# Active Reduction of Bearing Forces for Short Stroke Linear Actuators

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***Abstract*—Linear magnetic actuators are used in industry for a wide area of applications. In order to reach high values for the axial thrust force, the mover commonly features a permanent magnet excitation. Thus, in combination with a slotted stator layout, these actuators exhibit a high destabilizing stiffness in the direction perpendicular to the direction of motion which has to be compensated by the linear bearings. This paper introduces a linear short stroke actuator with an integrated active bearing force compensation to minimize the forces acting on the mechanical bearings. The potential and limits of a bearing force compensation is analyzed based on a short stroke linear actuators with an E-shaped stator layout. It is outlined how far bearing forces can be compensated even for simple actuator layouts. Especially for linear drives oscillating with a high speed this compensation can significantly reduce wear and thus improve lifetime and efficiency as well as reduce the size of the mechanical bearings.**

## I. Introduction

As soon as the mover of a linear motor is equipped with a permanent magnet (PM) excitation, attractive forces towards the stator exist at least if the mover is not in a perfect central position. With the need for high thrust forces, which can be achieved with slotted stator layouts, these forces are becoming a problem and a limiting factor for the bearing life time. A common measure to reduce or ideally cancel these forces is a symmetric design which can be tubular or planar. For an ideally centric mover no resulting forces will occur, but however, the often required dry running linear bush bearings are a subject to wear what results by the time in an eccentricity. While the wear increases the eccentricity, the bearing forces rise even more because of the destabilizing stiffness of the magnetic forces. This increases the friction and deteriorates the efficiency.

This problem has been investigated in [1] for a pick and place robot. There the attractive forces of the linear motor have been counteracted passively by fixing PMs underneath a soft magnetic plane.

In [2] the control of the rolling motion of a magnetically levitated linear motor is investigated. Small additional electromagnetic actuators are used to stabilize the bearingless linear motor.

Other examples for linear motors with active magnetic bearings are given in [3] and [4]. In these cases the linear motor is integrated in a free piston compressor and features two active magnetic bearings to compensate for the radial motor and gravity forces.

The objective of this paper is to analyze the potential and limits to reduce bearing forces of a short stroke actuator with single phase characteristics, based on E-shaped stator elements. These actuators are intended to be used as oscillating drives integrated into the respective target application which could be a previously mentioned free piston compressor. Thus it is assumed that a dry running linear bearing is available in the system but need not be exposed to the large attractive PM forces resulting from the magnetic actuator. However this bearing allows to overcome axial positions where no or only a weak compensation of the PM attractive forces is possible. Hence, the second objective of the paper is to keep the system as simple as possible.

## II. System layout

The system layout is given in Fig. 1. It features a two-sided planar E-shaped stator with one or alternatively two individual coils on each side. The planar mover is mounted to an axis which features a bearing ring at each end. It holds two permanent magnets (PM) on each side. The system is designed to cancel the lateral forces when it is in a perfect central position. External forces are assumed only to act along the $y$-axis.

However, as soon as wear or initial eccentricities in the $x$-direction cause an imbalance of the attractive forces, bearing forces occur. To measure the bearing forces the bearing rings run each in a radial force measurement gauge. Additionally the axial and lateral position of the axis are measured to determine the eccentricity and tilting angle.

The actuator has been designed to show a constant force/current ratio in $y$-direction for a stroke $s = \pm 5mm$ around the center position. Exceeding the maximal stroke the force/current ratio decreases rapidly. Hence a mechanical stop is required to keep the mover in the desired region of operation.

## III. Bearing forces

From the six degrees of freedom of the rigid body motion one degree of freedom is actively controlled to perform the required axial motion in $y$-direction. Two degrees of freedom are stabilized passively by choosing the active width of the mover in $x$-direction larger than the height in $z$-direction. This refers to the linear motion in $z$-direction and the tilting around the $y$-axis. The tilting motion around the $x$-axis is held by the mechanical bearings. Hence remain the attractive

