Analysis of Nonlinear Behavior in Power Amplifier of AMB System

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***Abstract*—In order to make the AMB system satisfied the high energy-efficiency demands, the nonlinear behavior of H-bridge inverter, which is a power amplifier for AMB system, is studied in this paper. The discrete mapping model of the H-bridge inverter is established with some improvement using stroboscopic method. Based on this model, the dynamic evolution process of the inverter from stable state to chaos is directly exhibited, and the stable variation ranges of feedback gain and coil inductances are determined. Furthermore, a new simulation nonlinear model for an AMB system based on nonlinear H-Bridge inverter amplifier is set up, the influence of H-bridge inverter' s nonlinear behavior on the entire AMB system with two alternate change parameters is investigated. The simulation and experimental results demonstrate the existence of chaos phenomenon in AMB system and provide a stable range for AMB coil and control system design. The new AMB system model may have some benefits to the study of chaos control method for avoiding bifurcation and chaos phenomenon in AMB system.**

***Index Terms*—H-bridge inverter, power amplifier, discrete mapping model, chaos phenomenon, stable variation range, nonlinear model, chaos control method**

I. INTRODUCTION

Nowadays, in order to acquire high energy-efficient drives, the development trend of rotating machinery is high speed, which makes active magnetic bearings (AMB) a crucial element because of negligible friction resulting from non-contact suspension capability and the ability of unbalance vibration control. AMB system has been used in many industrial fields such as vacuum and cleanroom systems, medical devices and turbo-machinery [1]. However, new problems emerge as the AMB system applied in certain applications such as compressor and air blower etc. Sometimes the rotor's volume and weight of the compressor or air blower are very considerable, which means the rotor exists large unbalance vibration while rotating. In order to suppress large unbalance vibration, AMB system demands high-power electromagnetic actuator, which needs large and rapidly changing current. As a result the power amplifier of the AMB system operates in the nonlinear section more frequently. The nonlinear problems of AMB system have attracted more and more attention in the last decade. Much work has been done in several aspects such as analyzing nonlinear phenomena in the AMB system [2-3], establishing nonlinear models of the AMB system [4-5] and designing control method for nonlinear model [6-8], which brought great improvement to AMB system. Recently, the study of nonlinear problems of AMB system is focusing on the instability of the AMB system caused by time delays of digital controllers. Zheng Kai and Yu Lie proposed a robust fuzzy logic-base control scheme for a nonlinear magnetic bearing system that is subject to time delays in feedback loop [9]. Yuan Ren and Jiancheng Fang present an ingenious design, proportional-differential current-sensing resistor networks, to solve the common problems associated with the digital control delay [10].

With the rapid development of the study of nonlinear circuits, another nonlinear issue in AMB system attracts our attention. H-bridge inverter, as the power amplifier for active magnetic bearing system, is considered as a proportional component for most of time. However, as a kind of typical piecewise smooth circuit [11], the H-bridge inverter has many nonlinear features, which is ignored by most of AMB models. The study of the nonlinear behavior of H-Bridge inverter starts from 1997. Robert et al. introduced the bifurcation and chaos in the proportion controlled circuit for the first time [12-16], and they prove that the time-delayed feedback control (TDFC) method is an effective method to stabilize chaotic H-Bridge inverter. Hiroyuki Asahara continues studying on the chaotic evolution process of the H - bridge inverter with a width coefficient range [17-18]. Wang Xue-Mei points out that the fundamental cause of chaos in H-bridge inverter is the boundedness of duty cycle [19]. The anterior researchers have given a quiet accurate description and study method of the H-bridge inverter's nonlinear behavior. However, all the work above considered H-bridge inverter as an independent system rather than a component of the entire AMB system.

In this paper, we establish a discrete mapping model of the H-bridge inverter with some improvements, which makes the model more suitable for a genuine AMB power amplifier circuit. Furthermore, we analyze the H-bridge inverter's nonlinear behavior, not only as an independent system, but also as a component of the entire AMB system.

The brief is organized as follows. In Section II, we establish the discrete mapping model of the H-bridge inverter. The stable variation range of feedback gain and coil inductance is analyzed in Section III. In Section IV, we set up a new nonlinear model for the AMB system based on nonlinear H-bridge inverter amplifier and analyze nonlinear behavior with two alternate change parameters. Finally, we provide an experimental result of static state suspension with appropriate feedback gain and unsuitable feedback gain.

