotted according to the above functions.

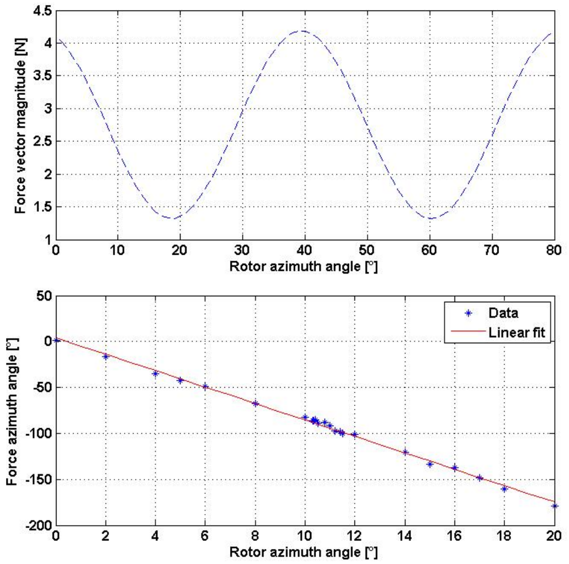

Figure 4. Analysis of radial force on the rotor of Whirl Imposer according to rotor azimuth angle.

For azimuth of the rotor φ=0°, simulation has been made as well. Magnitude of F_R is plotted in Figure 5 for radial displacement varying from 0 to 4.5mm.

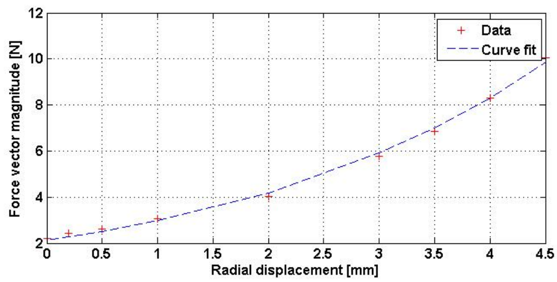

Figure 5. Analysis of radial force on the rotor of Whirl Imposer according to radial displacement.

Function of F_R in this case could be written as following with Curve Fitting:

$$F_R = 2.113e^{0.343d_x} \quad (17)$$

The results obtained with FEM calculations show that the magnitude of force vector on the rotor is no longer a fixed value if the rotor moves off the nominal position, instead, it varies as a sinusoid wave according to the azimuth of the rotor. It varies as an exponential function according to the radial displacement. The azimuth of the radial force is proportional to the azimuth of the rotor with a coefficient κ close to -9 (the value from equation (14)), which shows the relationship between the angular speed of the radial force and the rotor angular speed.

D. A passive magnetic rotor bearing system

Fig.6 shows the bearing system with combination of the negative axial stiffness compensation and two whirl imposers. The whirl imposers compensate the negative axial stiffness and introduce negative radial stiffness, the overall radial stiffness is thus negative.

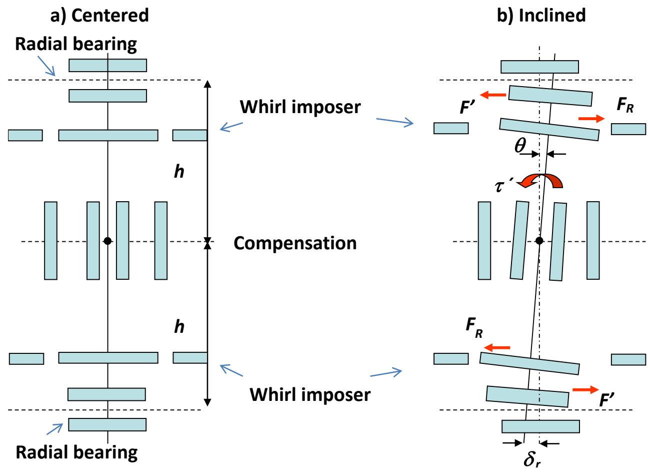

Figure 6. A bearing system combined with the negative axial stiffness compensation and two whirl imposers.

The whirl imposers could be located between the two radial bearings or outside of them. In order to achieve a torque onto the system axis, the azimuthal orientation of whirl imposers will be such that the rotating force of the upper imposer will be shifted with respect to the lower one by 180 degrees. These forces should have the same magnitude but opposite orientation (Fig. 6b). Thus the torque will be exerted onto the shaft about the central point of shaft symmetry (i.e. at the middle of the shaft). The torque produces precession, which always changes its orientation in accordance with the torque vector. Thus an unremitting **conical mode** arises. For a given rotational speed the cone angle (with respect to the vertical axis) remains constant.

