Comparison of a Model-based and a Model-free Unbalance Control Methods in Magnetic Bearing System

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***Abstract*—Synchronous vibration caused by rotor imbalance is an obvious vibration problem in rotating machinery. Since active magnetic bearing has been proved to be an effective approach to solve this problem, many researchers have proposed various methods to reduce synchronous vibration and those methods can be classified into two categories: model-based method and model-free method. In the paper, we use a model-free method, iterative learning control (ILC) and a model-based adaptive forced balancing methods (MBAFB) to attenuate synchronous vibration respectively and then compare their performances in several aspects. The results of this paper can provide references for the users to choose an appropriate unbalance control algorithm.**

I. Introduction

Compared with traditional bearings, magnetic bearings possess several advantages, such as, contact free, no lubrication, and the unique ability of active control. Those advantages promote the developments and applications of magnetic bearings. At present, they are widely used in vacuum and clean room technology, turbo machinery and centrifuges [1].

Rotor imbalance results from materials or manufacture errors is inevitable. For a rotating system, the imbalance may generate a centrifugal force which would cause undesirable noise, rotor run-out and housing vibration. More specially, the centrifugal force is proportional to the square of rotating speed. Therefore, when the rotor operates at high speed, the synchronous vibration caused by rotor imbalance is obvious and even has an effect on the stability of the system. Many mechanical measures have been presented and applied to balancing the rotor. However, those measures are always costly and time consuming. Take dynamic balancing for example, rotor imbalance is reduced by adding or removing a small mass from the rotor. During the balancing process, the system need to start first then stop for several times and the rotor need to be re-balanced when the operation speed changes.

Since the active magnetic bearing has been proved to be an effective approach to solve this problem, many researchers have proposed several effective control methods to reduce synchronous vibration and those methods can be classified into two categories: model-based method and model-free method [2], [3], [4], [5]. In the paper, first of all, we build the model of the whole system. It is worth mentioning that a flexible rotor model is built using lumped mass method which promote the accuracy of objective system model. Then, synchronous vibration control of the system is achieved using a model-free method and a model-based method respectively. Finally, we compare the performances of those unbalance control methods.

II. Model of the objective system

An AMB system is proposed of several components: rotor, magnetic bearings, sensors, controller, amplifier. In this section, we build the model of each components and the details are described below

A. Model of the Flexible Rotor

It is well-known that motion of the rotor can be described by following equation:

$$M\ddot{X} + \Omega G\dot{X} + KX = F \tag{1}$$

Where M, G, K are respectively mass matrix, gyroscopic matrix, stiffness matrix. Here, X is the rotor displacements vector, Ω is rotating speed and F is the magnetic bearing forces vector. If we choose $Z = \begin{bmatrix} X^T & \dot{X}^T \end{bmatrix}^T$ as state variables, then the rotor model can be rewritten in the state space form:

$$\begin{aligned} \dot{Z} &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -\Omega M^{-1}G \end{bmatrix} Z + \begin{bmatrix} 0 \\ I \end{bmatrix} F \\ Y &= \begin{bmatrix} 0 & I \end{bmatrix} Z \end{aligned} \tag{2}$$

Where, Y is the output vector and I is identity matrix.

Since, magnetic bearings are widely applied in high speed rotating machinery. The rotor, in generally, operates beyond the rigid modes and several flexible modes. In this case, the simple rigid model of the rotor is ill-suited and a flexible rotor model is required. In the paper, parameters of rotor model is obtained and rotor model is established using lumped mass method. The basic principle of this method is to simplify rotor as several discs with mass and the moment of inertia lumped and massless elastic rods and the details about this method can be seen in literatures [6], [7]. To obtain a flexible rotor model, several flexible modes are maintained during the model reduction process.

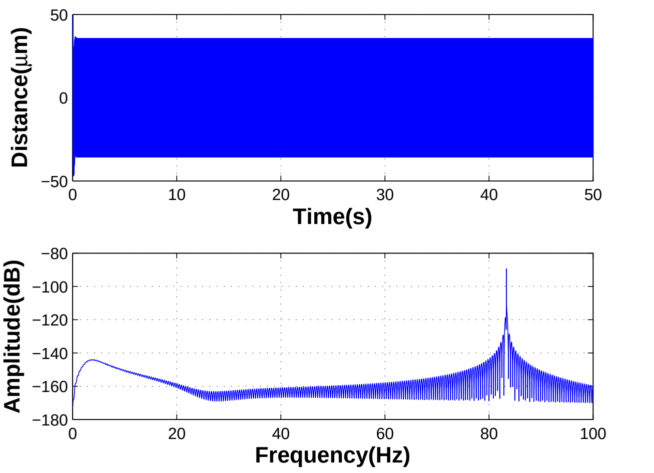

Figure 1. Rotor runout curve at axis X1 without unbalance control at $5000rpm$.