II. MODELING OF H-BRIDGE INVERTER

The idea circuit of the H-bridge inverter and its load is shown in Fig. 1. The circuit consists of a voltage source (E), four switch transistors ($VT1 \sim VT4$), an inductor (L) and a resistor (R). It is running in a fixed switching period (T). The circuit working principle can be described as follows: At the beginning of each period the current is sampled and compared to the reference current to give a pulse width modulated (PWM) signal, which controls the duty cycle of that period. The reference current is given by the AMB system controller, which usually contains displacement of the rotor loop and current of the coil loop in a genuine AMB system. In this section, we consider H-bridge inverter as an independent system; the reference current is given in two cases, direct current and sinusoidal wave, corresponding to static state suspension and rotary state suspension.

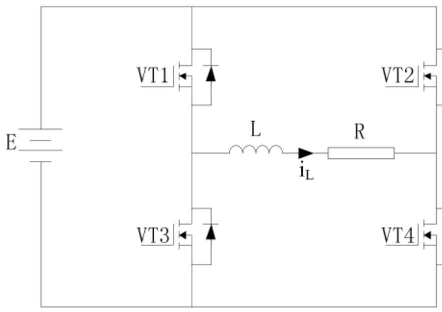

Figure 1. Ideal H-bridge inverter and its load

Depending on the state of switch transistors, the H-bridge inverter has two working states, denoted as S_1 and S_2. S_1 represents current increasing state, while S_2 represents current decreasing state. When the circuit is working on S_1, PWM signal is "high" and two switch transistors ($VT1$ and $VT4$) are turned on. When the circuit is working in S_2, PWM signal is "low", and the other two switch transistors ($VT2$ and $VT3$) are turned on. In both states, $VT1$ and $VT4$ are always in complementary states to $VT2$ and $VT3$. The state variable is the inductor current i_L. The relationship of $VT1$ and i_L is shown in Fig. 2. (As $VT4$ is operating same as $VT1$, $VT2$ and $VT3$ are operating opposite to $VT1$, the relationships between $VT2$, $VT3$, $VT4$ and i_L are no longer need to exhibit.) During one period T the H-bridge inverter involves a sequence of three switch states $S_1 - S_2 - S_1$. ($t1$ denotes the first switching time, $t2$ denotes the second switching time.) The duty cycle d_n is the proportion of time duration for S_1 to the switching period. It is obvious that d_n must take value between zero and one. (The subscript n denotes the values at the beginning of the nth switching period.) The inductor current's dynamics can be described by

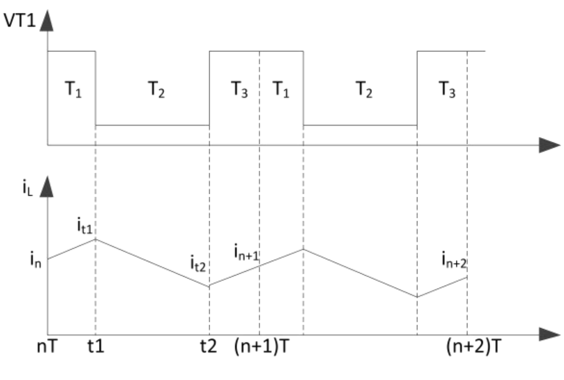

Figure 2. The relationship of $VT1$ and i_L

$$S_1 : \frac{di_L}{dt} = -\frac{Ri_L}{L} + \frac{E}{L} \tag{1}$$

$$S_2 : \frac{di_L}{dt} = -\frac{Ri_L}{L} - \frac{E}{L} \tag{2}$$

The inductor current can be described by the following equations according to Fig. 2, Eq. (1) and Eq. (2).