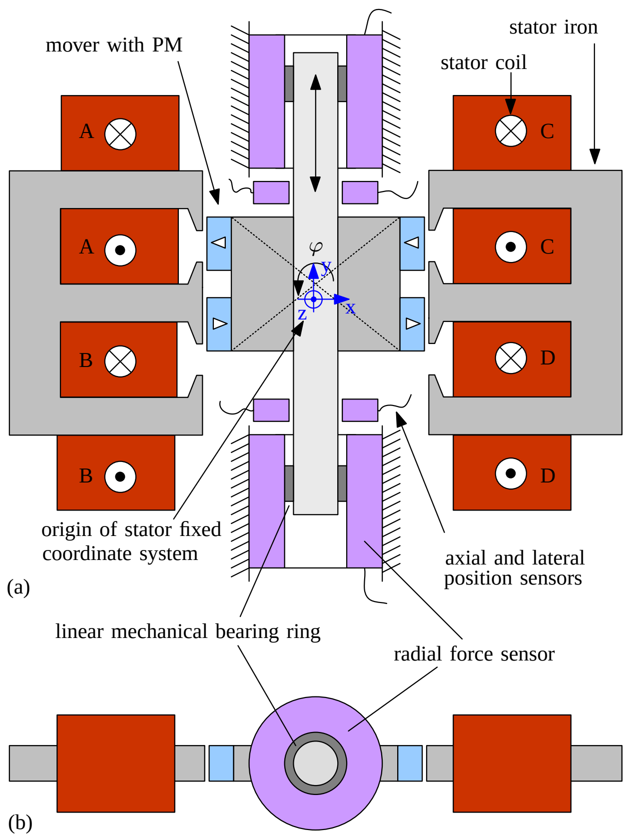


Figure 1. (a) Top view and (b) front view of the investigated linear actuator system with 4 phases labeled with A-D.

forces of the PMs in $x$-direction and the tilting around the $z$-axis that require to be compensated actively. Remaining forces that cannot be compensated are stabilized by the mechanical bearings.

### A. Wear of linear bearings

Compensating bearing forces is of main interest when the actuator requires to be integrated into a system. Here a typical requirement is often a small system size and high system overall efficiency. For oscillating actuators this results in the need for dry running bearings using a durable bearing material. However in the end wear limits the life time of the mechanical bearings. Assuming an oscillating actuator with the mean mover velocity $v_m$ and the mean bearing pressure $p_m$ the differential equation for wear is empirically deduced and given as

$$\dot{w}(t) = K_0 p_m(t) v_m \tag{1}$$

with the parameter $K_0$ having the unit $\mathrm{ms}^2/\mathrm{kg}$ which is the wear parameter of the selected bearing material.
It is assumed that all forces acting on the bearing are negligible compared to the resulting passive PM forces perpendicular to the direction of motion. Hence, the mean bearing pressure

$$p_m(t) = f(w(t)) \tag{2}$$

is a function of the wear, which results in an eccentricity of the mover.

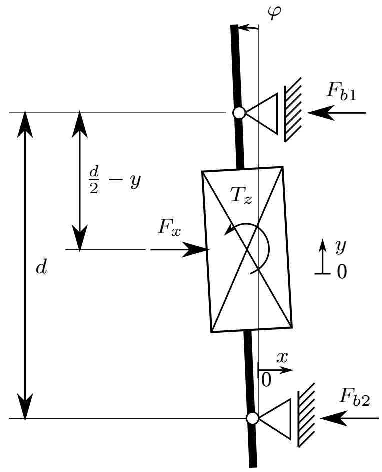


Figure 2. Model of static bearing forces under influcence of eccentricity and tilting.

For the investigated symmetrical electromagnetic actuators with small air-gap width, the forces perpendicular to the direction of motion can be approximated to vary linear with the eccentricity or wear respectively. The linear dependency is defined by the stiffness $k_x$. This leads to the expression

$$p_m(t) = \frac{k_x w(t)}{n h_b d_b \pi / 3} \tag{3}$$

for the mean bearing pressure. The installed $n$ bearing rings all have the same geometric dimensions of length $h_b$ and diameter $d_b$. It is assumed that each ring bears only on one third of its circumferential surface. Given an inevitable initial eccentricity of $w_0$ the wear results in:

$$w(t) = w_0 e^{\frac{3K_0 k_x t v_m}{n d_b h_b \pi}}. \tag{4}$$

From (4) follows, that for fixed geometrical dimensions of the bearing rings the stiffness $k_x$ directly influences the lifetime

$$t_{life} = \frac{n d_b h_b \pi \ln\left(\frac{w_{max}}{w_0}\right)}{3 K_0 k_x v_m}. \tag{5}$$

of the bearing, that is reached as soon as $w$ reaches $w_{max}$.