B. Models of Other Parts

The models of other parts of the objective system is given below:

Sensors and amplifiers are simplified into proportional components and represented by K_S, K_A respectively. Magnetic bearings are described by the classical linear form as following:

$$F_M = -K_X X + K_I I_C \tag{3}$$

Where F_M is magnetic bearing forces matrix, K_X is the force-displacement factor matrix, and K_I is the force-current factor matrix. Here, I_C is control current vector. As we all know that, the open loop of magnetic bearing system is inherently unstable. Therefore, a PID controller is introduced as negative feedback control to realize the stable of the system.

III. Unbalance Analysis and Compensation

A. Unbalance Analysis

When rotor rotates at speed ω, there will be a centrifugal force F_C results from the misalignment between the mass center and the geometric center of the rotor. Assume that the rotor imbalance is m and the distance between the mass center and the geometric center of the rotor is e. Then this centrifugal force can be described as:

$$F_C = me\omega^2 \sin(\omega t + \varphi) \tag{4}$$

However, this periodical centrifugal force acting on the rotor could induce sinusoidal disturbance to the displacement of the rotor if only the PID controller applied. The simulation result in Fig.1 also confirms this phenomenon. The sinusoidal disturbance can be written as:

$$d(t) = \alpha_d(t)\sin(\omega t) + \beta_d(t)\cos(\omega t) \tag{5}$$

Here, $\alpha_d(t)$, $\beta_d(t)$ are the fourier coefficients of the sinusoidal disturbance.

In generally, the sinusoidal disturbance is the mainly part of rotor vibration. It obstructs the applications of AMBs in high-lever rotating accuracy conditions. In these cases, unbalance compensation techniques can be applied to reduce the rotor runout. Among various of methods, open loop feed-forward control is most widely used. These methods add additional control signals to the controller outputs according to the displacements of the rotor. The structure of AMB system with unbalance compensation is illustrated in Fig.2. In this paper, the additional control signals are generated based on ILC and MBAFB respectively.

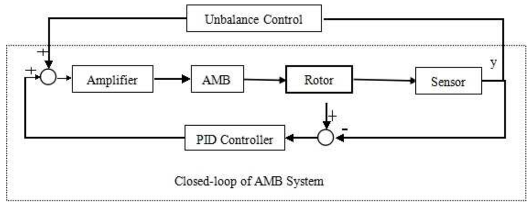

Figure 2. Block diagram of closed-loop control AMB system with unbalance control.

B. Iteration learning control

The original idea of iterative learning control was first raised by Uchiyama in 1978 [8] and further proposed by Arimoto and others. Iterative learning control is a model-free system synthesis method which yields the control signal u_{k+1} for the next cycle using the current control signal u_k and an error correction item, the product of learning gain and e_k, the difference between desired output y_d and the current output y_k, or the derivative of e_k. There are a series of ILC control methods, P-type, D-type, PID-type and so on. In the paper, we will use P-type ILC law which yields u_{k+1} by adding u_k and error correction item, the product of learning gain and e_k only. Therefore, this method has significant advantages: physically easy to implement and insensitive to measurement noises. The open loop P-style learning law in time domain can be described as:

$$\begin{aligned} &u_{k+1}(n) = u_k(n) + Le_k(n) \\ &n \in 0, 1, \cdots, n_f - 1 \end{aligned} \tag{6}$$

Where, $n_f = 2\pi/(T_s\omega)$, T_s is the sample time. $u_k(n)$ is the current control input, and $u_{k+1}(n)$ is the control input of next cycle at sample point n. Here, L is the iterative learning gain and convergence condition can be found in [9]. The error defined as:

$$e_k(n) = y_d(n) - y_k(n) \tag{7}$$

Here, $y_d(n)$ is the desired output.

C. Model Based Adaptive Forced Balancing

The simplified AMB system with MBAFB control is illustrated in Fig.3. The block of plant denotes the close loop of AMB system. And, it's transfer function $G(s)$ can be derived according to Fig.2. Here, we assume that $G(j\omega) = Ae^{j\theta}$ and the synchronous compensation signal has the following time domain representation:

$$r(t) = \alpha_r(t)\sin(\omega t) + \beta_r(t)\cos(\omega t) \tag{8}$$

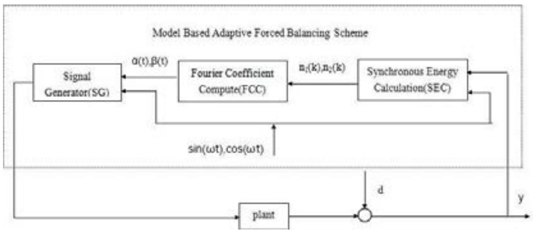

Figure 3. Block diagram of closed-loop AMB system with AFB.

