LQG Control of an Active Magnetic Bearing with a Special Method to consider the Gyroscopic Effect

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***Abstract*—It could be very difficult to find an acceptable feedback path for coupled differential equations like a FIVE degree of freedom (DOF) AMB system. This paper describes a linear quadratic Gaussian (LQG) approach on this problem. Due to the time variant gyroscopic part of the AMB system equation the controller could only be optimized for a specific angular velocity. For this reason a time variant state feedback is developed and added to the LQG controller in a way that the system is nearly equal for all angular velocities. The feedback matrix which is the result of the LQG problem uses all feedback terms. Compared to a decentralized controller which only uses the diagonal terms, the LQG method also uses the cross feedback terms which are able to provide instability. After developing the LQG method was compared to a decentralized PID controller.**

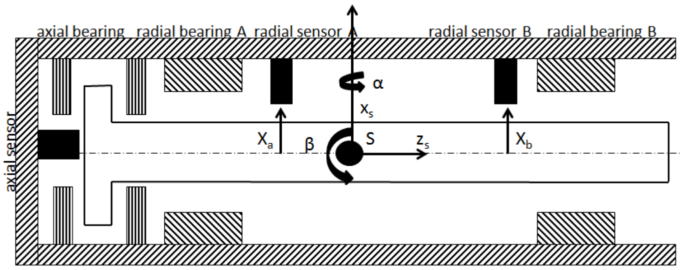


Figure 1. FIVE-DOF AMB SYSTEM

I. INTRODUCTION

Magnetic Bearings are used in many industrial applications, because of their big advantages compared to other bearing types. The main advantages are that they operate frictionless and wearless, they need no lubricant and are maintenance free. In the last years also a few methods for sensorless control of AMB systems were developed, like the INFORM method which is described in [1] and [2]. The stable levitation is provided by magnetic forces exerted by electromagnets in a closed loop system. A FIVE DOF AMB system is described as a coupled MIMO structure, where it could be difficult to design a controller with a good performance. The most straightforward method is a decentralized control method, where each sensor is fed back to the actuator of the measured degree of freedom, which is used in [3]. A drawback of this method is that only the diagonal terms of the feedback matrix are used. To eliminate that drawback a central MIMO controller can be designed. For such a coupled MIMO control problem a LQR controller is a good solution. A way to solve that problem is described in [4]. But it has to be considered that the AMB system is a time variant system, because of the gyroscopic term. So the controller would only be optimal for one operating point and this could lead to instability for other rotational speeds [5]. In [6] a time variant cross coupling for a decentralized controller is developed, which canceled the time variant term. The aim of this paper is to combine the LQR controller with the time variant feedback path to design a robust control structure for a wide angular velocity range. For the LQR method alone all states have to be known. If the states cannot be measured, the states could be estimated with an observer. When a LQR controller is combined with a Kalman observer it is called LQG controller. In this paper the experimental results of this new control structure are compared to the results of a decentralized controller.

II. FIVE DOF AMB SYSTEM

A FIVE DOF AMB system consists of 2 radial bearings and one axial bearing as shown in Fig.1. Every DOF needs a position sensor for the feedback path. Under certain assumptions the radial and the axial bearings can be treated seperatly from each other. Here the seperated case is used. The equation to describe the radial part of a linear AMB-System is mathematically derived in [7] and is given by:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{B}\mathbf{K}_s\mathbf{B}^T\mathbf{x} = \mathbf{B}\mathbf{K}_i\mathbf{i} \tag{1}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{2}$$

with

$$\mathbf{M} = \begin{bmatrix} I_x & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_x & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} c & 1 & 0 & 0 \\ d & 1 & 0 & 0 \\ 0 & 0 & c & 1 \\ 0 & 0 & d & 1 \end{bmatrix}$$

$$\mathbf{K}_s = \begin{bmatrix} k_s & 0 & 0 & 0 \\ 0 & k_s & 0 & 0 \\ 0 & 0 & k_s & 0 \\ 0 & 0 & 0 & k_s \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} a & b & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{K}_i = \begin{bmatrix} k_i & 0 & 0 & 0 \\ 0 & k_i & 0 & 0 \\ 0 & 0 & k_i & 0 \\ 0 & 0 & 0 & k_i \end{bmatrix} \quad \mathbf{i} = \begin{bmatrix} i_{xA} \\ i_{xB} \\ i_{yA} \\ i_{yB} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \beta \\ x_S \\ -\alpha \\ y_S \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & I_p\Omega & 0 \\ 0 & 0 & 0 & 0 \\ -I_p\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} x_{seA} \\ x_{seB} \\ y_{seA} \\ y_{seB} \end{bmatrix}$$

