Nonlinear Nonparametric Identification of Magnetic Bearing System Based on Support Vector Regression

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Abstract

The mathematical model of components in an active magnetic bearing system plays important roles in system designing and adjusting. The linearized models are widely applied; however, sometimes these models are not precise enough. In this paper, a nonlinear modeling method based on Support Vector Regression (SVR) is proposed. The proposed model treats the rotor acceleration as a nonlinear function of the rotor displacement, rotor velocity and the coil currents; the model parameters are determined by a training procedure based on observed samples, no mechanism, computational or empirical models are needed in the modeling procedure. The proposed method is validated by a series of experiments.

1 Introduction

Compared with conventional bearings, active magnetic bearings (AMBs) [1,2] possess several attractive advantages, such as no friction, no need of lubrication, and the ability of long-term high speed running. Nevertheless, unlike the well-standardized sliding bearings and rolling element bearings, no ready-to-use AMBs are available yet. In other words, for each application case, the AMB system should be specially designed, installed and adjusted. This procedure is quite costly and time-consuming. The mathematical models of the components in the an AMB system, such as rotors, electromagnets, sensors, controllers and power amplifiers, play important roles in designing and adjusting the whole system and the precision of models significantly affects the performance and efficiency of the designing and adjustment procedure.

The characteristics of magnetic bearings are inherently nonlinear, so that in some cases (e.g. when the displacement of rotor is large or the operation frequency is high) the well-known linearized models [2] are not precise enough [3,4]. Therefore it is of great significant to establish a precise large range nonlinear model of the magnetic bearing system.

Some researches on the nonlinear modeling of magnetic bearing system are reported [3-6], all these works are based on parametrical regression technique, namely some mechanism and/or empirical models are utilized in modeling. Unlike these methods, this paper proposes a modeling method of the magnetic bearing-rotor systems. The proposed method utilizes the Support Vector Regression (SVR) technique [7,8]. In the proposed model, the rotor acceleration is treated as a nonlinear function of the rotor displacement, rotor velocity and the coil currents and the model parameters are determined by a training procedure based on training samples, no mechanism, computational or empirical models are needed in the modeling procedure.

An experimental system with a five degrees-of-freedom (DoF) levitated rotor (about 3.5m long and 630kg heavy) is utilized to perform a series of sinuous-swept experiments and the proposed method is validated by the experiment data. The experiment results show the validity of the proposed method.

In this paper, the displacement of the rotor will be denoted by x, the velocity of the rotor (i.e. the derivation of x) will be denoted as $\dot{x}$ and the acceleration of rotor as $\ddot{x}$. The notations i_+ and i_- stand for the currents in the plus

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and minus coils of the magnetic bearing. For a time instant h, define the sample as $\mathbf{z}_h = \begin{bmatrix} x_h & i_{+,h} & i_{-,h} & \dot{x}_h \end{bmatrix}^{\mathrm{T}}$, or simply $\mathbf{z} = \begin{bmatrix} x & i_+ & i_- & \dot{x} \end{bmatrix}^{\mathrm{T}}$.

2 Support Vector Regression

Support vector regression (SVR) is a novel nonparametric modeling method. The SVR possesses the following attractive features: 1) A large class of modeling problems can be treated with SVR, since by SVR modeling the only mathematical assumption on the actual model is the Lipschitz continuity. 2) SVR is a nonparametric modeling method, that is, no prior model is needed in modeling. 3) The computational complexity of a SVR merely depends on the dimension of the problem. 4) SVR is a distribution-free method, i.e. no prior-knowledge or assumptions on the distribution of the samples are needed other than that the training samples and test samples are generated independently from the same distribution. 5) The generalization ability of SVR is ensured theoretically. The generalization ability is the precision (in the statistical sense) of a modeling method when only finite samples are available. Based on the statistical learning theory [9], the generalization ability of SVR can be estimated, that is, the boundary on modeling error of a SVR can be calculated in the statistical sense. With these features, SVR provides solutions to many complex modeling problems.