For $nT < t < t1$, state S_1

$$i_{t1} = (i_{t2} - \frac{E}{R})e^{-\frac{T1}{\tau}} + \frac{E}{R} \tag{3}$$

$$T1 = t1 - nT \tag{4}$$

For $t1 < t < t2$, state S_2

$$i_{t2} = (i_{t1} + \frac{E}{R})e^{-\frac{T2}{\tau}} - \frac{E}{R} \tag{5}$$

$$T2 = t2 - t1 \tag{6}$$

For $t2 < t < (n+1)T$, state S_1

$$i_{n+1} = (i_{t2} - \frac{E}{R})e^{-\frac{T3}{\tau}} + \frac{E}{R} \tag{7}$$

$$T3 = T - T1 - T2 = T1 \tag{8}$$

$$d_n = \frac{T1 + T3}{T} \tag{9}$$

By solving the corresponding equation, i_{n+1} can be expressed in terms of i_n and d_n by

$$i_{n+1} = i_n e^{-\frac{T}{\tau}} + 2\frac{E}{R} e^{-\frac{-T}{2\tau}} [2\sinh(\frac{d_n T}{2\tau}) - \sinh(\frac{T}{2\tau})] \tag{10}$$

where $\tau = \frac{L}{R}$. Then, we have to find out the feedback relationship that connects the duty cycle to the state variable. As the duty cycle is computed by the controller of AMB system, we need to know something about the AMB system's features. A typical single channel schematic diagram of the decentralized controlled AMB system is shown in Fig. 3. A contact-less position sensor measures the displacement x between rotor and AMB, and then feeds this information into the controller. The controller sends out a desired current signal i_{ref} to the electromagnetic actuator which transforms this signal in the electromagnetic coil to generate the desired magnet force. As a result the AMB system contains two feedback loops, the displacement loop and the current loop. In this section we focus on the current loop. The current loop contains power amplifier (H-bridge inverter), electromagnetic

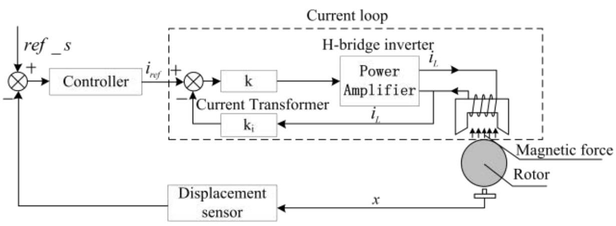

Figure 3. Single channel schematic diagram of decentralized controlled AMB system

coil, current transformer and feedback gain k. The duty cycle derived from Fig. 3 is written as

$$d_n = \frac{1}{2} + k(i_{ref} - k_i i_n) \tag{11}$$

where k_i is the proportional coefficient of current transformer, k is the feedback gain of current loop.

III. Simulation of Current Loop

The simulations of current loop are based on the equations in Section II. Firstly, for each set of parameter values, time-domain cycle-by-cycle waveforms are generated by solving Eq. (10) and Eq. (11) in a sub-interval of time. Secondly, we capture the steady-state time-domain waveforms of the state variable (inductor current) under each set of parameter values to obtain bifurcation diagram. The primary circuit parameters used in simulations are shown in TABLE I. There are two notes of caution here. Firstly, duty cycle d_n is strained between 0.1 and 0.9, which is necessary in a real AMB system. Secondly, the carrier frequency F is the three times of the reciprocal of switching period, which is applied in the practical system.

TABLE I. Parameters used in simulations of the current loop

	Parameter	Value
T	Switching Period	0.15 ms
E	Input Voltage	90 V
L	Coil Inductance	10 mH
R	Coil Resistance	2 Ω
F	Carrier frequency	20KHz
d_n	Duty cycle	$0.1 < d_n < 0.9$
k	Feedback gain	0.8
k_i	Proportional coefficient	0.9

A. Bifurcations with amplification coefficient k

Considering the feedback gain k as the bifurcation parameter, we study the dynamics of the state variable i_L. The obtained bifurcation diagram is shown in Fig. 4 and Fig. 5, corresponding to static state suspension and rotary state suspension respectively. The diagrams summarize the values of i_L as k increases in the two suspension conditions. From the diagrams, we observe that the current loop subsystem exhibits a T periodic orbit when k is relatively small. This orbit exists whatever k is, but it is stable when $k < 1.48$. When $k > 1.48$, a $2T$ periodic orbit appears and then chaos is observed. The change of dynamical behavior is due to a nonstandard bifurcation known as border-collision