### B. Calculation of static bearing forces

According to Fig. 2 the equilibrium of the bearing forces $F_{b1}$, $F_{b2}$ and the force $F_x$ resulting from eccentricities in $x$-direction

$$F_{b1}(y) + F_{b2}(y) = F_x(y) \tag{6}$$

and the torque equilibrium

$$0 = F_{b2}(y)d + T_z(y) - F_x(y)\left(\frac{d}{2} - y\right) \tag{7}$$

yield

$$F_{b1}(y) = \frac{F_x(y)d + 2T_z(y) + 2F_x(y)y}{2d} \tag{8}$$

$$F_{b2}(y) = \frac{F_x(y)d - 2(T_z(y) + F_x(y)y)}{2d} \tag{9}$$

with a distance $d$ between the two bearings. The torque $T_z$ and the force $F_x$ are both dependent on $y$ and are acting on the center of the mover (Fig. 2).

## IV. System Model

### A. Force and torque model

The generalized forces

$$\mathbf{Q}_F = \begin{bmatrix} F_x \\ F_y \\ T_z \end{bmatrix} \tag{10}$$

with respect to $\mathbf{q} = [x, y, \varphi]^T$ can be developed into a Taylor series at $\mathbf{i} = \mathbf{0}$ and $x_0 = 0$ what results in the definition of the forces in the form

$$\mathbf{Q}_F = \mathbf{M}_L \cdot \mathbf{i} + \mathbf{M}_C \tag{11}$$

if all terms of higher order with respect to $\mathbf{i}$ are neglected. The terms $\mathbf{M}_L$ holds the coefficients linear to the coil currents and terms $\mathbf{M}_C$ is independent of the currents. The vector $\mathbf{i}$ has the dimension $m \times 1$ with $m$ being the number of phases. The applied superposition requires the model to be linear with respect to the phase currents $\mathbf{i}$. However, a nonlinear behavior can occur in the direction of motion $y$. According to [5] the linear model can be expressed as

$$\mathbf{Q} = \mathbf{T}_m \mathbf{i} = \mathbf{Q}_F - \mathbf{M}_C. \tag{12}$$

The entries of the matrices $\mathbf{M}_L$ or $\mathbf{T}_M$ and $\mathbf{M}_C$ are calculated using a finite element (FE) analysis of the investigated system given in Fig. 1.

### B. Dynamic model

Assuming current control a model based on applied phase currents can be derived. For the given setup the mover of the linear actuator should be operated a the point $x = 0$ and $\varphi = 0$ whereas the motion along the $y$-axis has no limitation as long as the previous set assumption of linearity with respect to the current holds true.
Thus, the equation of motion

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{Q} - \mathbf{Q}_l. \tag{13}$$

with the generalized mass matrix $\mathbf{M}$, and the generalized load force $\mathbf{Q}_l = [0, F_{ly}, 0]^T$ can be developed into a Taylor series. Neglecting all terms with a higher order as one this results in

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} = & \left.\frac{\partial(\mathbf{M}_L \mathbf{i})}{\partial \mathbf{q}}\right|_{x_0, y, \varphi_0, \mathbf{i}_0} \mathbf{q} + \left.\frac{\partial \mathbf{M}_C}{\partial \mathbf{q}}\right|_{x_0, y, \varphi_0, \mathbf{i}_0} \mathbf{q} + \\ & + \mathbf{M}_L\left(x_0, y, \varphi_0\right)\mathbf{i} + \left.\frac{\partial \mathbf{M}_C}{\partial \mathbf{i}}\right|_{x_0, y, \varphi_0}. \end{aligned} \tag{14}$$

The term $\mathbf{M}_C$ is per definition independent of the current $\mathbf{i}$ and the term $\mathbf{M}_L\mathbf{i}$ can approximately be seen as independent from $x$ and $\varphi$. With the stiffness matrix

$$\mathbf{k}_q(y) = \left.\frac{\partial \mathbf{M}_C}{\partial \mathbf{q}}\right|_{x_0, y, \varphi_0, \mathbf{i}_0} \tag{15}$$

and (12) the equation of motion in (14) can be simplified to

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{k}_q\left(y\right)\mathbf{q} + \mathbf{T}_{m0}\left(y\right)\mathbf{i}. \tag{16}$$