Suppose the actual model of some relationship can be described as follows:

$$\zeta = \varphi(\boldsymbol{\xi}) + \upsilon \tag{1}$$

where $\zeta \in \mathbb{R}$ is the observation of output of φ, $\boldsymbol{\xi} \in \mathbb{R}^n$ is the n-dimensional variable, $\varphi : \mathbb{R}^n \mapsto \mathbb{R}$ is a real-valued function and υ is the observation noise. Suppose m two-tuples $\{\boldsymbol{\xi}_k, \zeta_k\}, k = 1, \cdots, m$ are observed, the SVR model is in the following form:

$$\hat{\zeta} = \rho + \sum_{k=1}^{m} \alpha_k K(\boldsymbol{\xi}_k, \boldsymbol{\xi}), \tag{2}$$

where $\hat{\zeta}$ is the estimation of $\zeta(\boldsymbol{\xi})$, α_k are coefficients and ρ is the bias, $K(\circ,\circ) : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ is a predefined bivariable real-valued function called as kernel function. The parameters α_k and ρ will be determined in the training procedure. The SVR method makes a tradeoff between the model complexity (i.e. the smoothness of function (2)) and the estimation error, this feature ensures the boundedness of the generalization ability of SVR.

3 Experiment set up

3.1 The magnetic bearing system

The magnetic bearing system used in the experiments is shown as Figure 1.