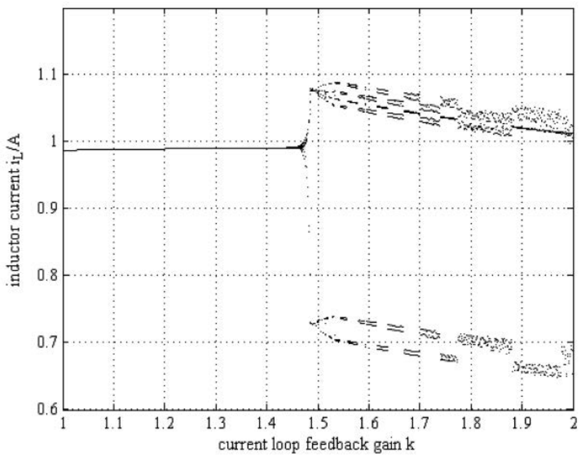

Figure 4. Bifurcation diagram of the coil current and feedback gain with direct current reference

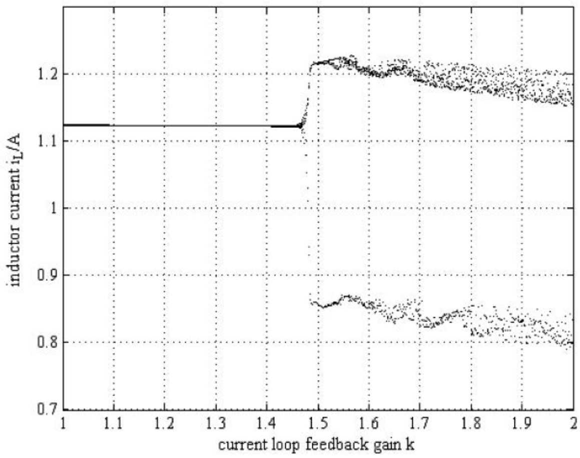

Figure 5. Bifurcation diagram of the coil current and feedback gain with sinusoidal wave current reference

bifurcation, as studied in [12] and [18]. As a result, in order to stabilize the current loop we need to choose feedback gain in the stable range from 0 to 1.48.

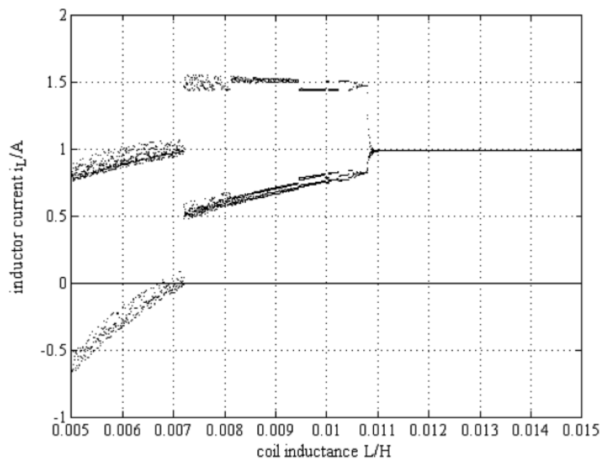

Figure 6. Bifurcation diagram of the coil current and coil inductance with direct current reference

B. Bifurcations with coil inductance L

Considering coil inductance L as the bifurcation parameter, we obtain bifurcation diagrams which are shown in Fig. 6 and Fig. 7. The diagrams show that a relatively small coil inductance L will cause bifurcation. The bifurcation is observed when L is about 11mH with parameters in Table I. Similarly, we have the stable range for L, which is bigger than 11mH.

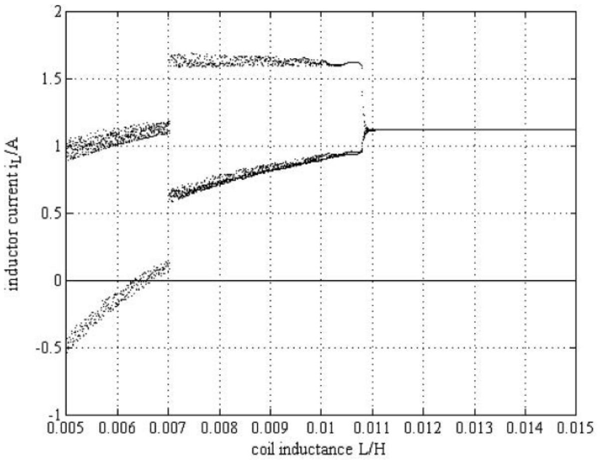

Figure 7. Bifurcation diagram of the coil current and coil inductance with sinusoidal wave current reference

In this section, we obtained bifurcation diagrams of i_L with k and L, and parameter stable range. The simulation result may be helpful for designing AMB's control and physical parameters. But all the simulations and analyses above consider the current loop as an independent module. In the next section we analyze the H-bridge inverter as a component of the entire AMB system.