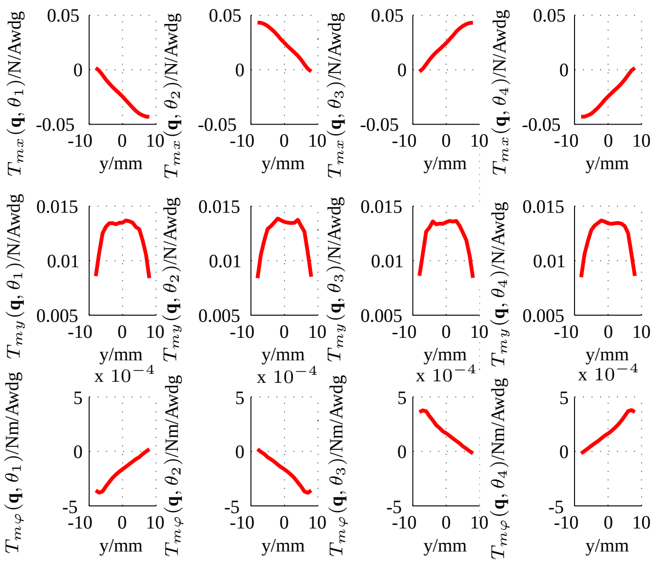


Figure 3. $\mathbf{T}_m(\mathbf{q}, \theta)$ matrix of the actuator with four phases and $\mathbf{q} = [x_0, y, \varphi_0]^T$ as a result from a FE analysis.

Neglecting all minor couplings in the stiffness matrix it results in the structure

$$\mathbf{k}_q(y) = \begin{bmatrix} k_{xx} & 0 & k_{x\varphi} \\ 0 & k_{yy} & 0 \\ k_{\varphi x} & 0 & k_{\varphi\varphi} \end{bmatrix} \tag{17}$$

showing a decoupling of the direction of motion from the other directions. The $x$- and $\varphi$-coordinates are coupled with $k_{x\varphi}$ and $k_{\varphi x}$. In general all entries of $\mathbf{k}_q$ are dependent on $y$.

## V. Bearing force reduction

To reduce both bearing forces and to achieve the desired motion in axial direction independently the $\mathbf{T}_m$ matrix has to fulfill the rank condition [5]

$$\mathrm{rank}(\mathbf{T}_m) = 3. \tag{18}$$

Hence the system requires to have at least three phases. The currents in these phases have to be controlled in a way to generate the desired force and torque components $\mathbf{Q}$. Thus (12) requires to be solved for $\mathbf{i}$ what results in

$$\mathbf{i} = \mathrm{inv}\left(\mathbf{T}_m\right)(\mathbf{q})\mathbf{Q} = \mathbf{K}_m(\mathbf{q})\mathbf{Q} \tag{19}$$

with $\mathrm{inv}\left(\mathbf{T}_m\right)$ being a generalized matrix-inverse of $\mathbf{T}_m$.

### A. Four coil actuator

The classic matrix-inverse can with (19) only be implemented for a three phase system. For the investigated case of four phases further constraints can be applied. In [6] this problem is solved to achieve minimum copper losses and the sum of all phase currents being zero. Hence the inverse of $\mathbf{T}_m$ can be calculated as

$$\mathbf{K}_m(y) = \mathbf{A}\left(\mathbf{T}_m\mathbf{A}\right)^{-1} \tag{20}$$

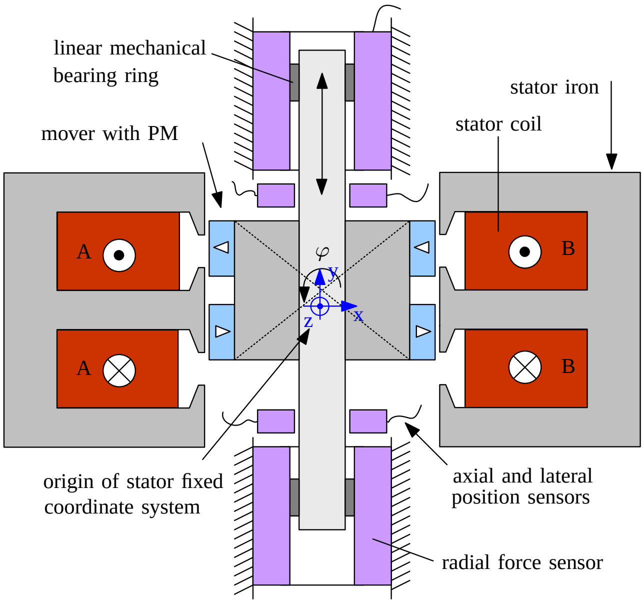


Figure 4. Top view of the actuator system reduced to two coils.