Where k_i is the linearized force current factor, k_s is the linearized force/displacement factor, m is the mass of the rotor, I_x is the equatorial moment of inertia, $\mathbf{G}$ is the time variant gyroscopic term, I_p is the polar moment of inertia, Ω is the angular velocity, and $\mathbf{i}$ is the current from the current controller. The equation of an AMB system is usually more complicated and nonlinear, but with k_i and k_s the nonlinear electrical equation is linearized to a linear one. The index S means that the axes are in the center of gravity and seA or seB that the axes are at the position sensors. The mechanical system has to fulfill a few assumptions that the linear equation (1) can be used [5]:

- The rotor is modeled as rigid body
- Deviations from the reference position are small compared to the rotor dimension
- Non-diagonal terms of the matrix of inertia are small compared to the diagonal terms
- The cross sectional area of the rotor is symetric ($I_x = I_y$)
- The bearing forces in two directions vertical to each other are independent
- The angular velocity around the longitudinal axis is constant or is changing slowly compared to the dynamic of the AMB system

III. Design of the Controller

A. Time variant feedback path

The result of the LQR method is an optimal controller only for a linear time invariant system. From this context it can be seen that the time variant path of equation (1) needs to be eliminated. If the control law

$$\mathbf{i} = (\mathbf{BK}_i)^{-1}\mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{v} \tag{3}$$

is inserted to equation (1)

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{BK}_s\mathbf{B}^T\mathbf{x} = \mathbf{BK}_i(\mathbf{BK}_i)^{-1}\mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{BK}_i\mathbf{v} \tag{4}$$

the result is a time variant system in the form with a new input $\mathbf{v}$:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{BK}_s\mathbf{B}^T\mathbf{x} = \mathbf{BK}_i\mathbf{v} \tag{5}$$

In [6] it is suggested that a complete elimination of the gyroscopic term is not very robust, and therefore a factor C_{att} is introduced. After that step the control law has the following form:

$$i = (\mathbf{BK}_i)^{-1}C_{att}\mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{v} \tag{6}$$

Because a constant or a slowly changed angular velocity is assumed, this feedback term could be treated as a linear time invariant system for stability analyses. With this feedback law it is possible to find an optimal controller for a wide angular velocity range. The angular velocity can be measured with a sensor or is able to estimated with a special unbalance observer.

B. LQR Controller

The LQR method is a powerful technique for designing controllers for complex systems, for which it is hard to find a good solution with classical control theory. Compared to other control design techniques the LQR seeks to find an optimum that minimizes a cost function. In [8] this theory is described clearly and understandable. Because the controller is usually implemented in a Digital Processor, the cost function of the discrete system has to be minimized. The first step is to convert equation (5) in a time continuous state space representation.

$$\begin{aligned} \dot{\mathbf{z}} &= \mathbf{Az} + \mathbf{Bv} \\ \mathbf{y} &= \mathbf{Cz} \end{aligned} \tag{7}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{E} \\ -\mathbf{M}^{-1}\mathbf{BK}_s\mathbf{B}^T & \mathbf{0} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{BK}_i \end{bmatrix} \tag{8}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \tag{9}$$

Where $\mathbf{E}$ is the identity matrix and $\mathbf{0}$ is the zero matrix. This continuous state space system is transformed to a discrete system with the following Form:

$$\begin{aligned} \mathbf{z}_{k+1} &= \mathbf{\Phi}\mathbf{z}_k + \mathbf{\Gamma}\mathbf{v}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{z}_k \end{aligned} \tag{10}$$

To have a controller which is able to eliminate constant disturbances like the weight force the system is extended with an integration action.

