# Controller Structure for Magnetic Bearing for Achieving both Unbalance Suppression and Stabilization


Yuichi Ariga[a], Yohei Saitoh[b], Keiji Watanabe[a]

[a] Faculty of Engineering, Yamagata University, 4-3-16 Jonan, Yonezawa, Japan, y_ariga@yz.yamagata-u.ac.jp
[b] Matsue College of Technology, 14-4 Nishiikuma-cho, Matsue, Japan



***Abstract*— This study proposes a control system in which the controller structure is divided in consideration of the unstable zeros of the controller. This controller can both inhibit a sine wave disturbance intended to produce unbalanced vibrations and stabilize the system with desired bearing stiffness. The effectiveness of the proposed method is verified through a simulation.**


## INTRODUCTION

One of the features of active magnetic bearings is the ability to control unbalances. Already, many studies have focused on unbalance compensation methods [1]. Because "generalized notch filters" [2] can be used within rigid body critical speeds, these are very effective methods. However, they require the measurement of the closed-loop sensitivity function after designing the stabilized controller.

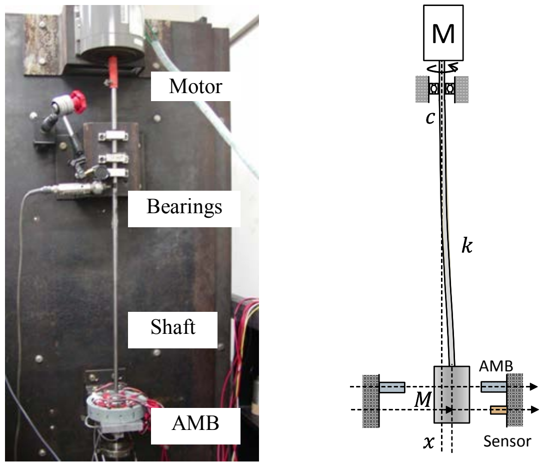
At the same time, many control methods have also been proposed to solve various control specifications, such as $H_\infty$ control [3][4] and linear matrix inequality (LMI) [5]. However, because a determination of a weight function which satisfies simultaneously various control specification is difficult, more advanced techniques are required to design a good controller. To solve this problem, model bridge control (MBC), which is based on internal model control [6], is proposed [7][8]. MBC is a modification of the generalized stabilizer. It has a control structure in which adjustable models bridge over the gaps between the external signals and the desired outputs. MBC has been used to configure an unbalance control system for a magnetic bearing, and this was verified through a simulation [9]. The simulation results show that the proposed control method makes it possible to pass the rigid body critical speed safely, control housing vibrations, and avoid amplifier saturation. However, it is necessary to design a high supported stiffness controller for inhibiting a large overshoot of displacement for levitating. This is because the zeros of the unbalanced disturbance compensator closer to the origin cause a large overshoot.

This study proposes a controller structure that is divided in consideration of the unstable zeros of the controller. By this method, without changing the performance of the sine wave disturbance suppression, a response for levitating and good support desired rigidity are obtained.

## CONTROLLED OBJECT

The configuration of a vertical-type rotor system supported by magnetic bearings is shown in Figure 1. The nomenclature used in this paper is shown in Table 1.

A controlled object is assumed to be a one-degree-of-freedom system under decentralized control. This system is unstable when a constant attractive force produced by a bias current is greater than the spring force of a shaft. The coil current is considered a control input and mass displacement, the output. The state equation is as given below.




(a) Experimental equipment (b) Model

Figure 1. Controlled object

TABLE I. PARAMETER DEFINITION

| | Parameter | Value |
|---|---|---|
| $M$ | Equivalent mass | 1.47 kg |
| $F_0$ | Constant attractive force | 3.5 N |
| $X_0$ | Air gap | 2.0 mm |
| $I_0$ | Bias current | 0.35 A |
| $k$ | Spring constant | 1.28 kN/m |
| $d_c$ | Damping constant | 0.016 μNs/m |

$$\dot{X}_g = A_g X_g + B_g u$$
$$= \begin{bmatrix} 0 & 1 \\ \left(4\frac{F_0}{X_0} - k\right)\Big/M & d_c\!\!\Big/\!M \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 2\frac{F_0}{I_0}\frac{1}{M} \end{bmatrix} u, \tag{1}$$

$$y = C_g X_g = [1 \quad 0] X_g \tag{2}$$

The transfer function of a controlled system is expressed as follows.

$$G(s) = C_g(sI - A_g)B_g = \frac{k}{(s-\alpha)(s+\beta)} \tag{3}$$

where $\alpha$ is an unstable pole and $\beta$, a stable pole.