If the shaft is inclined (Fig. 6b) in conical mode, then there will be the attractive force from the whirl imposers as well as restoring force due to the radial bearings. Even a small torque due to the whirl imposer is enough for modulating the restoring torque due to the radial bearings, thus the conical mode will be ensured.

In the rotating system, the spin angular momentum L is along the rotation axis, but the torque due to F_R about the centre of the shaft is in a direction perpendicular to the angular momentum. The torque produces a change in L which is perpendicular to L. Such a change causes a change in direction of L but not a change in its size. This motion is the so called torque-induced precession.

For a spinning top (or a gyroscope), the torque is induced by gravity and it rotates at the same speed as precession. In our case, the torque τ_p is induced by F_R. Thus the rotating speed of τ_p is basically proportional to the spinning speed of the rotor with a coefficient κ. Besides, the magnitude of τ_p varies periodically due to spinning of the rotor and varies exponentially with the radial displacement.

Analysis of the rotordynamics of this system is done with the FEM code DYNROT. Some realistic parameters are chosen for a numerical simulation. The radial bearings, the whirl imposer and the negative axial stiffness compensation are as described above.

Assume that the system is axially stable, which means enough axial stiffness has been provided. Gravity can easily be compensated for a vertical rotor, so it is not considered at this moment.

The rotor of the system is modelled with 1-D beam elements, while the rotating parts of the two whirl imposers are modelled with rigid mass elements.

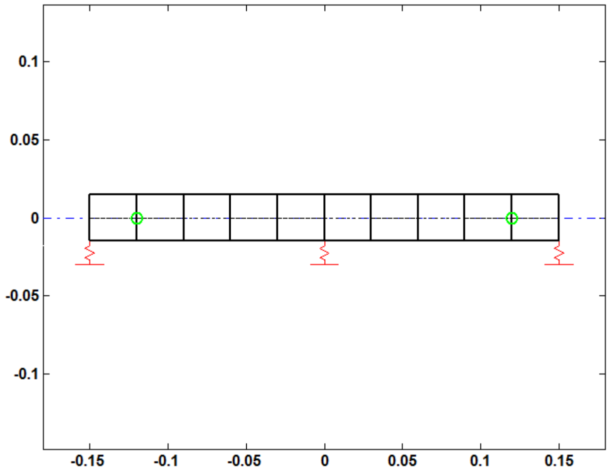

Figure 7. Modelling of the rotor system in DYNROT.

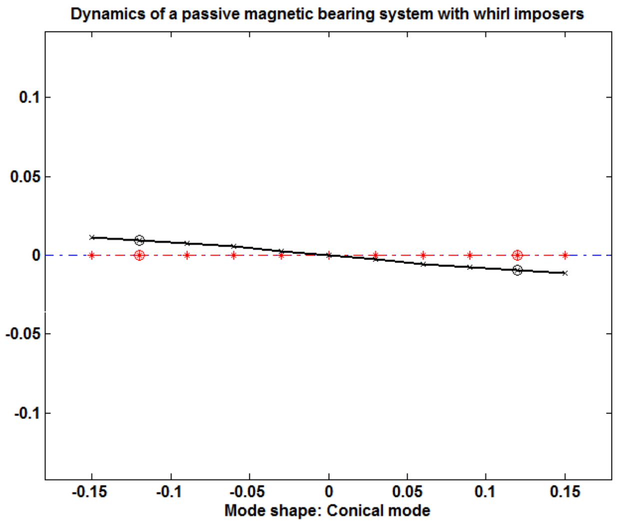

Figure 8. The expected conical mode of the system.

Figure 8 shows the desired conical mode.

The radial forces due to whirl imposers vary not only with radial displacement, but also with the azimuth of the rotor. This kind of force is not easy to handle in commercial FEM codes which assume rotational symmetry.

As shown in Figure 4, F_R varies in a function of the radial displacement. Thus, for simplicity, the maximum and minimum values are used in a first analysis for prediction of approximate dynamic behaviour. A more complete simulation is underway.

III. Conclusion

This paper introduces a novel design of a passive magnetic bearing system using solely permanent magnets. On the base of a rotating system equipped with radial bearings, two subsystems have been designed: Negative axial stiffness compensation and Whirl Imposer. Conical stiffness can thus be increased without modifying radial and axial stiffness of the system. The new component proposed here is the "whirl imposer", consisting of a number of stator and rotor permanent magnets, which induce precession and at the same time compensate for the residual axial negative stiffness.

Although the system is radially unstable at standstill, a conical mode is expected to stabilize the system in a certain working speed range.

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