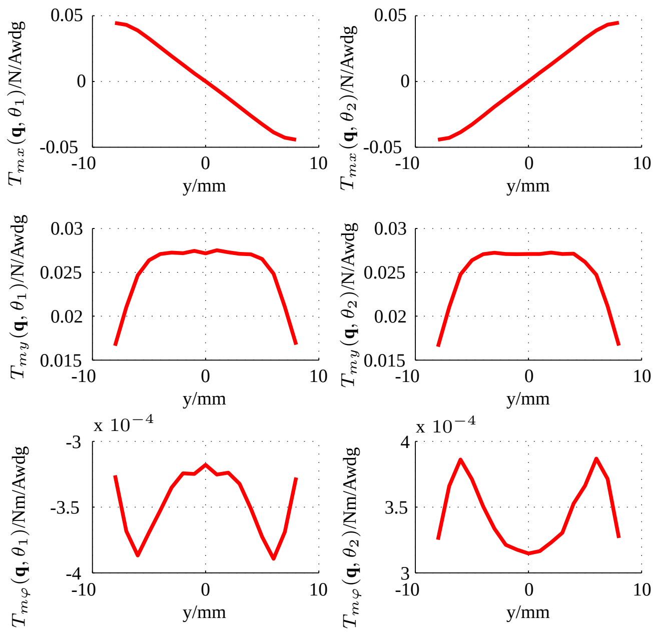

Figure 5. $\mathbf{T}_m(\mathbf{q}, \theta)$ matrix of the actuator with two phases and $\mathbf{q} = [x_0, y, \varphi_0]^T$ as a result from a FE analysis.

with

$$\mathbf{A} = \left(\mathbf{T}_m^T - \frac{1}{m}\mathbf{1}\mathbf{1}^T\mathbf{T}_m^T\right). \tag{21}$$

The $\mathbf{T}_m$ matrix for the investigated system is visualized in Fig. 3, fulfills the rank condition of (18) and has the structure

$$\mathbf{T}_m = \begin{bmatrix} T_{mx1} & T_{mx2} & -T_{mx1} & -T_{mx2} \\ T_{my} & T_{my} & T_{my} & T_{my} \\ T_{m\varphi 1} & T_{m\varphi 2} & -T_{m\varphi 1} & -T_{m\varphi 2} \end{bmatrix} \tag{22}$$

with all entries dependent on $y$. Thus, the system described in (16) can be separated into a subsystem describing the axial motion and an independent subsystem concerning the bearing forces. With this setup it is possible to cancel both bearing forces $F_{b1}$ and $F_{b2}$. As disadvantage remains the fact that the system needs four separate phases. This requires a realization with a minimum of four coils, what is double the amount as originally required for the $y$-direction motion.

### B. Two coil actuator

Without the need for bearing forces reduction, the actuator system of Fig. 1 would simply be constructed with two coils connected in series, one on each side. Controlling the two coils independently, this system allows a bearing force reduction, too. The system with two phases is shown in Fig. 4. This setup still requires with two full bridges a power electronics of the same size as the star connected four-coil system. However, the mechanical setup remains the same as for an actuator without bearing force reduction.

In this case again a $\mathbf{T}_m$ matrix can be calculated according to (12). As a matter of fact the rank of $\mathbf{T}_m$ cannot exceed 2 in this case. A visualization of the $\mathbf{T}_m$ matrix featuring $\text{rank}(\mathbf{T}_m) = 2$ is given in Fig. 5. Two actuating variables now have to control the motion in the three directions of $\mathbf{q}$. The structure of the $\mathbf{T}_m$ matrix

$$\mathbf{T}_m(y) = \begin{bmatrix} T_{mx} & -T_{mx} \\ T_{my} & T_{my} \\ T_{m\varphi} & -T_{m\varphi} \end{bmatrix} \tag{23}$$

with all entries dependent on $y$ and the stiffness matrix (17) yield that the direction of motion $y$ can again be independently controlled from the bearing forces. To control the bearing forces now only one actuating variable is available. The matrix $\mathbf{K}_m(y)$ is calculated pointwise numerically as the Moore-Penrose pseudo-inverse of $\mathbf{T}_m(y)$ for each position $y$.
The pseudo-inverse satisfies with the following four conditions

$$\begin{aligned} \mathbf{T}_m\mathbf{K}_m\mathbf{T}_m &= \mathbf{T}_m \\ \mathbf{K}_m\mathbf{T}_m\mathbf{K}_m &= \mathbf{K}_m \\ \mathbf{T}_m\mathbf{K}_m & \text{ is hermitian} \\ \mathbf{K}_m\mathbf{T}_m & \text{ is hermitian} \end{aligned} \tag{24}$$

most of the requirements to an inverse matrix. However, the product

$$\mathbf{T}_m\mathbf{K}_m \tag{25}$$

need not be identical to the unity matrix and indicates which of the variables $x$ and $\varphi$ can be controlled with two phases. From the visualization of this matrix product (25) in Fig. 6 it can be concluded that the $y$-position can be controlled all time, whereas the $x$-position can be controlled for all $y$-positions except the center, where only the angle $\varphi$ can be influenced. From (16), (23) and (17) follows that a current in the form

$$\begin{aligned} i_A &= i_y + \Delta i \\ i_B &= i_y - \Delta i \end{aligned} \tag{26}$$

