Identification of magnetic bearing system using a novel subspace identification method

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***Abstract*—Active magnetic bearings (AMBs) are widely applied to support high-speed rotating machineries, especially in special environment. Identification is a useful method to obtain the mathematical model of an AMB system. Frequency domain subspace identification method (FDSIM) is an attractive identification technique. However, in practice one usually expects that the identified model possesses some properties, for example stability. FDSIM cannot ensure these expected properties. This paper firstly proposes a novel subspace identification method, FDSIM-λ. This method can guarantee that the resultant model possesses predefined poles. Moreover, an optimization procedure is introduced to obtain the optimal poles based on FDSIM-λ, the initial estimation of system poles and constraints on the system poles. The proposed method is then applied to identify an AMB system and validated by the experiments on a real AMB system.**

I. Introduction

Compared with conventional bearings, active magnetic bearings (AMBs)[1], [2] possess several attractive advantages, such as no friction, no need of lubrication, and the ability of long-term high speed running. Therefore AMBs are widely applied to support high-speed rotating machineries, especially in special environment. Nevertheless, for each application case, the AMB system should be specifically designed, installed and adjusted. This procedure is quite costly and time-consuming. The mathematical model of an AMB system plays an important role in designing and adjusting the whole system. The models of parts in an AMB system can be computed numerically [1], [2], [3]. However, the practical parts are usually so complex that the computational models of flexible rotors and AMB systems are not precise enough.

Identification is the procedure of estimating the model of a system based on measured input-response data of this system. It is an useful method to obtain the system model. From the viewpoint of control system analysis, an AMB system with a flexible rotor possesses several special features. First, it is well-known that an AMB system is open-loop unstable. This obstructs the application of most well-developed identification methods where the stability of system is assumed. Second, the flexible rotor lacks of internal damping, which results in a series of numerical problems in computation. Based on these reasons, frequency domain test and identification are often utilized to attain the model of an AMB system.

Frequency domain subspace identification method (FDSIM)[4], [5], [6] is an attractive identification technique and an improved version of FDSIM proposed in [5] and the so-called w-operator is introduced in this method to improve computational aspects. In this method, the continuous-time state space description $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ of a system is utilized and the system model is then transferred to a w-domain description $(\mathbf{A}_w, \mathbf{B}_w, \mathbf{C}_w, \mathbf{D}_w)$. The column space of the observation matrix $\begin{bmatrix} \mathbf{C}_w^\top & (\mathbf{C}_w\mathbf{A}_w)^\top & \cdots & (\mathbf{C}_w\mathbf{A}_w^{i-1})^\top \end{bmatrix}^\top, i \in \mathcal{N}$ is estimated based on frequency domain response (FDR) data and estimation of the model $(\mathbf{A}_w, \mathbf{B}_w, \mathbf{C}_w, \mathbf{D}_w)$ can be attained. The FDSIM possesses excellent numerical stability and generality, therefore it is applied to many identification problems, including the region of AMB systems[7].

However, the FDSIM possesses a serious defect, namely no prior knowledge and assumption is involved in the identification. In other words, the stability and any other properties of the resultant model cannot be guaranteed by this method. To overcome this defect, efforts on subspace identification method with guaranteed stability are reported [8], [9]. Nevertheless, as mentioned, an AMB system is open-loop unstable and the damping ratios of the bending modes are relatively small, these characteristics obstruct the direct application of these modified methods. It is important to notice that the poles of an AMB system can be roughly estimated by the numerical computations and the FDR measurements. In this paper, a novel FDSIM method is proposed, the proposed method ensures that the resultant model possesses predefined poles. Then, an optimization procedure is introduced to modify the estimated poles. The initial estimation of the system poles and constraints on the system poles will be introduced in this optimization procedure. In this way, the properties of the poles of the resultant system can be ensured. The proposed method is then applied to identify an AMB system and validated by the experiments on a real AMB system.

In this paper, the notation † is used to denote the Moore-Penrose generalized inverse of a matrix, $\|\circ\|$ denotes the Frobinius norm of a matrix. The notation $R_{(m_1,m_2)}(\mathbf{U})$ reflects the matrix composed of the m_1-th to m_2-th rows of matrix $\mathbf{U}$, $\Leftrightarrow$ denotes that two optimization problems are equivalent. For matrixes $(\mathbf{A}, \mathbf{C})$ and some integer i, the generalized observation matrix is defined as

$$\mathbf{G}_o(\mathbf{C},\mathbf{A},i)=\begin{bmatrix}\mathbf{C}\\ \mathbf{CA}\\ \vdots\\ \mathbf{CA}^{i-1}\end{bmatrix}. \quad (1)$$