IV. Simulations of AMB System

The simulations of AMB system are based on the Simulink Module of Matlab. Firstly, we established AMB system model based on nonlinear H-bridge inverter amplifier as shown in Fig. 8. Secondly, with each set of parameter values, time-domain cycle-by-cycle waveforms are generated by running the AMB system nonlinear model. Thirdly, we capture the steady-state time-domain waveforms of the state variable (inductor current) under each set of parameter values to obtain a bifurcation diagram.

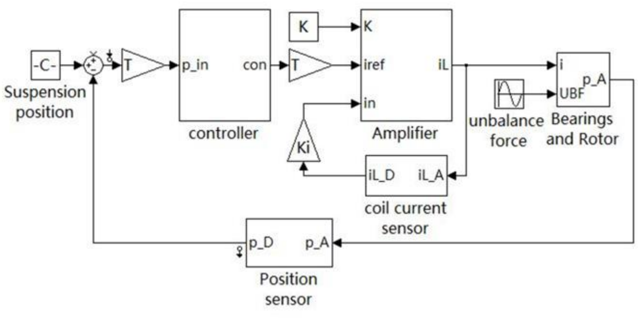

Figure 8. AMB system nonlinear model

Once the AMB system is assembled, the carrier frequency, bus voltage and coil inductance are already determined, we can't adjust these parameters while the AMB system is operating. Therefore, we pay more attentions to the adjustable parameters while the system is running, which is the proportional coefficient of current transformer k_i and the feedback gain of the current loop k. Fig. 9 is the bifurcation diagram of the coil current with proportional coefficient and current transformer adjusting at the same time while the AMB system is working on the rotating state. The primary parameters used in our simulations are same as TABLE I. The simulation result shows that the bifurcation point of i_L with changing k decreases while k_i increases. The stable zone for feedback gain k and current transformer k_i is achieved. Fig. 10 is the contrast of coil current with stable parameters and unstable parameters. We can notice that unstable parameters caused oscillation of coil current is obvious in Fig. 10(b). The current oscillation caused by the nonlinearity of H-bridge inverter is the performance of bifurcation behavior in the time-domain.

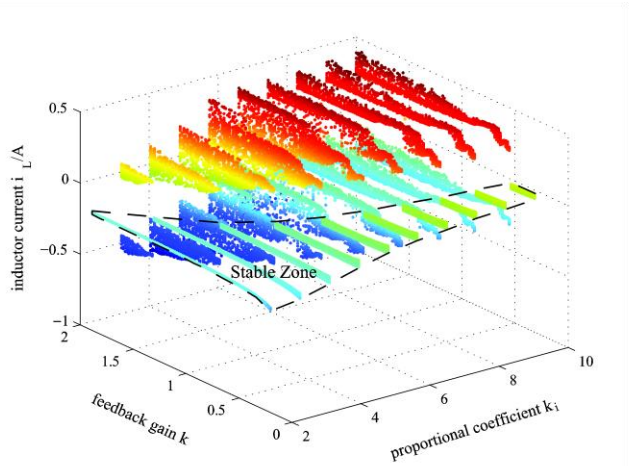

Figure 9. Bifurcation diagram of the coil current, proportional coefficient and current transformer while rotary state suspension

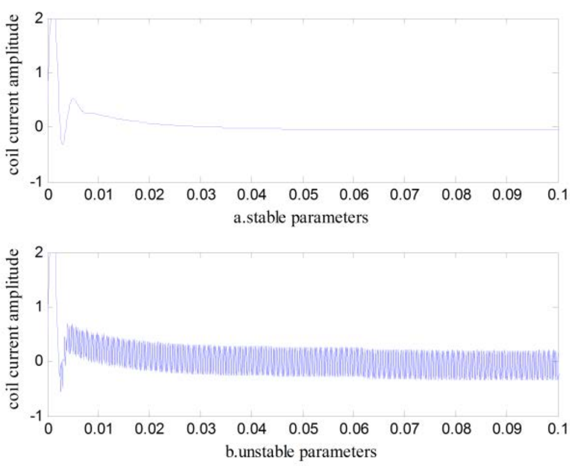

Figure 10. Coil current with stable parameters and unstable parameters

V. EXPERIMENT OF STATIC STATE SUSPENSION

The static state suspension experimental investigations with different proportional coefficient of current transformer k_i and the feedback gain of current loop k are presented in this section. The AMB system is applied to a 100Kw high speed permanent magnet motor as an experimental system which is shown in Fig 11.

