

# Statistic Errors of Different INFORM Evaluation Methods applied to Magnetic Bearings

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**Abstract**—Active magnetic bearings offer many advantages compared to conventional bearings. But due to economic reasons their field of application is relatively small. Self-sensing magnetic bearings can contribute to a significant decrease of the system costs, of course at limited accuracy in position detection compared to systems with sensors.

The later on described INFORM method is one approach to self-sensing magnetic bearings. The position depending reluctance is measured through high frequent voltage injections and the resulting current slope. This paper discusses how different evaluation methods capable of calculating the current slope from the raw measurement data, differ in sensitivity and noise rejection. Therefore first the theoretical background of the used self-sensing method is explained. Next the error propagation of different evaluation methods is analysed and finally compared to measurements.

## I. INTRODUCTION

Major field of research for self-sensing magnetic bearings is to improve the position detection to facilitate more accurate and stable position control algorithms. This paper focuses on how different evaluation methods for the INFORM-measurement affect the position detection accuracy.

The INFORM method is well known from self-sensing synchronous electrical drives [1]. Basic idea is to measure the inductance of a phase coil by injection of high frequent voltage impulses and measuring the resulting slope of the phase current. If the reactance of the iron path and leakage fluxes are neglected, then the coil inductance solely depends on the air-gap between rotor and stator. Hence the INFORM method can be used for self-sensing magnetic bearings as proved in several studies [2][3].

A first study on the statistical behaviour of INFORM position detection [4] showed that the resolution was approximately one tenth of the air gap. This means that the quality of self-sensing position measurements has to be improved if used in highly unstable system like a active magnetic bearing with bias flux.

## II. THE INFORM METHOD

The so called INFORM method (Indirect Flux Detection by Online Reactance Measurement) was originally developed for the rotor position detection of synchronous drives [1] and is well known for the sensorless control of synchronous drive applications. It relies on measuring the position by high frequency voltage injection pulses. If the iron path

and leakage fluxes are neglected, then the coil inductance is solely dependent on the air-gap between rotor and stator. Assuming a three pole stator arranged in Y-shape, then the coil inductance can be calculated from:

$$L(x)|_{y=0} = \frac{\mu_0 N^2 A}{\frac{3}{2} (l_0 - \frac{x}{2})} \quad (1)$$

with the number of coil turns  $N$ , the air gap area  $A$ , the nominal air gap length  $l_0$  and the displacement  $x$  at  $y = 0$ . If the inductances of the three coils are known, the 2D radial displacement  $(x,y)$  can be calculated. In the following paragraph it is described how these inductances are obtained.

### A. 3-Active Working principle

In the used prototype, the so called 3-Active INFORM is implemented. It is a enhanced variant of the classical INFORM. The INFORM pulse sequence is produced according to [5], which is a combination of three active test voltage space phasors, approximating arbitrary voltage space phasors within a certain limit. Hence, INFORM measurement information and desired space phasors according to current control can be realized at the same time.

The zero-voltage space vector  $u_0$  is generated if all three ON-times are equal within a PWM cycle. The corresponding voltage and current curves are shown in figure 1.

Using equation 2, the coil inductance can be calculated out of the known dc link voltage and the measured current slope ( $r_s = 0\Omega$ ).

$$u = L \cdot \frac{di(t)}{dt} \rightarrow L = u \cdot \frac{\Delta\tau}{\Delta i} \quad (2)$$

When the coil inductances are known the position can be calculated using calculation rules basing upon the mathematical model described in [5].

## III. THEORY OF ERROR PROPAGATION

### IV. OBTAINING THE INCLINATION

In order to obtain a high Signal to noise ratio (SNR) for the current slope measurement, the current curve is oversampled several times. Result is a measurement vector with a sample series of the phase current curves. In a first approach the

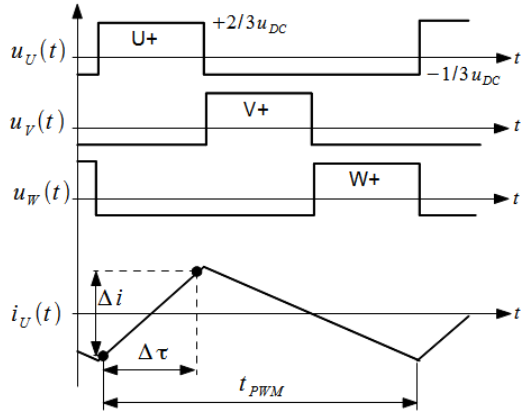


Figure 1. Voltage and current curves in 3-Active PWM modus

single measurement points can be seen as not correlative with equidistant time stamps. The following chapters will describe four different methods to obtain the position dependent inclination of the current slope.