Similarly, for $(\mathbf{A},\mathbf{V})$ and i, the generalized input matrix is defined as

$$\mathbf{H}_i(\mathbf{V},\mathbf{A},i)=\begin{bmatrix}\mathbf{V} & \mathbf{AV} & \cdots & \mathbf{A}^{i-1}\mathbf{V}\end{bmatrix}. \quad (2)$$

II. Subspace identification with predefined poles

A. Frequency domain subspace identification method

For a system $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D})$ with $\mathbf{A}\in\mathcal{R}^{n\times n},\mathbf{B}\in\mathcal{R}^{n\times h},\mathbf{C}\in\mathcal{R}^{m\times n},\mathbf{D}\in\mathcal{R}^{m\times h}$, for some $i\geq n$, the generalized observation matrix $\mathbf{G}_o(\mathbf{C},\mathbf{A},i)$ plays the center role in the subspace identification methods [4], [5], [6]. More specifically, in the subspace identification methods, the column space of $\mathbf{G}_o(\mathbf{C},\mathbf{A},i)$ can be estimated based on the time domain or frequency domain measurement of system inputs and outputs. Then the system matrixes $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D})$ are evaluated based on the estimation of $\mathbf{G}_o(\mathbf{C},\mathbf{A},i)$. As an improvement of the standard FDSIM, [5] introduced the so-called w-operator to prevent numerical illness or ill-condition. This paper concentrates on the w-FDSIM (namely FDSIM with w-operator) and this paper mainly focuses on the evaluation of w-domain system matrixes $(\mathbf{A}_w,\mathbf{C}_w)$ based on the estimation of column space of $\mathbf{G}_o(\mathbf{C}_w,\mathbf{A}_w,i)$. The estimation of the column space of $\mathbf{G}_o(\mathbf{C}_w,\mathbf{A}_w,i)$ and evaluation of $(\mathbf{B}_w,\mathbf{D}_w)$ are realized by the algorithm proposed in [5].

B. FDSIM with predefined poles

In the standard w-FDSIM, the column space of the observation matrix $\mathbf{G}_o(\mathbf{C}_w,\mathbf{A}_w,i)$ is estimated as a matrix $\hat{\mathbf{U}}_n$, which is constructed based on FDR measurement. The estimations $\hat{\mathbf{C}}_w$ and $\hat{\mathbf{A}}_w$ are evaluated directly by

$$\hat{\mathbf{C}}_w=R_{(1,m)}\left(\hat{\mathbf{U}}_n\right), \quad (3a)$$

$$\hat{\mathbf{A}}_w=R_{(1,m(i-1))}\left(\hat{\mathbf{U}}_n\right)^{\dagger}R_{(m+1,mi)}\left(\hat{\mathbf{U}}_n\right). \quad (3b)$$

However, no prior knowledge and assumptions about $\hat{\mathbf{C}}_w$ and $\hat{\mathbf{A}}_w$ are involved in (3). An obvious problem is that the estimated $\hat{\mathbf{A}}_w$ may be unstable, no matter $\mathbf{A}_w$ is stable or not. Some modified methods with guaranteed stability are reported [4], [8], [9]. Nevertheless, an AMB system is open-loop unstable and these methods cannot be applied directly.

To overcome these defects, we propose a novel method to estimate $\hat{\mathbf{C}}_w$ and $\hat{\mathbf{A}}_w$ based on $\hat{\mathbf{U}}_n$. Since a state space model is invariant under similar transformation, we can define

$$\hat{\mathbf{A}}_w=\mathbf{U}\bar{\mathbf{A}}_w\mathbf{V},\hat{\mathbf{C}}_w=\bar{\mathbf{C}}_w\mathbf{V}, \quad (4)$$

where $\bar{\mathbf{A}}_w$ is a real block diagonal matrix, whose eigenvalues equal the predefined poles $\boldsymbol{\lambda}=\{\lambda_k\}$ and $\mathbf{V}=\mathbf{U}^{-1}$ is a matrix to be optimized. In this paper, for given $\boldsymbol{\lambda}$, the following optimization problem is solved to obtain $\mathbf{V}$ and $\bar{\mathbf{C}}_w$

$$\min_{\mathbf{V},\bar{\mathbf{C}}_\mathbf{w}}\left\|\mathbf{G}_o\left(\hat{\mathbf{C}}_w,\hat{\mathbf{A}}_w,i\right)-\hat{\mathbf{U}}_n\right\|^2. \quad (5)$$