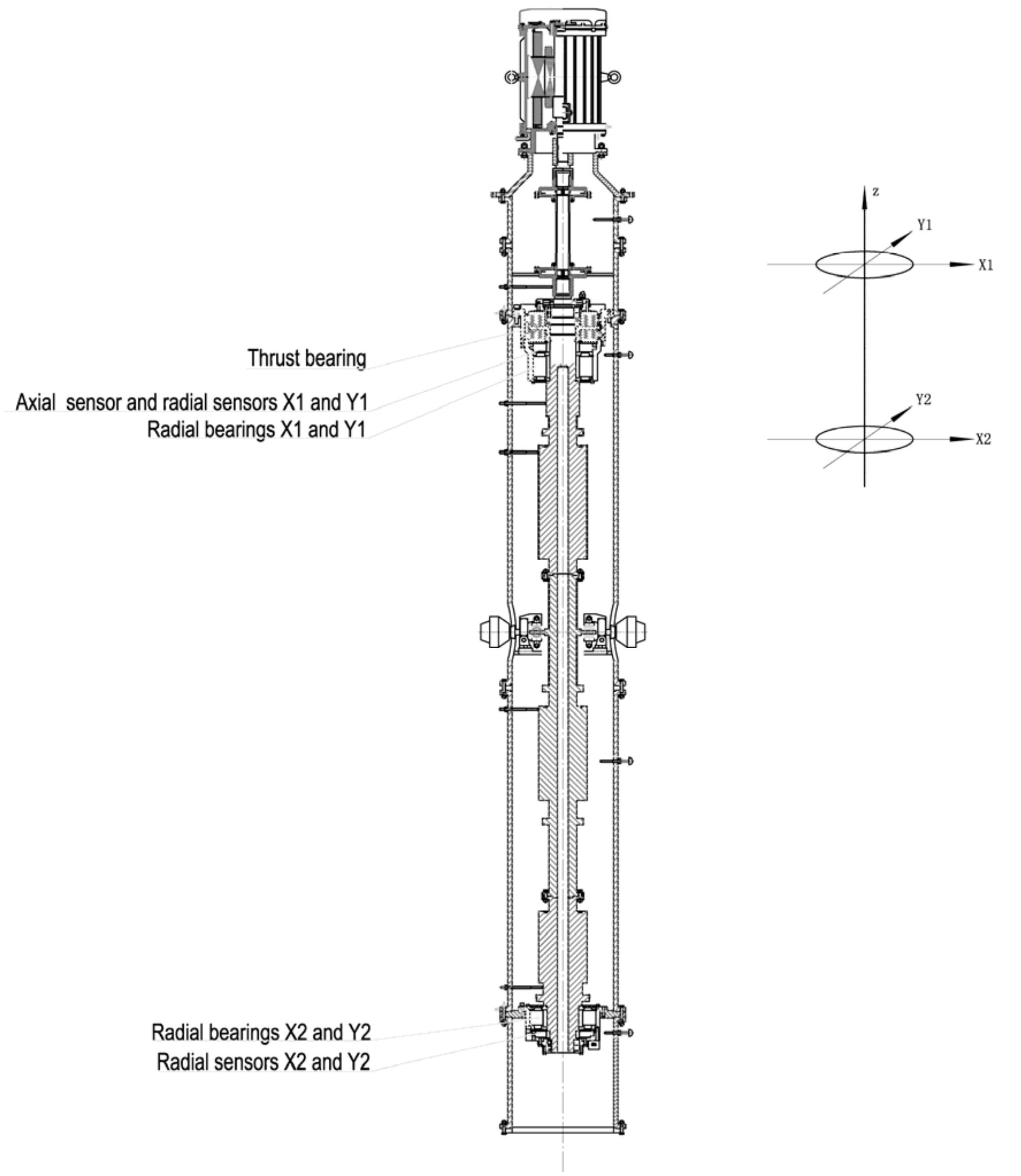

Figure 1: Magnetic bearing-rotor system

This system is characterized by an about 3.5m long and 630kg heavy rotor. Four radial electromagnetic bearings and an axial one are mounted to levitate the rotor. The displacement of the rotor is measured by five sensors. Based on the disvplacement signal, a DSP controller computes dynamically the adequate current to levitate the rotor and an amplifier produces the corresponding current. In this system, the bearings work in the differential mode, thus a bearing contains two electromagnets. The definition of coordinate system in this bearing system is illustrated in Figure 1. In the reminder part of this paper, the electromagnet driving the rotor to direction X1+ is called as “bearing X1+”, and so on.

The radial gap between the rotor and the auxiliary bearing is 0.3mm in diameter. In other words, support that the levitation position of the rotor exactly matches the center of the auxiliary bearing and no significant bending of the rotor occurs, the range of radial motion of the rotor is $\pm 0.15\mathrm{mm}$.

3.2 Experiment procedure

Throughout all experiments, the rotor is levitated in five DoF, the bias current of bearings X1 and Y1 is 4A. Sinusoidal disturbance is added into the current of bearing Y1- to excite the rotor. The frequency of the disturbance signal is fixed as 23.3Hz and the magnitude varies in different experiments. Four experiments are performed with 0.8A, 1.6A, 2.4A and 3.2A disturbance are performed. In every experiment, the duration of the disturbance signal is 5.29 seconds, i.e. one second plus 100 periods of the signal. The disturbance signal, the displacement of the rotor and the current of the bearings are sampled and recorded every 0.1ms. The relationship between the rotor displacement and the phase of disturbance is shown in Figure 2.

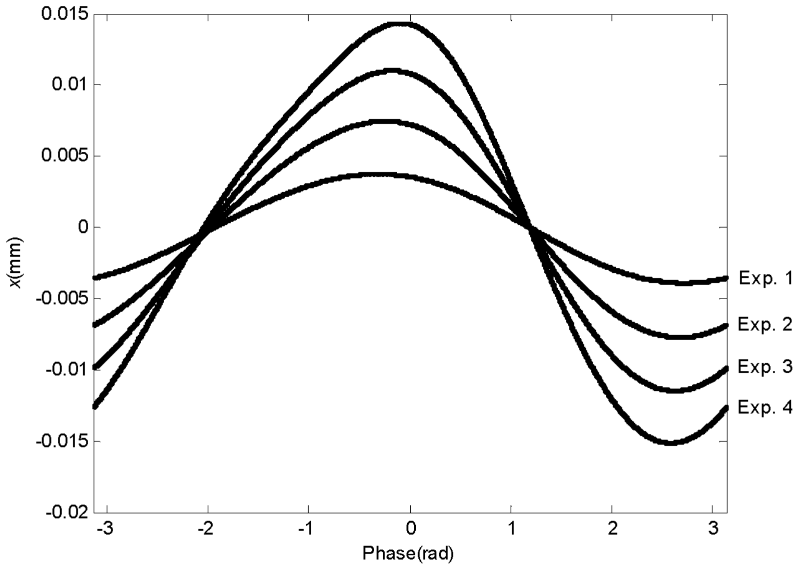

Figure 2: Rotor displacements vs. disturbance phase in various experiments

3.3 Data pre-processing

The proposal of data pre-processing is to recover the physical quantities, such as rotor displacement, velocity and acceleration and bearing currents, from the noisy sampled data. To prevent the effect of system transition, in every experiment the data in the first one second are neglected, namely only the last 100 periods of data are applied. In every experiment 42919 data are applied.