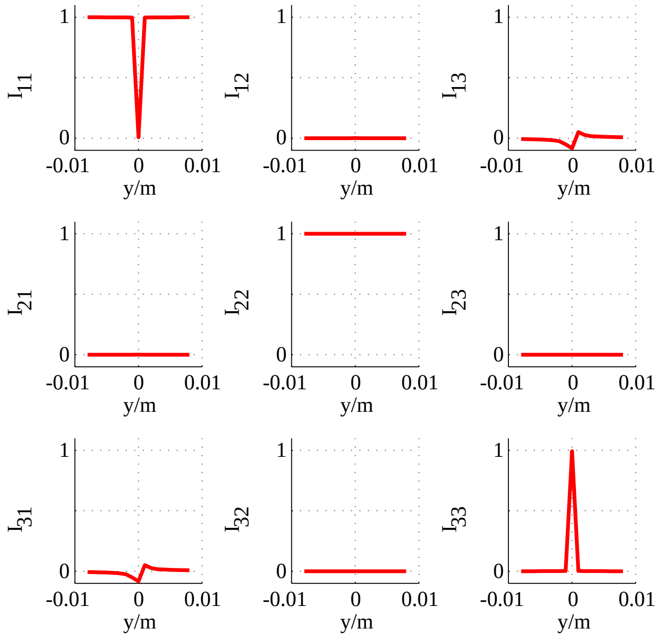


Figure 6. The product of $\mathbf{T}_m\mathbf{K}_m$ has a singularity point at $y = 0$.

allows to control the force in $y$ direction with the current component $i_y$ and the bearing forces with the component $\Delta i$. Consequently follows from (9) and (16) with (26) for the static bearing forces

$$\begin{aligned} F_{b1} &= \frac{dk_{xx}+2k_{\varphi x}+2k_{xx}y}{2d}x + \frac{2dT_{mx}+4(T_{m\varphi}+T_{mx}y)}{2d}\Delta i + \\ &+ \frac{dk_{x\varphi}+2(k_{\varphi\varphi}+k_{x\varphi}y)}{2d}\varphi \\ F_{b2} &= \frac{dk_{xx}-2k_{\varphi x}-2k_{xx}y}{2d}x + \frac{2dT_{mx}-4(T_{m\varphi}+T_{mx}y)}{2d}\Delta i + \\ &+ \frac{dk_{x\varphi}-2(k_{\varphi\varphi}+k_{x\varphi}y)}{2d}\varphi. \end{aligned} \tag{27}$$

Defining the movement in $x$-coordinate as eccentricity $e$ resulting from wear, the max. tilting angle can be expressed as

$$\varphi_{max} = \arctan\left(\frac{d}{e}\right) \approx \frac{d}{e}. \tag{28}$$

Using the maximum tilting angle to compare the terms influenced by $\varphi$ in (27) shows a negligible influence of the tilting angle on the bearing forces of the investigated actuator. Together with the result of (25) follows that the bearing forces can be reduced controlling the eccentricity in $x$-direction, while the tilting can be neglected.

## VI. Optimal Reduction of Bearing Forces

For the two coil actuator follows from (27) that both bearing forces cannot be canceled completely. Thus the function

$$f(y) = F_{b1}(y)^2 + F_{b2}(y)^2 \tag{29}$$

is introduced to minimize both bearing forces simultaneously. Solving

$$\frac{\mathrm{d}f}{\mathrm{d}\Delta i}(y) = 0 \tag{30}$$

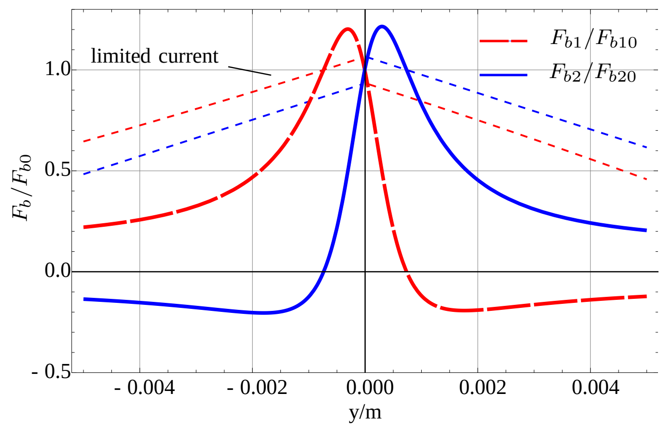


Figure 7. Relative force reduction for the two phase system with minimum bearing forces (thick) and a current limit of 5% of the rated current (dashed).