The optimization problem (5) can be further reduced. In fact, suppose that

$$\hat{\mathbf{U}}_n=\begin{bmatrix}\mathbf{U}_1\\ \vdots\\ \mathbf{U}_i\end{bmatrix}, \quad (6)$$

with $\mathbf{U}_k\in\mathcal{R}^{m\times n},k=1,\cdots,i$. Define the block rearrangement of $\hat{\mathbf{U}}_n$ as

$$\hat{\mathbf{U}}'_n=\begin{bmatrix}\mathbf{U}_1 & \cdots & \mathbf{U}_i\end{bmatrix}, \quad (7)$$

then it can be verified that

$$\begin{aligned}\min_{\mathbf{V},\bar{\mathbf{C}}_\mathbf{w}}\left\|\mathbf{G}_o\left(\hat{\mathbf{C}}_w,\hat{\mathbf{A}}_w,i\right)-\hat{\mathbf{U}}_n\right\|^2 &\Leftrightarrow \min_{\mathbf{V},\bar{\mathbf{C}}_\mathbf{w}}\left\|\begin{bmatrix}\bar{\mathbf{C}}_w\mathbf{V}\\ \bar{\mathbf{C}}_w\bar{\mathbf{A}}_w\mathbf{V}\\ \vdots\\ \bar{\mathbf{C}}_w\bar{\mathbf{A}}_w^{i-1}\mathbf{V}\end{bmatrix}-\hat{\mathbf{U}}_n\right\|^2\\ &\xLeftrightarrow{\mathbf{H}=\mathbf{H}_i\left(\mathbf{V},\bar{\mathbf{A}}_w,i\right)}\min_{\mathbf{V},\bar{\mathbf{C}}_\mathbf{w}}\left\|\bar{\mathbf{C}}_w\mathbf{H}-\hat{\mathbf{U}}'_n\right\|^2\\ &\xLeftrightarrow{\bar{\mathbf{C}}_w=\hat{\mathbf{U}}'_n\mathbf{H}^{\dagger}}\min_{\mathbf{V}}\left\|\hat{\mathbf{U}}'_n\mathbf{H}^{\dagger}\mathbf{H}-\hat{\mathbf{U}}'_n\right\|^2. \quad (8)\end{aligned}$$

The last optimization problem in (8) is a nonlinear optimization problem associated with the invertible matrix $\mathbf{V}$. This optimization problem is not easy to solve in practice. In this paper the random optimization technique is utilized to obtain a "good enough" solution of $\hat{\mathbf{V}}$. That is, for a large enough number N, N invertible candidate matrixes $\{\mathbf{V}_k\}_{k=1}^N$ are generated from some probability distribution, then the "good enough" estimation $\hat{\mathbf{V}}$ is given by

$$\hat{\mathbf{V}}=\left\{\mathbf{V}_\kappa:\nu\left(\mathbf{V}_\kappa\right)=\min_{1\leq k\leq N}\nu\left(\mathbf{V}_k\right)\right\}, \quad (9)$$

where

$$\nu\left(\mathbf{V}\right)=\left\|\hat{\mathbf{U}}'_n\mathbf{H}^{\dagger}\mathbf{H}-\hat{\mathbf{U}}'_n\right\|^2,\mathbf{H}=\mathbf{H}_i\left(\mathbf{V},\bar{\mathbf{A}}_w,i\right). \quad (10)$$

Finally,

$$\bar{\mathbf{C}}_w=\hat{\mathbf{U}}'_n\mathbf{H}_i\left(\hat{\mathbf{V}},\bar{\mathbf{A}}_w,i\right)^{\dagger}, \quad (11)$$

$$\hat{\mathbf{A}}_w=\hat{\mathbf{V}}^{-1}\bar{\mathbf{A}}_w\hat{\mathbf{V}},\hat{\mathbf{C}}_w=\bar{\mathbf{C}}_w\hat{\mathbf{V}}. \quad (12)$$

In this paper, the proposed FDSIM with predefined poles will be abbreviated as "FDSIM-$\boldsymbol{\lambda}$".

III. Identification based on FDSIM-$\boldsymbol{\lambda}$ with initial estimation of system poles

For a system, suppose the FDR data

$$\mathbf{G}_m\left(\mathrm{j}\omega_k\right),k=1,2,\cdots,M \quad (13)$$

is obtained by the swept-sine measurement. Moreover, suppose the poles of the subsystem can be roughly estimated as $\boldsymbol{\lambda}_0$.