## STABILIZING CONTROLLER, AND ITS STRUCTURE

### A. *Basic-type controller*

A feedback control system using a controller $G_c(s)$ is formed as shown in Figure 2. $y(s)$ is the output, $r(s) = 1/s$ is a step reference input, and $d(s)$ is a sine wave disturbance expressed as follows.

$$d(s) = \omega/(s^2 + \omega^2) \tag{4}$$

A controller for stabilizing the system of Figure 2 is generally represented by the following internal model parameterization (IMP).

$$G_c(s) = \frac{P(s)}{1 - G_c(s)P(s)} \tag{5}$$

Here, $P(s)$ is a free parameter that is expressed as follows.

$$P(s) = \frac{(s-\alpha)(s+\beta)}{k(1+\tau s)^2}\frac{a + bs + cs + ds^2}{(1+\tau s)^3} \tag{6}$$

$\tau$ is the time constant of the closed loop system, and the response of this system is determined by its value. If its value is small, quick levitation response and high bearing stiffness can be obtained. Contrarily, if a larger value is set, slow levitation response and low bearing stiffness are obtained. The coefficients a, b, c and d are calculated by a simultaneous equation based on the following conditions of an internal stabilization (See APPENDIX).

$$1 - G(s)P(s)\big|_{s=\alpha} \tag{7}$$

$$1 - G(s)P(s)\big|_{s=0} \tag{8}$$

$$1 - G(s)P(s)\big|_{s=\omega j} \tag{9}$$

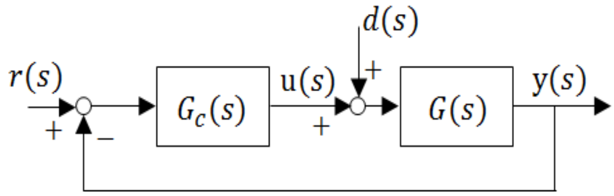



Figure 2. Feedback control system

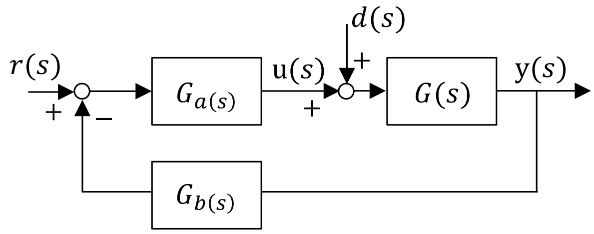



Figure 3. Separated-type feedback control system

$P(s)$ as calculated by the above method has zeros owing to a sine wave disturbance. If these zeros are close to the origin, they worsen a step response. Therefore, it had to be set to a small value of $\tau$ for inhibiting a large overshoot of displacement levitating. This constraint causes that cannot be set only high bearing stiffness.

### B. *Separated-type controller*

The structure of the controller $G_c(s)$ obtained in the previous section can be changed to a distributed structure, as shown in Figure 3. $P(s)$ for this structure is defined as follows:

$$P(s) = P_a(s)P_b(s)$$
$$P_a(s) = \frac{(s-\alpha)(s+\beta)}{k(1+\tau s)^2} \tag{10}$$
$$P_b(s) = \frac{a + bs + cs + ds^2}{(1+\tau s)^3}$$

$P_b(s)$ contains unstable zeros of $G_c(s)$. Based on equation (10), the controller $G_c(s)$ is divided into two elements.

$$G_c(s) = G_a(s)G_b(s)$$
$$G_a(s) = \frac{P_a(s)}{1 - G_c(s)P(s)}, \quad G_b(s) = P_b(s) \tag{11}$$

Thus, the step response can be improved without changing the disturbance suppression performance.

## SIMULATION RESULTS

The controllers are designed to suppress a sine wave disturbance of 5Hz, and their control performance is verified through a simulation.

Basic-type controllers were designed for two conditions—a low bearing stiffness of τ = 0.05 and a high bearing stiffness of τ = 0.02. Figure 4 shows a comparison of the results for the levitation response. A large overshoot occurs if τ is large. This causes sustained vibrations between the touchdown bearing in the experiment.

A separated-type controller is configured to separate the basic-type controller designed based on Equations 10 and 11. Figure 5 shows a comparison of the results for the levitation response. Regardless of the value of τ, a separated-type controller shows smooth levitation without overshoot.