#### A. Mean-difference method

The first is to average all the points of the first half of the  $n$  measurement points to the mean value  $\bar{x}_1$ , and all points of the second half to  $\bar{x}_2$  as shown in figure 2. Goal is to fit these two

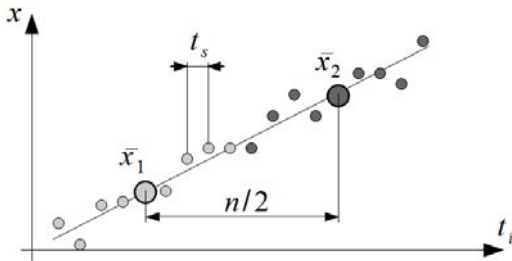


Figure 2. Sample points for the mean-difference calculation rule

measurement points in a linear approximation function which can be described by:

$$x = a + bt \quad (3)$$

As the offset value  $a$  is not of interest, the focus relies only on the calculation of the slope coefficient  $b$ . The current slope can be calculated by:

$$b = \frac{\bar{x}_2 - \bar{x}_1}{\frac{n}{2} \cdot t_s} \quad (4)$$

with the number of samples  $n$  and the sampling rate time  $t_s$ .

#### B. Linear regression method

Second considered method is a linear regression. A linear curve is fitted into the measurement vector by minimizing the

sum of the squares of the errors. The slope coefficient  $b$  can be calculated by

$$b = \frac{SS_{tx}}{SS_{tt}} = \frac{\sum_{i=1}^n (t_i - \bar{t})(x_i - \bar{x})}{\sum_{i=1}^n (t_i - \bar{t})^2} \quad (5)$$

$SS_{tx}$  is the empirical covariance of  $t$  and  $x$  and  $SS_{tt}$  the empirical variance of  $t$ . For the sake of simplicity the mean time  $\bar{t}$  is set to zero (see figure 3).

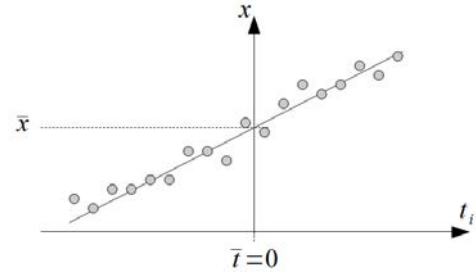


Figure 3. Sample points for the linear regression calculation rule

This simplifies equation (5) to

$$b = \frac{\sum_{i=1}^n t_i x_i}{\sum_{i=1}^n t_i^2} \quad (6)$$

#### C. Reverse mean-difference method

The working principle of the calculation rules before was to first take samples from the mid point and let the observation horizon grow from "inside to outside" with an increasing number  $n$ . The reverse approach is to first take the most outer sample points and grow from "outside to inside" with an increasing number  $n$ . Hence the calculation rule changes to

$$b = \frac{\bar{x}_2 - \bar{x}_1}{(n_{max} - n/2 + 1)t_s} \quad (7)$$

with the number of totally sampled points  $n_{max}$

#### D. Reverse linear regression method

The reverse linear regression is similar to the above described reverse mean-difference method, where first the outer sample points are utilized growing from "outside to inside" with an increasing number  $n$ . The resulting calculation rules for  $b$  is very similar and just defers in the indices of the sum operator:

$$b = \frac{\sum_{i=n_{max}-n}^{n_{max}} t_i x_i}{\sum_{i=n_{max}-n}^{n_{max}} t_i^2} \quad (8)$$

### V. ERROR PROPAGATION WITH MULTIPLE INPUTS

#### A. Basic considerations

Assuming small errors  $|\Delta x| \ll |x|$ , a common approach is an approximation to a first-order Taylor series of the function  $b(x)$  [6]. For a single input system the resulting equations would be

$$b(x + \Delta x) = b(x) + \frac{db(x)}{dx} \Delta x \quad (9)$$

$$b(x + \Delta x) - b(x) = \Delta b = \frac{db(x)}{dx} \Delta x \quad (10)$$

with the input error  $\Delta x$  and the output error  $\Delta b$ .