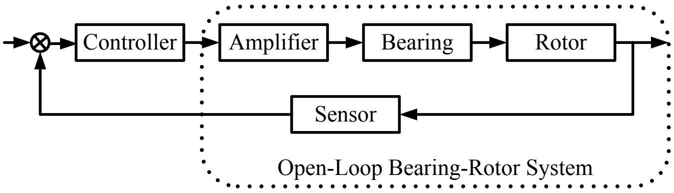

Figure 1: Scheme of an AMB system

For given pole set $\boldsymbol{\lambda}$, the identified model by FDSIM-$\boldsymbol{\lambda}$ is $\hat{\Psi}(\boldsymbol{\lambda}) = (\mathbf{A}(\boldsymbol{\lambda}), \mathbf{B}(\boldsymbol{\lambda}), \mathbf{C}(\boldsymbol{\lambda}), \mathbf{D}(\boldsymbol{\lambda}))$ and the corresponding frequency responses are

$$\hat{\mathbf{G}}_i(\boldsymbol{\lambda}, \mathrm{j}\omega_k), k = 1, 2, \cdots, M. \tag{14}$$

Then the pole set is estimated by solving the following optimization problem:

$$\hat{\boldsymbol{\lambda}} = \underset{\boldsymbol{\lambda}}{\operatorname{argmin}} \sum_{k=1}^{M} \left\| \hat{\mathbf{G}}_i(\boldsymbol{\lambda}, \mathrm{j}\omega_k) - \mathbf{G}_m(\mathrm{j}\omega_k) \right\|^2. \tag{15}$$

The initial estimation of poles $\boldsymbol{\lambda}_0$ is used as the initial value in this optimization problem. Some constraints can be incorporated into this optimization problem to ensure the resultant optimal poles obey some known properties. This optimization problem can be solved by standard nonlinear optimization algorithm and the details of solving is ignored in this paper.

IV. Experiment: identification of an AMB system

In this section, the proposed method is applied to identify an AMB system.

A. *System model and problem description*

This paper concentrates on AMB systems with fully suspended rotors. This paper supposes that the axial and radial dynamic characteristics of the rotor can be decoupled. Moreover, this paper considers static identification, that is, during the identification experiments the rotor does not rotate. Thus gyroscopic effects [3] are not involved in the experiments and the dynamic features in two orthogonal radial planes can also be decoupled. In this paper we only consider the dynamic model of the rotor in one radial plane.

Since AMB systems are open-loop unstable, only closed-loop experiments can be performed. However, the model of the controller is known, thus it is not difficult to compute the open-loop model by block transformations. The system scheme of a typical AMB system is shown in Fig. 1. As shown in Fig. 1, this paper mainly considers the open-loop system consisting of the power amplifier, the magnetic bearing, the flexible rotor and the sensor. In the following part of this paper, we use the term "Open-Loop Bearing-Rotor System (OLBRS)" to refer to this system. In a radial plane, the OLBRS is a (2×2)-dimensional system.

B. *Experiment setup and data preprocessing*

The magnetic bearing system used in the experiments is shown as Fig. 2. A detailed description of this system can be found in [10]. This system is characterized by an about 3.5m long and 630kg heavy rotor. Four radial electromagnetic bearings and an axial one are mounted to suspend the rotor. The nominal maximum displacement of rotor, restricted by the clearance of auxiliary bearing, is 150μm. The definition of coordinate system in this bearing system is illustrated in Fig. 2. The nature frequencies of the first three free-free bending modes of the rotor are approximately 45Hz, 122Hz and 239Hz, respectively. A maximum rotational speed of 230Hz is achieved on this system, namely beyond the second bending mode.