$$\mathbf{z}_{Ik+1} = \mathbf{z}_{Ik} + (\mathbf{r}_k - \mathbf{C}\mathbf{z}_k) \tag{11}$$

Herein $\mathbf{r}_k$ is the reference signal. This integrator can be considered in the discrete state space representation.

$$\begin{aligned} \begin{bmatrix} \mathbf{z}_{k+1} \\ \mathbf{z}_{Ik+1} \end{bmatrix} &= \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ -\mathbf{C} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{z}_{Ik} \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{0} \end{bmatrix} \mathbf{v}_k + \begin{bmatrix} \mathbf{0} \\ \mathbf{E} \end{bmatrix} \mathbf{r}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{z}_k \end{aligned} \tag{12}$$

With this equation the cost function

$$J(z_0) = \sum_{k=0}^{N-1} \left(\mathbf{z}_k^T\mathbf{Q}\mathbf{z}_k + \mathbf{u}_k^T\mathbf{R}\mathbf{u}_k\right) + \mathbf{z}_k^T\mathbf{S}\mathbf{z}_k \tag{13}$$

with the weighting matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{S}$ can be minimized. From this equation it can be seen that $\mathbf{Q}$ weigths the states. When a diagonal matrix is used, the state which has the biggest entry in the matrix will decay very fast to zero compared to the other states. The $\mathbf{R}$ matrix weights the manipulated variables. When $\mathbf{R}$ increases the manipulated variables will decrease. The result of this optimization problem is:

$$\mathbf{u}_k = \mathbf{K}_k\mathbf{x}_k \tag{14}$$

$$\mathbf{K}_k = -\left(\mathbf{R} + \mathbf{\Gamma}^T\mathbf{P}_{k+1}\mathbf{\Gamma}\right)^{-1}\left(\mathbf{\Gamma}^T\mathbf{P}_{k+1}\mathbf{\Phi}\right) \tag{15}$$

$$\mathbf{P}_k = \left(\mathbf{Q} + \mathbf{\Phi}^T\mathbf{P}_{k+1}\mathbf{\Phi}\right) - \left(\mathbf{\Gamma}^T\mathbf{P}_{k+1}\mathbf{\Phi}\right)^T \left(\mathbf{R} + \mathbf{\Gamma}^T\mathbf{P}_{k+1}\mathbf{\Gamma}\right)^{-1} \left(\mathbf{\Gamma}^T\mathbf{P}_{k+1}\mathbf{\Phi}\right) \quad (16)$$

Equation (16) is a discrete Riccati equation, which operates backwards. With this constellation the feedback matrix is time variant and the end time has to be known. To overcome this issue the end time is set to infinite. Now the Riccati equation is an algebraic equation and the feedback matrix is time invariant. To get a stable system the matrices of (16) have to fulfill two assumptions:

- The eigenvalues ot the pair $(\mathbf{\Phi}, \mathbf{\Gamma})$ outside the unity circle need to be controllable.
- The eigenvalues ot the pair $(\boldsymbol{C}, \mathbf{\Phi})$ outside the unity circle need to be observable

In [6] it is stated that a LQR controller is not very robust. It is possible to get a more robust system, if the system equation is modified in a specific way. The LQR method calculates a system, which has all poles of the system matrix $(\mathbf{\Phi} + \mathbf{\Gamma}_s)$ inside the unit circle. To increase robustness it is possible to get a system, which has the poles inside a circle with a radius of $r < 1$. The new system equation is

$$\mathbf{z}_{k+1} = \tilde{\mathbf{\Phi}}\mathbf{z}_k + \tilde{\mathbf{\Gamma}}\mathbf{v}_k \quad (17)$$

with

$$\tilde{\mathbf{\Phi}} = \frac{1}{r}\mathbf{\Phi} \quad \tilde{\mathbf{\Gamma}} = \frac{1}{r}\mathbf{\Gamma} \quad (18)$$