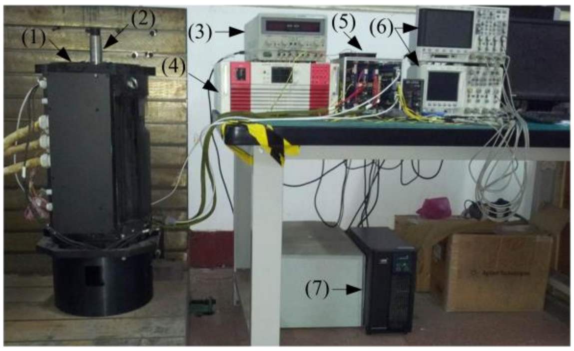

Figure 11. The experimental system. (1) Magnetically suspended motor. (2) Rotor. (3) Controller power supply 48V. (4) Amplifier power supply 90V. (5) Control system and Power amplifier. (6) Oscilloscope. (7) UPS.

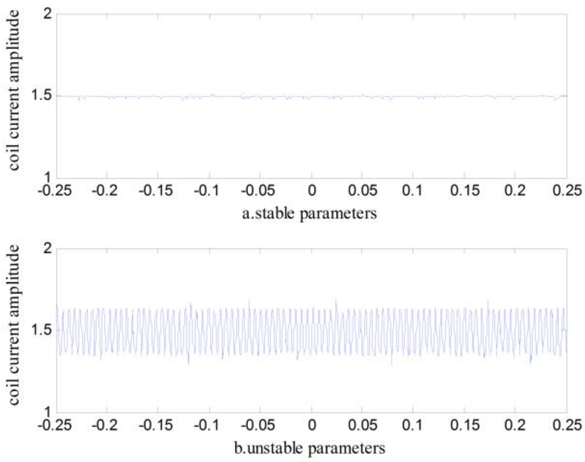

Figure 12. Coil current with stable parameters and unstable parameters

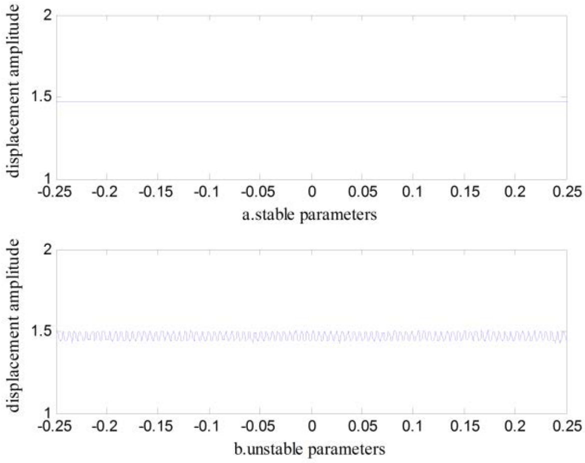

Figure 13. Rotor displacement with stable parameters and unstable parameters

Fig. 12 and Fig. 13 are the contrast of coil current and rotor displacement with stable parameters and unstable parameters of one radial suspension channel. The parameters of Fig. 12(a) and Fig. 13(a) are $k = 0.32$ and $k_i = 4.3$, the parameters of Fig. 12(b) and Fig. 13(b) are $k = 0.75$ and $k_i = 8.2$. The oscillation of rotor displacement and coil current are clearly observed in Fig. 12(b) and Fig. 13(b) respectively. The current oscillation is caused by the nonlinearity of the H - bridge inverter; the rotor displacement oscillation is caused by current oscillation which leads to oscillations of magnetic force.

VI. CONCLUSION

The simulation and experimental results demonstrate the existence of chaos phenomenon in power amplifier of AMB system, which will lead to oscillation of rotor displacement and affect the stability of the system. The simulation of current loop provides a stable range for feedback gain and coil inductance which may have some help with AMB coil's parameters design. The simulation of AMB system based on nonlinear power amplifier model achieves a stable zone for feedback gain k and current transformer k_i. The new AMB system model may have some benefits to the study of chaos control method for avoiding bifurcation and chaos phenomenon in AMB system.

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