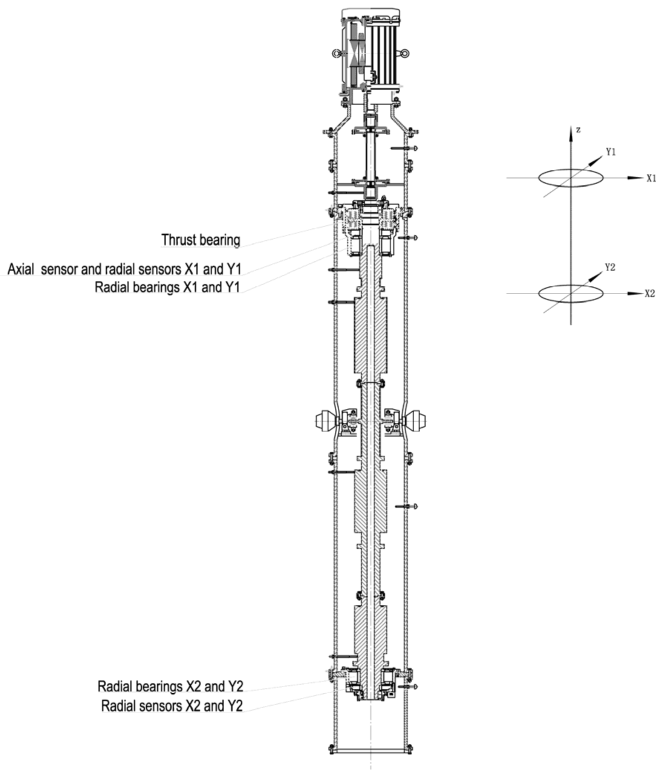

Figure 2: Magnetic bearing-rotor system

Closed-loop swept-sine measurements are performed on this AMB system and the FDR model of OLBRS is then calculated. For such a large-size AMB system, the system delay will significantly affect the FDR. However, the system delay model is not involved in a FDSIM model and will bring negative effect to identification, thus it should be eliminated. In this paper, the method proposed in [11] is applied to estimate and eliminate the system delay.

C. *Experiment results*

The experiment result is shown in Fig. 3. Here Bode plot is applied to express the magnitude of the frequency response gain and the phase shift. Since the OLBRMs discussed in this paper are $2{\times}2$ systems, four subplots are contained in the plot. The arrangement of the subplots corresponds to the arrangement of the transfer function matrix. In this Bode plot, blue dots denote the measured FDR and red lines denote the frequency response of identified model.

As shown in Fig. 3, the Bode plots of identified model fit the measured FDR precisely. This fact validates the proposed

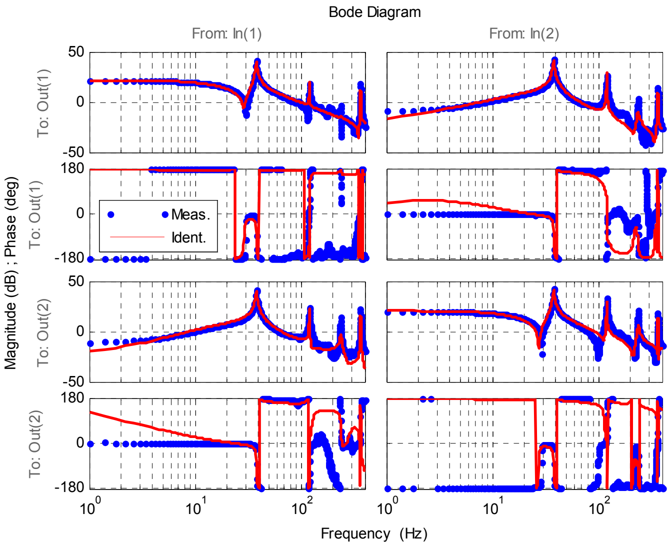

Figure 3: Experiment result

identification method. However, some antiresonances (e.g. the antiresonance of the second bending mode) are not correctly identified. In our opinion, the main reason is that FDSIM is essentially a least-square type method, the estimation error near antiresonances will be submerged by the error near resonances, as discussed in [11].

V. Conclusions and discussions

In this paper a novel FDSIM is proposed. The proposed method FDSIM-$\boldsymbol{\lambda}$ can ensure that the resultant model possesses predefined poles. Based on the proposed method, an initial estimation of the system poles and constraints on the poles, the system can be identified and it can be guaranteed that the resultant system possesses some predefined properties. The proposed method is applied to identify AMB systems. Experiments on a large-size AMB system validate the proposed method.

The future works of this paper include:

1) In this paper, a random optimization strategy is applied to find a "good enough" transform matrix $\hat{\mathbf{V}}$. However, this is quite inefficient and the exact optimum cannot be achieved. Analytical or efficient numerical solution of Eq. (8) should be developed.

2) As mentioned, the antiresonances are not precisely fitted by the proposed method. On the other hand, as thoroughly discussed in [12], [13], [11], the antiresonance frequencies play an important role in defining the rotor's flexible behavior, therefore one expects precise agreement of the identified model with the measured data near antiresonance frequencies. A simple solution to this problem is adding a cost term associated to the antiresonances to Eq. (15). However, this method will not affect the least-square nature of FDSIM. Researches on this problem should be performed.

3) In this paper, the parameters in FDSIM are chosen empirically. Moreover, for a stable system, the numerical stability can be ensured by the w-FDSIM, however, an AMB system is essentially instable, so that numerical illness or ill-condition may arise while applied w-FDSIM-based methods and the paramter α in w-FDSIM (see [5]) should be selected carefully. In future works, we expect to make detailed researches on the choice of these parameters and develop novel identification method to deal with instable systems.

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