Then, the output of the system can be derived as:

$$y(t) = \alpha_d(t)\sin(\omega t) + \beta_d(t)\cos(\omega t) + A\alpha_r(t)\sin(\omega t + \theta) + A\beta_r(t)\cos(\omega t + \theta) \quad (9)$$

As shown in Fig.3, the MBAFB control algorithm consists of three parts: synchronous energy calculation (SBC), fourier coefficient computer (FCC) and signal generater (SG) [10]. In the first part, the so-called synchronous energy signals are calculated according to the system output signal $y(t)$. In order to obtain $n_1(t)$, $n_2(t)$, $y(t)$ is demodulated and then filtered using low pass filter to filter out the non-DC components. With the assumption that high frequency components are canceled out, $n_1(t)$, $n_2(t)$ can be derived and then be used to generate the fourier coefficients of the synchronous compensation signal in the FCC part. FCC plays an important role in the adaptive algorithm, in this part, the fourier coefficients are calculated online and updated every sample interval. The adaptive laws of $\alpha(k)$, $\beta(k)$ can be written as follows

$$\alpha(k+1) = \alpha(k) - (cos(\theta)n_1(k) + sin(\theta)n_2(k))/A \quad (10)$$

$$\beta(k+1) = \beta(k) - (cos(\theta)n_2(k) - sin(\theta)n_1(k))/A \quad (11)$$

Where $\alpha(k) = \alpha(t)$, $\beta(k) = \beta(t)$ for $t = kTs$, $k = 0, 1, \cdots$. Finally, the synchronous compensation signal can be constructed using the following equation

$$r(t) = \alpha_r(t)\sin(\omega t) + \beta_r(t)\cos(\omega t) \quad (12)$$

IV. Simulation and Comparison of the Methods

A. Attenuation of Synchronous Vibration

To validate the algorithms introduced above, simulations with ILC and MBAFB control algorithm are carried out at $5000rpm$. The simulation results in Fig.4 and Fig.5 clearly demonstrated that either the ILC algorithm or the MBAFB control algorithm can attenuate the synchronous vibration to some degree. As shown in Fig.1, without unbalance compensation control, the maximum rotor runout is about $35\mu m$. With ILC control, the maximum rotor runout attenuate gradually and significantly with the increase of repetition number (see Fig.5). The steady-state of rotor runout can be achieved by setting stop condition of ILC algorithm. With MBAFB control, the steady-state rotor runout can be obtained after several times of repetitions and maximum rotor runout is about $30\mu m$. Compared with ILC algorithm, the MBAFB algorithm applied is less efficient (see Fig.4).

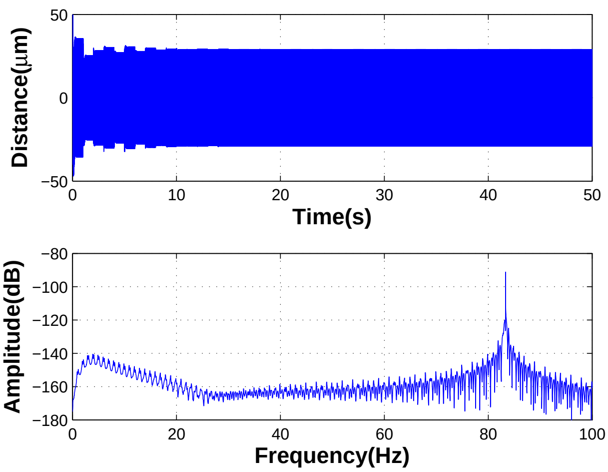

Figure 4. Rotor runout curve at axis X1 with MBAFB at $5000rpm$.