Moreover, as is well known, estimating the velocity and acceleration from noisy displacement data is merely realizable. However, in our case the periodicity of the signals can significantly improve the estimation. More specifically, since the disturbance signal is less noisy, the sampled data can be divided into periods according to the disturbance signal. Suppose that the period is scaled to 2π, then a signal (say x), due to its periodicity, can be expanded as trigonometric series [10]:

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(kt) + b_k \sin(kt) \right), t \in [-\pi, \pi]. \quad (3)$$

In practice (3) is approximated by the partial summation

$$x(t) = a_0 + \sum_{k=1}^{N} \left(a_k \cos(kt) + b_k \sin(kt) \right), t \in [-\pi, \pi]. \quad (4)$$

The coefficients $a_0, a_1, \cdots$ and $b_1, b_2, \cdots$ can be estimated by the sampled data. Once the expansion (4) is obtained, the velocity and acceleration can be estimated by

$$\dot{x}(t) = \sum_{k=1}^{N} \left(-a_k k \sin(kt) + b_k k \cos(kt) \right), t \in [-\pi, \pi] \tag{5}$$

and

$$\ddot{x}(t) = \sum_{k=1}^{N} \left(-a_k k^2 \cos(kt) - b_k k^2 \sin(kt) \right), t \in [-\pi, \pi]. \tag{6}$$

The relationship between the rotor acceleration and the disturbance phase is illustrated in Figure 3. The bearing current $i(t)$ can also be estimated in the similar way.

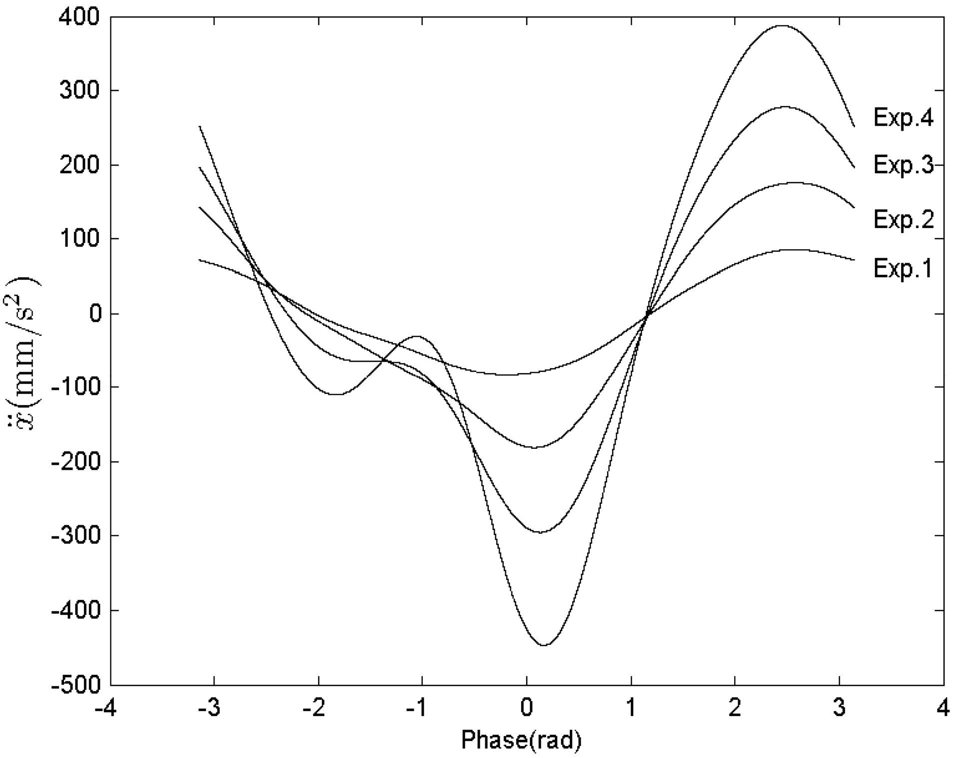

Figure 3: Rotor acceleration vs. disturbance phase

4 Support Vector Regression modeling of magnetic bearing

Suppose a training sample set $\left\{ \mathbf{z}_k^{\text{Train}}, \ddot{x}_k^{\text{Train}} \right\}_{k=1}^{m}$ with m samples is utilized, the dynamic of the bearing-rotor system is then described by the following model:

$$\hat{\ddot{x}} = f(\mathbf{z}) = \rho + \sum_{k=1}^{m} \alpha_k K\left(\mathbf{z}_k^{\text{Train}}, \mathbf{z} \right), \tag{7}$$

where α_k are coefficients and ρ is the bias, they are determined by the training process. In this paper, the LibSVM algorithm [11] is utilized for training.

The training and validation samples are collected in the following manner: for every experiment, 43 data (namely 1/10 of a period of data) are chosen to build training sample set and all 42919 data are chosen to build validation sample set. Since four experiments are involved, there are totally 172 training samples and 171676 validation samples. In this way, samples from all experiments are involved in the training procedure and hence the training result (7) is a general model which is validate for various magnitudes of disturbance.