C. Kalman observer

A disadvantage of the LQR method is that all states have to be known. Generally this is not the case, so an observer is needed to estimate the states. For a MIMO system it could be hard to find an acceptable observer with the method of pole placement. A powerful approach for this problem is a Kalman observer, which is an optimal observer in the sense of control sciences. For filter design a discrete, time invariant and linear system

$$\begin{aligned} \mathbf{z}_{k+1} &= \mathbf{\Phi}\mathbf{z}_k + \mathbf{\Gamma}\mathbf{v}_k + \mathbf{G}\mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{z}_k + \mathbf{H}\mathbf{w}_k + \mathbf{v}_k \end{aligned} \quad (19)$$

with the state vector $\mathbf{z}_k$, the output vector $\mathbf{y}_k$, the disturbances $\mathbf{w}_k$ and the measuring noise $\mathbf{v}_k$ is used. The Kalman observer has a few assumptions:

$$E(\mathbf{v}_k) = 0 \qquad E(\mathbf{w}_k) = 0 \qquad E(\mathbf{v}_k\mathbf{v}_j^T) = \mathbf{R}\delta_{kj}$$
$$E(\mathbf{w}_k\mathbf{w}_j^T) = \mathbf{Q}\delta_{kj} \qquad E(\mathbf{w}_k\mathbf{v}_j^T) = 0$$
$$E(\mathbf{w}_k\mathbf{x}_j^T) = 0 \qquad \text{for k} \geq \text{j}$$
$$E(\mathbf{v}_l\mathbf{x}_j^T) = 0 \qquad \text{for all l, j}$$

The equation of the Kalman filter is derived in [8] and is

$$\hat{\mathbf{z}}_{k+1} = \mathbf{\Phi}\hat{\mathbf{z}}_k + \mathbf{\Gamma}\mathbf{v}_k + \hat{\mathbf{K}}_k\left(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{z}}_k - \mathbf{D}\mathbf{v}_k\right) \quad (20)$$

$$\hat{\mathbf{K}}_k = \mathbf{\Phi}\mathbf{P}_k\mathbf{C}^T\left(\mathbf{C}\mathbf{P}_k\mathbf{C}^T + \mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R}\right)^{-1} \quad (21)$$

$$\mathbf{P}_{k+1} = \mathbf{\Phi}\mathbf{P}_k\mathbf{\Phi}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T - \mathbf{\Phi}\mathbf{P}_k\boldsymbol{C}^T \left(\boldsymbol{C}\mathbf{P}_k\boldsymbol{C}^T + \mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R}\right)^{-1} \boldsymbol{C}\mathbf{P}_k\mathbf{\Phi}^T \quad (22)$$

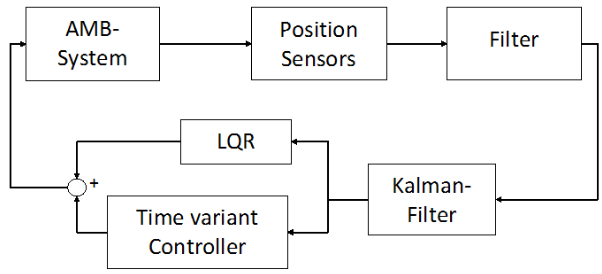


Figure 2. Control System of the AMB

Equation (22) is also a discrete Riccati equation, but compared to the LQR problem it runs forward. The feedback matrix is also time variant. If a time invariant feedback matrix is required, the end time could be set to infinite. The result of this method is a time invariant feedback matrix $\hat{\mathbf{K}}_k$. This feedback loop stabilizes the system under the following assumptions:

- The eigenvalues ot the pair $(\boldsymbol{C}, \mathbf{\Phi})$ outside the unity circle need to be observable.
- The eigenvalues ot the pair $(\mathbf{\Phi}, \mathbf{GQG^T})$ outside the unity circle need to be controllable.
- The marix $\mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R}$ needs to be positive definite.

Compared to the pole placement method the Kalman filter can affect the error system by the covariance matrix of the disturbances $\mathbf{Q}$ and the covariance matrix of the measuring noise $\mathbf{R}$. Where $\mathbf{R}$ considers the reliability of the sensors.

The total control system can be seen in Fig.2. Where LQR, time variant controller and Kalman filter are described in the sections above. The filter is a a second order IIR filter, which is used to stabilize bending modes at high frequencies.

D. Stabilization of the first bending mode

The rotor system has the first bending mode at 840 Hz. If the damping of the first bending modes are higher then zero the system is stable. Fig.3 shows the areas where the controller provides stability. The dead time of the sampling process would destabilize the system in the range of the first bending mode.