If every measurement point is considered as independent and mutually uncorrelated input variable, then the output error yields to

$$\Delta b = \frac{\partial b(x)}{\partial x_1} \Delta x_1 + \frac{\partial b(x)}{\partial x_2} \Delta x_2 + \dots + \frac{\partial b(x)}{\partial x_n} \Delta x_n \quad (11)$$

Equation (11) describes how to calculate the resulting absolute error. To evaluate how measurement uncertainties affect the result's uncertainty, the very similar Gaussian error propagation law offers an approach [6]:

$$S_b = \sqrt{\sum_{i=1}^n \left( \frac{\partial b(x)}{\partial x_i} \right)^2 (S_{x_i})^2} \quad (12)$$

where  $S_{x_i}$  is the absolute standard deviation of each measurement  $x_i$ , and  $S_b$  the absolute standard deviation of the slope coefficient  $b$ . For this law, the measurement noise needs to be a Gaussian distribution. Figure 4 illustrates a histogram of a typical current sample measurement. It can be seen that this assumption is roughly fulfilled.

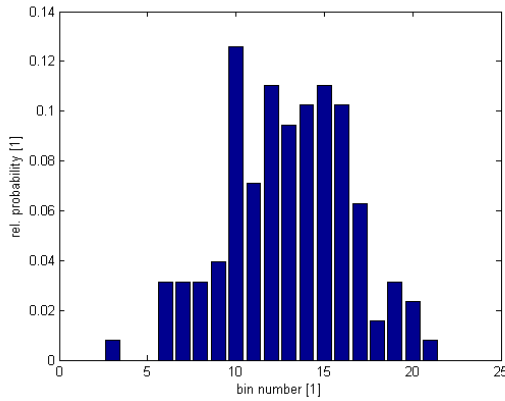


Figure 4. Histogramm of a measured current signal point in 3-Active PWM mode

This was also already shown in a previous study [4].

### B. Error propagation of the mean-difference method

To calculate how different numbers of samples used by the mean-difference method affect the result, the Gaussian error propagation law (12) has to be applied to the calculation rule (4). This yields to

$$S_b = \sqrt{\sum_{i=1}^2 \left( \frac{\partial}{\partial x_i} \frac{\bar{x}_2 - \bar{x}_1}{\frac{n}{2} \cdot t_s} \right)^2 (S_{\bar{x}_{1,2}})^2} \quad (13)$$

and further to:

$$S_b = \frac{2}{n \cdot t_s} \sqrt{2} S_{\bar{x}_{1,2}} \quad (14)$$

with the number of utilized sample points  $n$ , the sampling period time  $t_s$  and the standard deviation of the two average points  $S_{\bar{x}_{1,2}}$ . Thereby it was assumed that the standard deviation for all measurement points is equal. The standard deviation of the two points  $x_1$  and  $x_2$  yields from averaging

each with the half number of total sampling points. It can be calculated with:

$$S_{\bar{x}_{1,2}} = \frac{1}{\sqrt{n/2}} S_x \quad (15)$$

with the standard deviation of a single measurement point  $S_x$ . If equation 15 is applied to equation 14, the overall calculation rule leads to:

$$S_b = \frac{4}{\sqrt{n^3} \cdot t_s} S_x \quad (16)$$

### C. Error propagation of the linear regression method

Equivalent to the mean-difference method the Gaussian error propagation law (12) is applied to the calculation rule (6). This yields to

$$S_b = \sqrt{\sum_{j=1}^n \left( \frac{\partial}{\partial x_j} \frac{\sum_{i=1}^n t_j x_j}{\sum_{i=1}^n t_i^2} \right)^2 (S_{x_i})^2} \quad (17)$$

and further to:

$$S_b = \sqrt{\frac{1}{\sum_{i=1}^n t_i^2}} S_x \quad (18)$$

Again under the assumption the standard deviation for all sample points is equal.

### D. Error propagation of the reverse mean-difference method

When the Gaussian error propagation law is applied to the calculation rule 7, then the standard deviation of the reverse mean-difference method consequently yields to:

$$S_b = \frac{2}{\sqrt{n} (n_{max} - n/2 + 1) t_s} S_x \quad (19)$$

with the number of sample points  $n$  used for the calculation, and the total number of sampling points  $n_{max}$ .