Figure 6 shows the verification results of disturbance suppression performance of each type of controller. The controllers are designed with τ = 0.05. A sine wave disturbance is input from 4.0 [s] at the place that has levitated stably. Both controllers suppress the disturbance with identical performance.

## CONCLUSION

In this study, the effectiveness of the controller structure divided in consideration of the unstable zeros of the controller was shown. The future task is to experimentally verify the effectiveness of the proposed method in a two-degree-of-freedom system.

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## APPENDIX: CONTROLLER DESIGN METHOD

From Equations 5 and 6

$$1-G(s)P(s)$$
$$=\frac{(1-a)+(5\tau-b)s+(10\tau^2-c)s^2+(10\tau^3-c)s^3+5\tau^4 s^4+\tau^5 s^5}{(1+\tau s)^5} \quad \text{(A1)}$$

Than a condition of Equation 8

$$1-a=0 \quad \therefore a=1 \quad \text{(A2)}$$

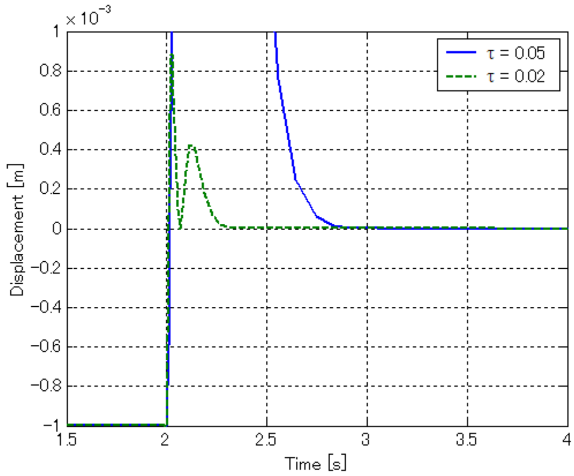


Figure 6. Levitation responses (basic-type controller)

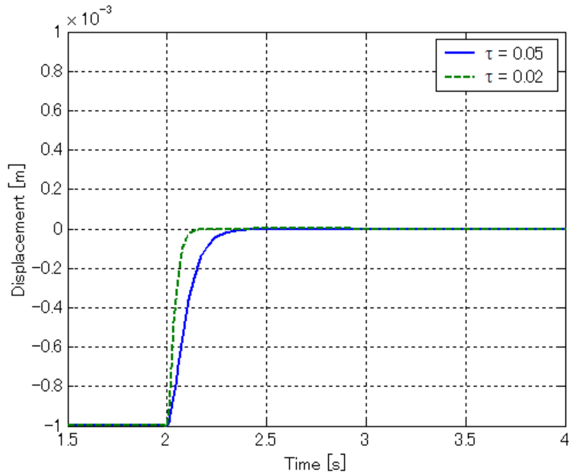


Figure 4. Levitation responses (separated-type contrller)

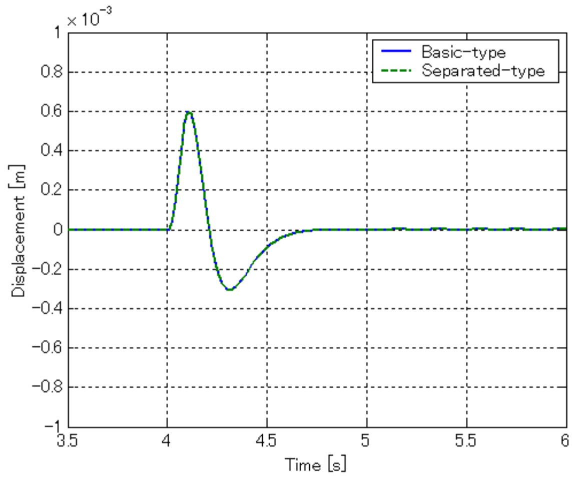


Figure 5. Responses of sine wave disturbance (τ = 0.05)

From Equation 7, which is a condition for the unstable pole α

$$1-G(s)P(s)\Big|_{s=\alpha} = \frac{(5\tau-b)\alpha+(10\tau^2-c)\alpha^2+(10\tau^3-c)\alpha^3+5\tau^4\alpha^4+\tau^5\alpha^5}{(1+\tau\alpha)^5} \tag{A3}$$

For the above equation becomes zero

$$(5\tau-b)\alpha+(10\tau^2-c)\alpha^2 +(10\tau^3-d)\alpha^3+5\tau^4\alpha^4+\tau^5\alpha^5=0$$