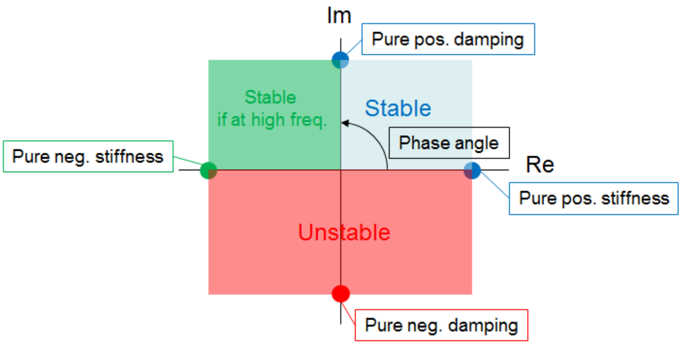


Figure 3. Requirement on the controller to provide stability [9]

Because no model for the flexible rotor was derived which could be used in the LQR design, a filter was designed to overcome this problem. This filter consists of an second order lowpass filter multiplied with a lead lag filter. The transfer function has the following form:

$$T(s) = \frac{1}{\frac{s^2}{(2\pi f_f)^2} + \frac{2D}{2\pi f_f}s + 1} \cdot \frac{\frac{s^2}{(2\pi f_n)^2} + \frac{2D_n}{2\pi f_n}s + 1}{\frac{s^2}{(2\pi f_d)^2} + \frac{2D_d}{2\pi f_d}s + 1} \qquad (23)$$

With this method the phase is shifted in the stable area at high frequencies, which can be seen from the Bode plot (Fig.4).

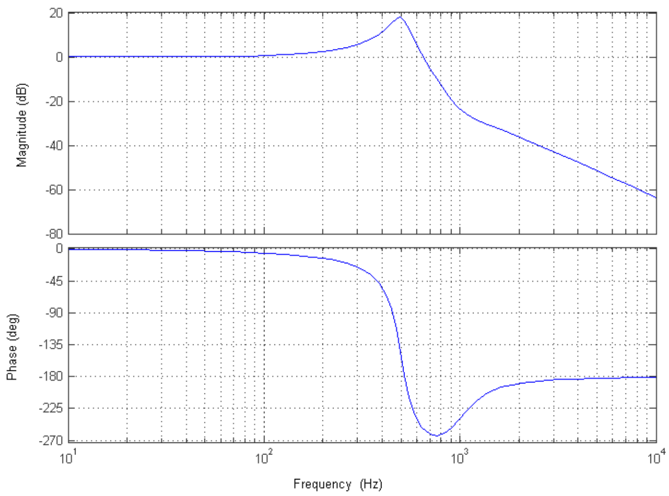


Figure 4. Bode plot of the filter to stabilize the first bending mode

A disadvantage of this method is that this filter limit the dynamic of the system and could lead to instability if this filter is not considered in the design of the PID or LQR Controller.

IV. Simulation results

For the simulation the rigid body model from section 2 is used and performed on MATLAB/SIMULINK. The controller and observer was implemented with a Matlab function block, where the digitization is considered. A white noise with an amplitude of 10 µm is applied on the measured signals. Fig.5 shows the response of the position signals controlled with the LQR and the decentralized PID controller after a disturbance force step of $40N$ at a time of $0.2s$ with speed of $3000rpm$. The LQR method shows a better performance than the PID and the states decrease almost equally fast to zero with the LQR. With the PID controller two states perform much faster than the other two. By tuning the value of r the robustness of the system is able to be changed. Another effect of decreasing r is the increasing stiffness of the system. To test the robustness of the system the dynamic matrix was changed to simulate a wrong model. The system has an acceptable performance from 0.1 times the dynamic matrix to 2.8 times the dynamic matrix. From these values it can be seen, that the AMB system is robust against model inaccuracies.

Fig.7 shows the Campbell diagram of the decentralized PID controller and Fig.6 for the LQG controller with the compensation of the gyroscopic effect. The natural frequencies of the decentralized PID controller are drifting away from each other, because of the gyroscopic effect. This can decrease the performance of the system and is able to destabilize the control structure. The time variant feedback path in combination with the LQG method on the other hand has constant natural frequencies, because of the compensation of the gyroscopic effect. This means that the system is the same for all angular velocities.