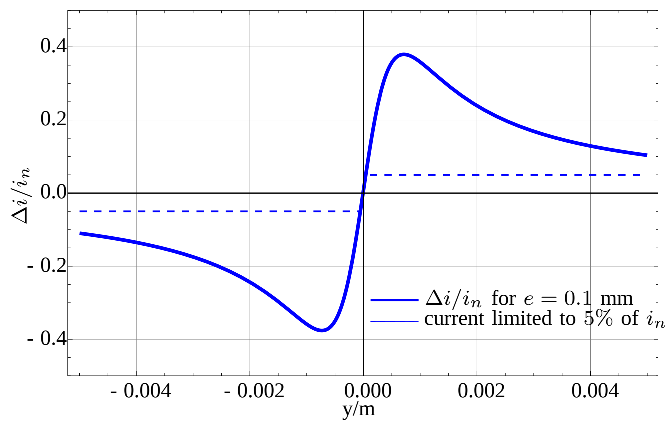


Figure 8. The current $\Delta i/i_n$ required for the two phase system to achieve minimum bearing forces for an eccentricity of 0.1mm due to wear is shown with a thick line and the current limited to 5% of the rated current is shown with a dashed line.

results in

$$\begin{aligned} \Delta i(y) = &-\frac{d^2k_{xx}T_{mx} + 4T_{ma}\left(k_{\varphi x} + k_{xx}y\right)}{2\left(d^2T_{mx}^2 + 4T_{ma}^2\right)}x - \\ &-\frac{d^2k_{x\varphi}T_{mx} + 4T_{ma}\left(k_{\varphi\varphi} + k_{x\varphi}y\right)}{2\left(d^2T_{mx}^2 + 4T_{ma}^2\right)}\varphi \end{aligned} \tag{31}$$

with

$$T_{ma}(y) = T_{m\varphi} + T_{mx}y,$$

where again the term proportional to $\varphi$ can be neglected for the investigated actuator.
The minimized bearing forces result in

$$\begin{aligned} F_{b1,2}(y) = \pm&\frac{dT_{mx} \mp -2T_{ma}}{d^2T_{mx}^2 + 4T_{ma}^2}\cdot \\ &\cdot\left(\left(k_{\varphi x}T_{mx} - k_{xx}T_{m\varphi}\right)x + \right. \\ &\left. + \left(k_{\varphi\varphi}T_{mx} - k_{x\varphi}T_{m\varphi}\right)\varphi\right) \end{aligned} \tag{32}$$

and will only disappear at selected $y$ positions as long as an eccentricity is present. Figure 7 shows the analytic calculation based on FE data of the resulting minimum bearing forces with respect to the original bearing forces for the investigated

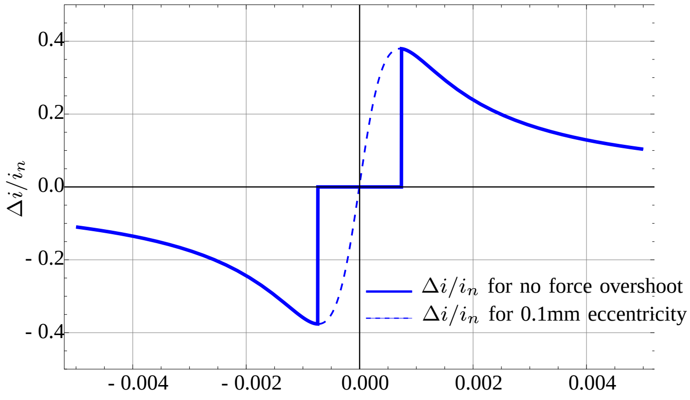


Figure 9. The current $\Delta i/i_n$ required for the two phase system to achieve minimum bearing forces for an eccentricity of 0.1mm due to wear is shown as dashed line and the current to avoid any overshoot of the bearing forces is shown with a thick line.