5 Modeling result and error analysis

The relationship between the actual rotor acceleration $\ddot{x}$ and the estimated rotor acceleration $\hat{\ddot{x}}$ is illustrated in Figure 4. If for a validation sample the estimated acceleration exactly equals to the actual one, then the corresponding point in Figure 3 falls on the line $y = x$. In fact, model (7) is quite precise since almost all of 171676 points lay on line $y = x$, as illustrated in Figure 4.

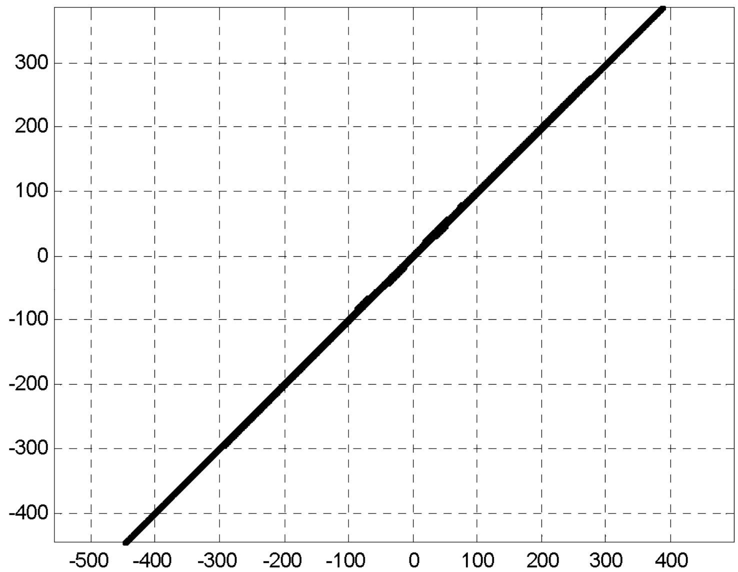

Figure 4: Observed vs. estimated rotor accelerations

Moreover, the error distributions in various experiments are shown in Figure 5, it is obvious that for most validation samples, the estimation error is less than $1\mathrm{mm/s}^2$.

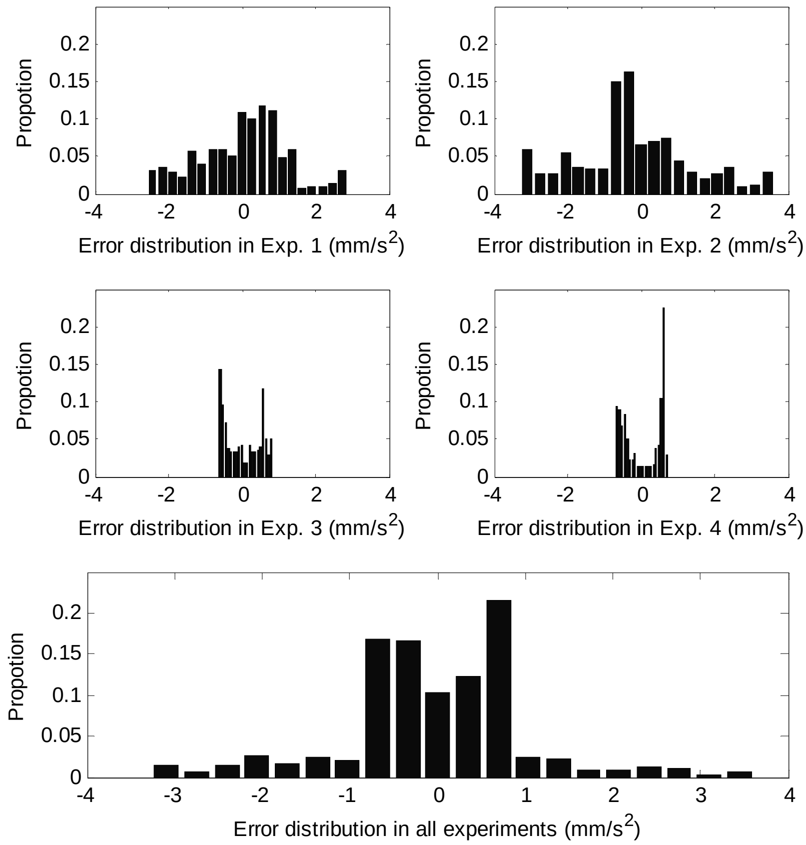

Figure 5: Error distributions in various experiments

Numerically, the precision of modeling is evaluated by the root mean square error:

$$\mathrm{ERR}\left(Z\right)=\sqrt{\frac{1}{l}\sum_{k=1}^{l}\left|\ddot{x}_k^{\mathrm{Validation}}-\hat{\ddot{x}}_k^{\mathrm{Validation}}\right|^2}=\sqrt{\frac{1}{l}\sum_{k=1}^{l}\left|\ddot{x}_k^{\mathrm{Validation}}-f\left(\mathbf{z}_k^{\mathrm{Validation}}\right)\right|^2}, \tag{8}$$

where

$$Z=\left\{\mathbf{z}_k^{\mathrm{Validation}},\hat{\ddot{x}}_k^{\mathrm{Validation}}\right\}_{k=1}^{l} \tag{9}$$

is the validation set with l samples. As mentioned, in this paper $l=171676$ and the total modeling error is

$$\mathrm{ERR}\left(Z\right)=1.06\mathrm{mm/s}^2 \tag{10}$$

Considering that the magnitude of the validation sample set is $\left\|Z\right\|=\sqrt{\frac{1}{l}\sum_{k=1}^{l}\left|\ddot{x}_k^{\mathrm{Validation}}\right|^2}=166.0\mathrm{mm/s}^2$, the relative error $\frac{\mathrm{ERR}\left(Z_k\right)}{\left\|Z_k\right\|}=0.0064$ is less than 1%. The modeling error associated with every experiment is listed in the following table:

Experiment number k	Disturbance magnitude (A)	$\mathrm{ERR}(Z_k)$ ($\mathrm{mm/s^2}$)	$\lVert Z_k \rVert$ ($\mathrm{mm/s^2}$)	$\frac{\mathrm{ERR}(Z_k)}{\lVert Z_k \rVert}$
1	0.8	1.20	58.2	0.0206
2	1.6	1.59	117.7	0.0135
3	2.4	0.49	179.0	0.0027
4	3.2	0.52	246.8	0.0021
Total	0.8/1.6/2.4/3.2	1.06	166.0	0.0064

Table 1: Modeling error in various experiments

The following conclusions can be drawn from the above figures and table:

1) The model (7) is valid for all four experiments.
2) For all experiments, the absolute modeling error is less than $1.6\mathrm{mm/s^2}$ the relative modeling errors are less than 2.1%.
3) The absolute error in experiments 1 and 2 is larger than that in experiments 3 and 4. The reason is that, the SVR modeling procedure makes a tradeoff between the smoothness and the error of the model, on the other hand, the function $\ddot{x}(t)$ in experiments 1 and 2 is inherently smoother than that in experiments 3 and 4, therefore the SVR modeling procedure slightly relaxes the error in experiments 1 and 2.

6 Conclusion

In this paper, a SVR-based modeling method of bearing-rotor system is proposed. In this model, the acceleration of the rotor is modeled as a nonlinear function of the rotor displacement, the rotor velocity and the coil currents, and the model parameters are determined by a training procedure based on training samples. The proposed modeling method is prior-model independent, that is, no mechanism, computational or empirical model is necessary. This characteristic is especially meaningful for complex bearing-rotor system, where no precise model is available.

An experimental system with a five DoF levitated rotor (about 3.5m long and 630kg heavy) is utilized to perform a series of sinuous-disturbance experiments. The experiment results show the validity of the proposed method.

The future works of this paper include:

1) Based on the proposed model of acceleration, more precise estimation of the rotor displacement can be achieved. More specifically, once the acceleration is available, some states estimators, e.g. Kalman filter, can be utilized to estimate rotor displacement. In this estimation, not only the observed displacement, but also the coil current is involved in the estimation, therefore more precise estimation can be excepted.

2) In the proposed method, mechanism, computational and empirical models are not necessary. However, in most cases these models are available and contain important information about the actual model; on the other hand, many prior-knowledge based modeling methods [12,13] can be utilized to incorporate these models into the sample based modeling method. This incorporation can improve the modeling performance.

3) Based on the proposed method the magnetic hysteresis model can be established. It is well-known that the magnetic hysteresis is an important source of the nonlinearity in the magnetic bearings, especially when the operation frequency is high (due to eddy loss). But it is difficult to build an analytical or a finite-element model of hysteresis, the proposed method provides a potential solution to describe the magnetic hysteresis.

4) Nonlinearity can degrade the stability, robustness and dynamic performance of the bearing-rotor system. If the above magnetic hysteresis model is obtained, it can be utilized improve the analyzing and designing procedure of the control method.

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