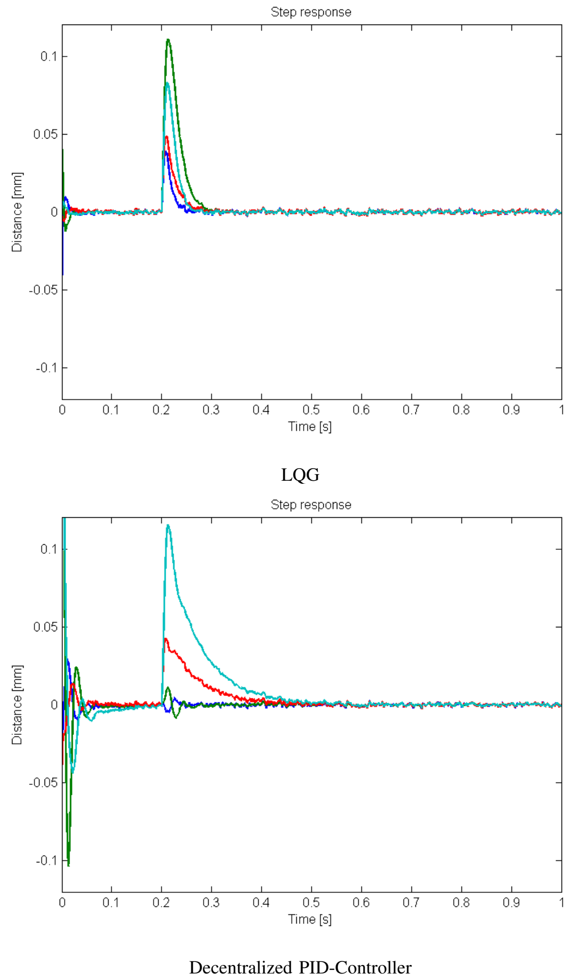


Figure 5. Disturbance-step-responses

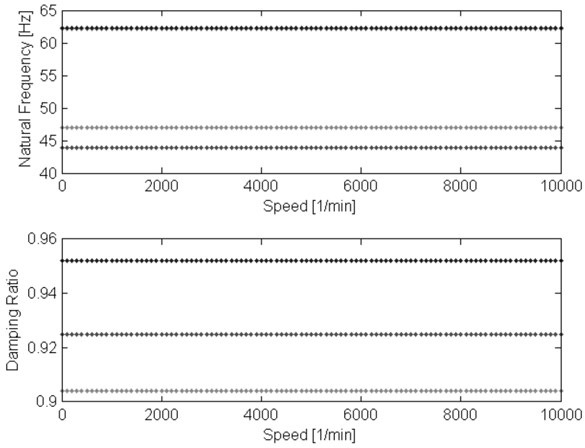


Figure 6. Campbell diagrams for LQG

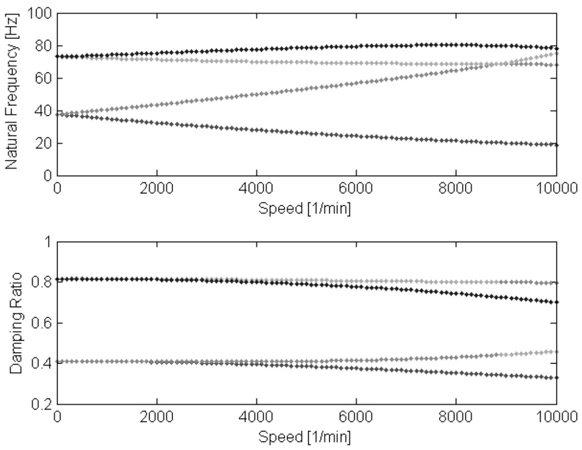


Figure 7. Campbell diagrams for decentralized PID-Controller

V. EXPERIMANTAL RESULTS

A. Behavior after a disturbance step

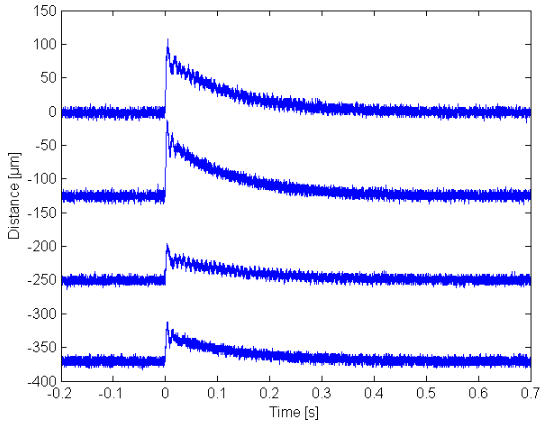
In this experiment a defined disturbance step of $26,5N$ at the upper bearings and $12N$ at the lower bearing was applied in both directions and the positions were measured. The LQR shows a better performance than the PID, because the step is lower and the settling time is faster. The reason is that the LQR uses the diagonal and the non-diagonal terms, the PID only uses the diagonal ones. The measurements are shown in Fig.5. The both higher signals are from the bottom bearing and the lower signals from the upper bearing. To plot all positions in one figure, offsets were used. In reality all signals starts at 0µm.

B. Comparison of the behavior at critical speed

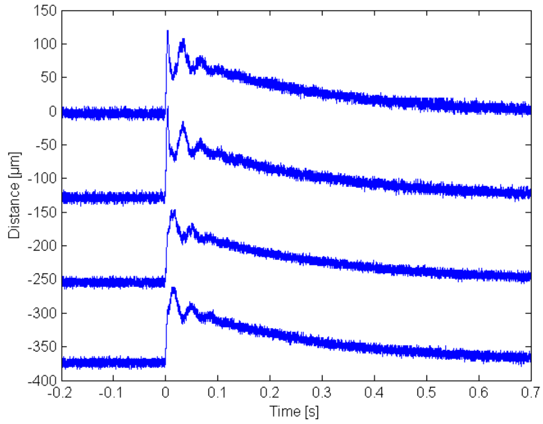
In this section both systems operate at the first critical speed. To compare them the amplitude of the positions were measured. The operation speed of the LQG controlled system was $3600rpm$ and of the PID controlled system $3900rpm$. With the PID controller the amplitude of the upper bearing is very low compared to the LQG controlled system (Fig.9). In the lower bearing occurs the opposite case. The ratio between the amplitude of lower and upper bearing is smaller with the LQG controller. As consequence the amplitude of both bearings is able to increase to higher values before the rotor touches its mechanical boundaries in one of the bearings.