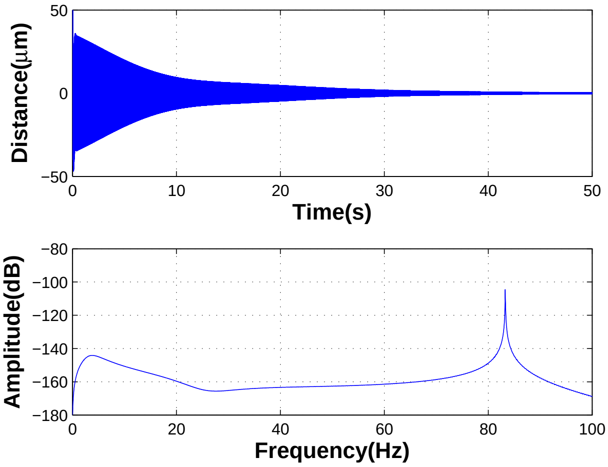

Figure 5. Rotor runout curve at axis X1 with ILC at $5000rpm$.

B. Tuning Lever of Controller Parameters

P-style ILC algorithm applied in this paper has a controller parameter L. The learning gain L is required to satisfy convergence condition below

$$\|I - LCB\| < 1 \quad (13)$$

Here, C and B are parameters of discrete-time close-loop system. However, the system parameters can be neglected when tuning the learning gain. Satisfied learning gain could be obtained by online adjusting[2]. In addition, according to [11], the range of learning gain L depend on the sample time as discrete-time close-loop system parameters come from continuous-time close-loop system. For MBAFB algorithm, there are no additional control parameters except the magnitude and phase of the close-loop system.

C. Sensitivity to Rotating Speed

In the simulation above, we assume that rotating speed of the rotor is known precisely. However, in practice, rotor

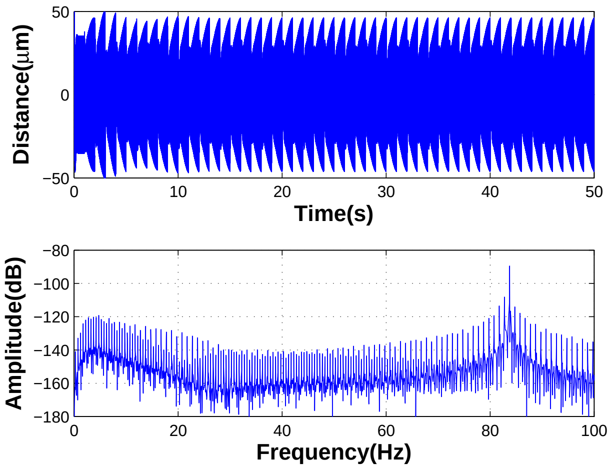

Figure 6. Rotor runout curve at axis X1 with MBAFB, 0.5% speed error

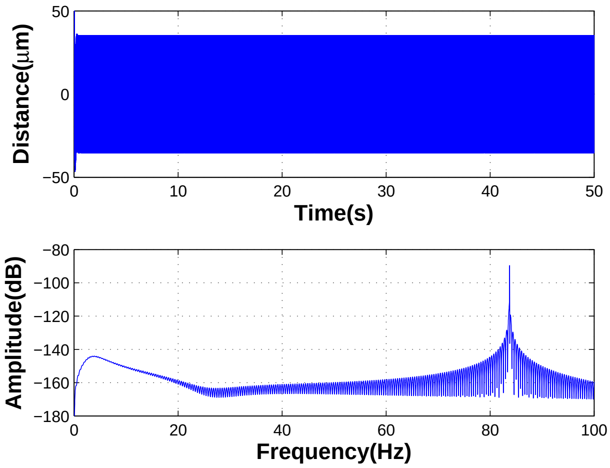

Figure 7. Rotor runout curve at axis X1 with ILC, 0.5% speed error

speed varies in a certain range. The performance of MBAFB algorithm with 0.5% speed error is illustrated in Fig.6. we find that, with speed error, MBAFB become invalid and even worsen the rotor runout. Fig.7 shows the performance of ILC algorithm with 0.5% speed error. ILC become invalid similarly. That's because synchronous vibration detection component is involved in the ILC algorithm to cancel out the affection of noises in practice[2]. From analysis above, we can see that both ILC and MBAFB algorithms are sensitive to the speed error. The algorithms should be developed to overcome this defect.

V. Conclusion

Synchronous vibration is a major vibration problem associated with rotating machinery. For a few years, researchers have devoted themselves to study unbalance compensation control algorithms using active magnetic bearing through active control. The algorithms presented can be divided into two categories: the model-free method and model-based method. In this paper, we establish the AMB system model and then use a model-free method, iterative learning control and a model-based adaptive forced balancing control to attenuate the unbalance vibration respectively. The performances of these unbalance control methods are compared from several aspects. The results of this paper reflect some properties of model-free and model-based unbalance compensation methods and can provide references for the users to choose an appropriate unbalance control algorithm.

VI. Acknowledgement

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