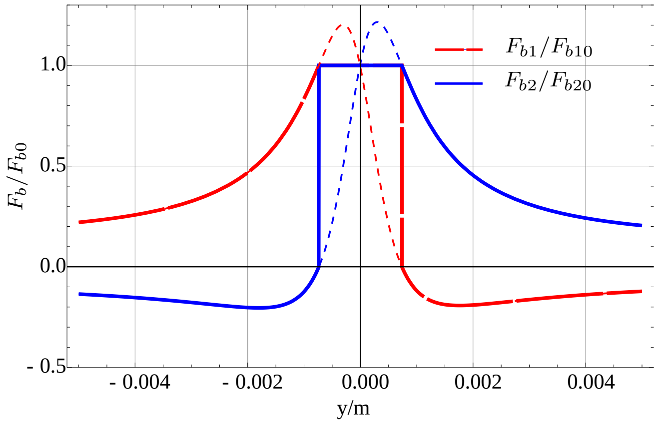


Figure 10. Relative force reduction of two phase system with minimum bearing forces (dashed) and no overshoot of bearing forces (thick).

system. It confirms the result derived from the $\mathbf{T}_m$ matrix, that it is not possible to cancel the bearing forces around the center position $y = 0$. The limit can be found from (32) as

$$y = \pm\left(\frac{d}{2} + \frac{T_{m\varphi}}{T_{mx}}\right). \tag{33}$$

Within this region only one bearing force can be reduced, while the other one increases.

### A. Influence of limits

The current $\Delta i$ that is required to reduce the bearing forces scales linear with the eccentricity. Thus, the maximum current density (here equal to the rated current) defines for a required force $F_y$, the maximum force reduction. Figure 8 shows the required current in relation to the rated current $i_n$ to achieve the force reduction of Fig. 7 with an eccentricity due to wear of 0.1mm as thick line. The peak amount of roughly 40% of the rated current is considered rather high. As an example the dashed line in Fig. 8 shows a current restricted to only 5% of the rated current, what leaves 95% of the rated current for the actuation in $y$-direction. Even with this restriction the bearing forces can considerably be reduced as shown in Fig. 7 with dashed lines. The terms $F_{b10}$ and $F_{b20}$ are the bearing forces if no compensation is applied.
To avoid any increase or overshoot of bearing forces in the area around the poorly controllable center position, the current $\Delta i$ has to be set to zero in the interval given in (33) as shown in Fig. 9. This results in a force reduction given in Fig. 10.

## VII. Conclusion and Outlook

In this paper the feasibility of compensating the bearing forces resulting from the PM attractive forces of a linear actuator are investigated based on analytical and numerical models. Though the bearing forces are canceled for an ideally centered position of the mover, wear and an eccentricity will occur by the time. Thus, the possibility is examined to reduce these emerging forces, to reduce the wear and increase the life time of the bearing ring.
The FE analysis shows that the actuator can be linearized with respect to current and magnetic stiffness. The linearized model has been derived and shows that at least four star connected coils are required to cancel both bearing forces for all $y$ positions completely. For the actuator with two phases the forces can no longer be canceled, but a significant reduction can be achieved. The required current and the limits of operation are derived analytically. This results in an approximation of the expected reduction of bearing forces.
The results show the feasibility of a bearing force reduction even for an actuator featuring only two coils.
Currently, a prototype of the system is built to allow an experimental validation of the presented results.

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## References

[1] J. J. H. Paulides, J. L. G. Janssen, and E. Lomonova, “Bearing lifetime of linear pm machines,” in *Energy Conversion Congress and Exposition, 2009. ECCE 2009. IEEE*, 2009, pp. 1083–1090.
[2] Woo-Yeon Kim, Jong-Min Lee, and Seung-Jong Kim, “Rolling motion control of a levitated mover in a permanent-magnet-type bearingless linear motor,” *Magnetics, IEEE Transactions on*, vol. 46, no. 6, pp. 2482–2485, 2010.
[3] Nan-Chyuan Tsai and Chao-Wen Chiang, “Design and analysis of magnetically-drive actuator applied for linear compressor,” *Mechatronics*, vol. 20, no. 5, pp. 596–603, 2010.
[4] ——, “High-frequency linear compressor and lateral position regulation,” *Control Systems Technology, IEEE Transactions on*, vol. 20, no. 1, pp. 127–138, 2012.
[5] S. Silber and W. Amrhein, “Force and torque model for bearingless pm motors,” *Proceedings of the International Power Electronics Conference (IPEC), Vol. 1, Tokyo*, Apr. 2000.
[6] ——, “Power optimal current control scheme for bearingless pm motors,” *7th International Symposium on Magnetic Bearings (ISMB), Zurich, Switzerland*, pp. 401–406, Aug. 2000.