C. System behavior at standstill and at high rotational speed

Here the system behavior at standstill and at high rotational speed is evaluated. With an exact compensation, the behavior is expected to be angular velocity independent. Both controller operates at standstill and at $6000rpm$. For comparison of the systems behavior the transfer function at standstill and $6000rpm$ was measured. For measuring the system is excited with a sweep sine at the position reference value and the positions from the sensors were measured. After the measurement in the time domain the signal was transformed in the frequency domain using the Fast Fourier Transformation. With



LQG



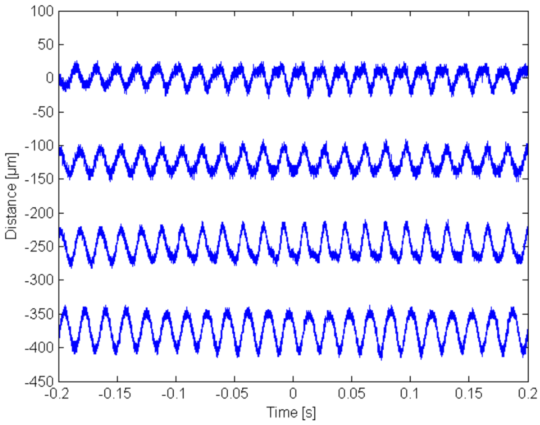
Decentralized PID-Controller

Figure 8. Disturbance step responce, x-y from lower bearing are the both higher signals, x-y from upper bearing are the both lower signals

the signals in the frequency domain the transfer functions was calculated (Fig.10). The LQG controlled system has nearly the same transfer functions for both angular velocities as expected. The peak in the range of $100Hz$ is not a resonance, its only a disturbance caused from the unbalance. In contrast the decentralized PID controller has different shapes for both operating points. These measurments established the simulation results of the compensation of the gyroscopic effect (Fig.6).