Thus

$$b+c\alpha+d\alpha^2 =5\tau+10\tau^2\alpha+10\tau^3\alpha^2+5\tau^4\alpha^3+\tau^5\alpha^4 \tag{A4}$$

Similarly, for an angular frequency of the agitation,

$$1-G(s)P(s)\Big|_{s=\omega j} = \frac{(5\tau-b)\omega j+(10\tau^2-c)\omega^2+(10\tau^3-c)\omega^3 j+5\tau^4\omega^4+\tau^5\omega^5 j}{(1+\tau\omega j)^5} \tag{A5}$$

From the condition that Equation A5 becomes zero, the following equations are provided.

$$-(10\tau^2-c)\omega^2+5\tau^4\omega^4=0 \tag{A6}$$

$$(5\tau-b)\omega-(10\tau^3-d)\omega^3+\tau^5\omega^5=0 \tag{A7}$$

From Equations A6

$$c=10\tau^2-5\tau^4\omega^2 \tag{A8}$$

The following equation is obtained and rearranging the equation A7.

$$b-\omega^2 d=5\tau-10\tau^3\omega^2+\tau^5\omega^4 \tag{A9}$$

Substituting Equation A8 in Equation A4

$$b+\alpha^2 d=5\tau+10\tau^3\alpha^2+5\tau^4\alpha(\omega^2+\alpha^2)+\tau^5\alpha^4 \tag{A10}$$

Subtracting Equation A9 from Equation A10.

$$(\omega^2+\alpha^2)d=10\tau^3(\omega^2+\alpha^2)+5\tau^4\alpha(\omega^2+\alpha^2) -\tau^5(\omega^2+\alpha^2)(\omega^2-\alpha^2)$$

$$\therefore d=10\tau^3+5\tau^4\alpha-\tau^5(\omega^2-\alpha^2) \tag{A11}$$

Substituting equation A11 in equation A9

$$b-\omega^2\left\{10\tau^3+5\tau^4\alpha-\tau^5(\omega^2-\alpha^2)\right\} =5\tau-10\tau^3\omega^2+\tau^5\omega^4$$

$$\therefore b=5\tau+5\tau^4\alpha\omega^2+\tau^5\alpha^2\omega^2 \tag{A12}$$

In the above, all coefficients of the free parameter are determined. The controller $G_c$ is obtained as the following form in the end.

$$G_c(s)=\frac{P(s)}{1-G_c(s)P(s)} =\frac{(s-\alpha)(s+\beta)(a+bs+cs^2+ds^3)}{k\left\{(5\tau-b)s+(10\tau^2-c)s^2+(10\tau^3-d)s^3+5\tau^4s^4+\tau^5s^5\right\}} \tag{A13}$$

The denominator becomes the following equation.

$$\begin{aligned}
&k\left\{(5\tau-b)s+(10\tau^2-c)s^2+(10\tau^3-d)s^3+5\tau^4s^4+\tau^5s^5\right\}\\
&=k\left[-\tau^4\omega^2\alpha(5+\tau\alpha)s+5\tau^4\omega^2s^2-5\tau^4\alpha s^3 -\tau^5(\alpha^2-\omega^2)s^3+5\tau^4s^4+\tau^5s^5\right]\\
&=k\tau^4s\left[-\omega^2\alpha(5+\tau\alpha)+5\omega^2s-5\alpha s^2 -\tau(\alpha^2-\omega^2)s^2+5s^3+\tau s^4\right]\\
&=k\tau^4s\left[5\omega^2(s-\alpha)+5s^2(s-\alpha) +\tau\left\{s^4+(\omega^2-\alpha^2)s^2-\omega^2\alpha^2\right\}\right]\\
&=k\tau^4s\left[5\omega^2(s-\alpha)+5s^2(s-\alpha)+\tau\left\{(s^2-\alpha^2)(s^2+\omega^2)\right\}\right]\\
&=k\tau^4s(s-\alpha)\left[5(s^2+\omega^2)+\tau(s+\alpha)(s^2+\omega^2)\right]
\end{aligned} \tag{A14}$$

Rearranging Equation A13 using Equation A14, the following controller $G_c$ is obtained.

$$G_c(s)=\frac{(s-\alpha)(s+\beta)(a+bs+cs^2+ds^3)}{k\tau^4s(s^2+\omega^2)\{\tau(s+\alpha)+5\}} \tag{A15}$$