### E. Error propagation of the reverse linear regression method

When the Gaussian error propagation law is applied to the calculation rule 7, then the standard deviation of the reverse mean-difference method consequently yields to:

$$S_b = \sqrt{\frac{1}{\sum_{i=n_{max}-n}^{n_{max}} t_i^2}} S_x \quad (20)$$

with the number of sample points  $n$  used for the calculation, and the total number of sampling points  $n_{max}$ .

## VI. EXPERIMENTAL RESULTS

To verify the theory, a set of slope gradient measurements was recorded, where the inclination was calculated by the micro controller using the mean-difference, the linear regression and their reverse variants. The signal to noise ratio SNR is defined as

$$\text{SNR} = 10 \cdot \log \left( \frac{\bar{x}^2}{\text{Var}(x)} \right) \quad (21)$$

and used to compare the results of the theoretical model with the measurements. The results for the inner to outer calculation rules are shown in figure 5 the results for the reverse outer to inner in figure 6.

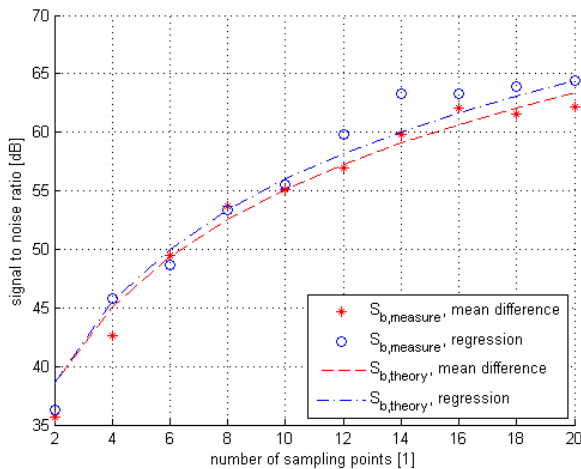


Figure 5. Comparison of the mean-difference / regression methods theoretical and measured SNR

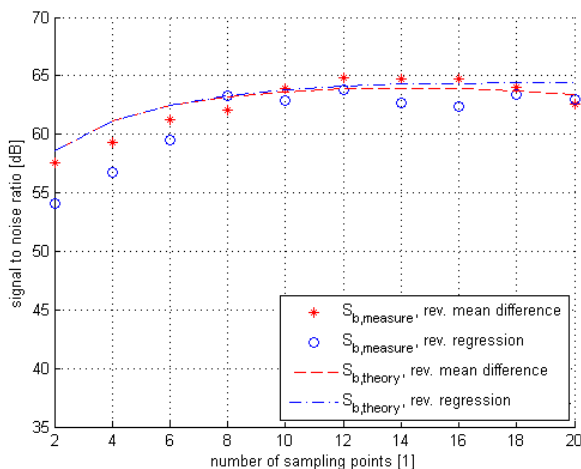


Figure 6. Comparison of the reverse mean-difference / regression methods theoretical and measured SNR ( $n_{max} = 20$ )

## VII. CONCLUSION

The curves show a good match between theory and measurement. Two main conclusions can thus be drawn. First the difference between the linear regression and the mean difference method is neglectable. The linear regression shows a little higher SNR with an increasing number of samples, but this does not stand against the drastically increased computation time due to the necessary multiplications. Second it can clearly be seen that the reverse approach is advantageous as the consideration of only a few sample points is enough to nearly reach the maximum SNR. Thus a lot of computation time can be saved. Figure 7 summarizes the results

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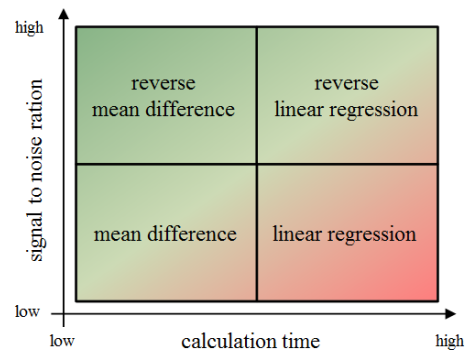


Figure 7. Comparison of the different calculation rules

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