VI. CONCLUSIONS

In this paper a LQG controller and a time variant feedback path has been designed. The resulting system of the combination of these two structures is a linear time invariant optimal system for a high angular velocity range. It is verified that the transfer functions of the AMB system are nearly the same for all angular velocities up to $6000rpm$. In the experiments a damping effect of the time variant feedback path was measured at the critical speeds. The combination of the filter to stabilize the first bending mode and the LQG controller is not ideal. If the feedback matrix of the Kalman observer is to high the system with the filter for the first bending mode gets unstable. Another solution would be to stabilize the first bending mode with the LQR controller by using a



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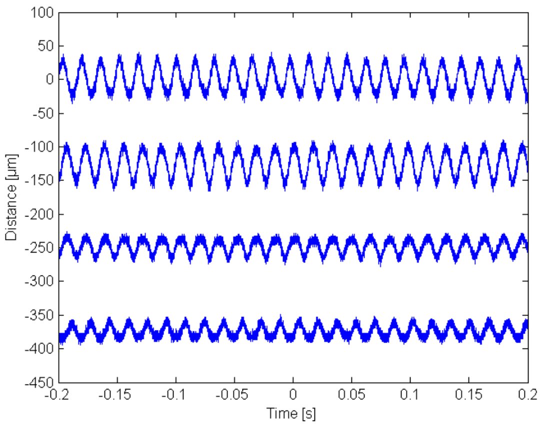
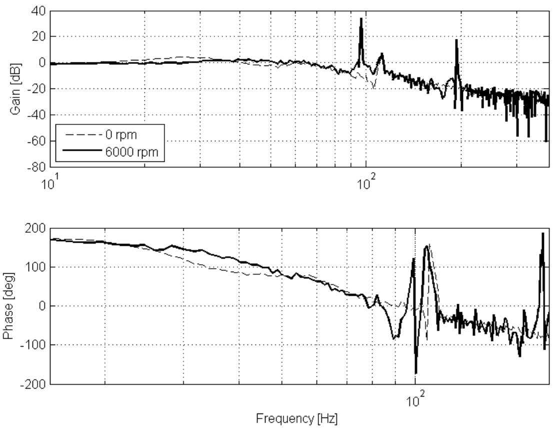


Decentralized PID-Controller

Figure 9. Behavior at critical speed, , x-y from lower bearing are the both higher signals, x-y from upper bearing are the both lower signals



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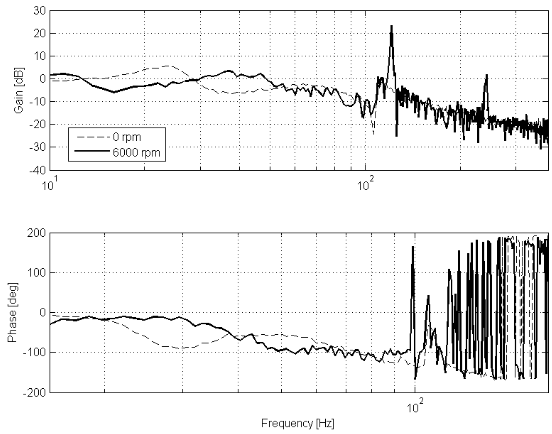

Decentralized PID-Controller

Figure 10. Transfer functions at standstill and 6000 rpm

flexible body model. If a Kalman observer is used for both, the LQR and to compensate the gyroscopic effect, a contradiction occurs, because the compensation of the gyroscopic effect for high frequencies depends on the dynamic of the Kalman filter. In summary, it can be stated, that it is easy to find a stable solution with the LQG method. If a system is required with a defiend stiffness and damping it is not so easy to adapt the solution with the LQG method compared to the decentralized PID controller. An advantage of the PID in this case is that the stiffness and damping can be changed directly by the control parameter. With the LQG controller only weighing matrices can be changed, which only changes the settling time of the states and not directly the stiffness and damping.

VII. Acknowledgments

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