Model-Based Fault Detection on Active Magnetic Bearings by Means of Online Transfer-Factor Estimation

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Abstract

This paper gives a deeper insight in model-based fault detection based on transfer-factor estimation on active magnetic bearings. The method aims at performing fault detection and diagnosis at minimal computational efforts. Hence, only a limited number of transfer-factors of the investigated system are considered. These are used as features for the diagnosis and represented by the magnitude and the phase at frequencies showing strong deviations between the examined states. Previously modeled faults are diagnosed by an online-estimation of the transfer-factors and a comparison to reference values. Three algorithms for this fault detection method are described and compared in detail: Recursive Least Squares (RLS), Least Mean Squares (LMS) and Goertzel-algorithm. For the comparison of the algorithms, their convergence behavior and updating rate are considered as well as the computational and implementation effort. Investigations on a test rig of a centrifugal pump in active magnetic bearings show that the RLS leads to slightly slower convergence with a higher computational effort than the LMS. The Goertzel-algorithm outperforms both in computational effort, but needs several samples for the computation and thus provides a lower updating rate. Based on their findings, the authors assess suitability of the algorithms for fault detection on active magnetic bearings.

1 Introduction

Common model-based fault detection and diagnosis methods can be categorized considering the usage of parameter estimation, neural networks, observers, state estimation and parity equations for feature generation [1]. To create analytical redundancy, methods based on parity equations simulate one or more models in parallel to the system and compare various signals from the system with the ones of the models - e.g., the outputs [1, 2]. Such methods are used in [3, 4] to identify and analyze deviations in the systems transfer behavior that are caused by different fault states. Further, the sensitivity of such methods could be improved by the introduction of balancing filters [5, 6, 7, 8]. In contrast, observer-based methods utilize common state observers and unknown-input, that both estimate the full system state, or diagnostic observers for reducing the system's order and estimating especially the output correctly [9, 10]. Parameter estimation based fault detection methods represent a third major group of methods. By performing an online identification of particular system parameters, features are extracted and compared to reference values of the examined states [11, 12, 13, 4]. Additionally, a model-based fault detection concept based on transfer-factor estimation on active magnetic bearings is presented in [7, 6]. Considering a limited number of transfer-factors of the investigated systems behavior, this method aims at comsuming minimum computational efforts.

This paper proposes and compares three algorithms for performing fault detection and diagnosis by means of online transfer-factor estimation on active magnetic bearings. In Section 2 the overall concept of this diagnosis method is described. Subsequently, in Section 3 the analytical background of Recursive Least Squares (RLS), Least Mean Squares (LMS) and the Goertzel-algorithm is given. For the comparison of the algorithms, their convergence behavior and updating rate is considered as well as the computational and implementation effort in the following sections. After a simulative

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evaluation in Section 4, experimental investigations on a centrifugal pump in active magnetic bearing are given in Section 5. Based on the results, the suitability of the algorithms for online fault detection on active magnetic bearings is assessed in Section 6. Finally, the findings of this study are concluded in Section 7.

2 Fault Diagnosis Concept

To achieve an efficient fault diagnosis strategy regarding computation effort, the diagnosis concept presented in [6] focuses on estimating a limited number of features online. Hence, the Fourier-coefficients - magnitude and phase - at certain points of the frequency response are chosen as features and identified online. By this, it becomes possible to detect temporal changes of these characteristic values of the system and generate reliable features for the fault diagnosis.

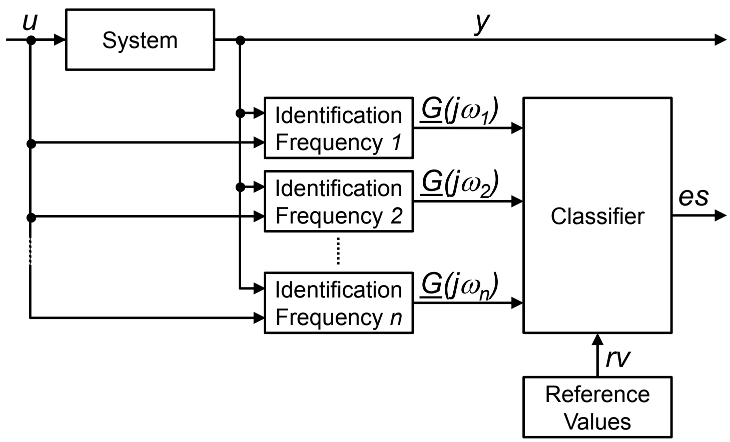

Figure 1: Block diagram of the diagnosis concept

As shown in figure 1, the input signal $u(t)$ and the output signal $y(t)$ of the system are used for the online identification. Based on these signals, the identification algorithms determine the transfer-factors of the system at the selected frequencies ω_k. The transfer-factors $\underline{G}(j\omega_k)$ are forwarded to a classifier, that compares those with several sets of reference values *rv*, representing the nominal state and previously modelled faults, to finally identify the current state *es*.

For the identification of the frequency response at selected frequency points ω_k, several methods can be applied. These can be divided into the classes of parametric identification methods - e.g., Recursive Least Squares and Least Mean Squares algorithm - and nonparametric methods - e.g. Goertzel algorithm [14]. With these, the Fourier-transforms $\underline{Y}(j\omega_k)$ and $\underline{U}(j\omega_k)$ of the input signal $u(t)$ and the output signal $y(t)$ are determined at selected frequency points. Based on these values the transer factors of the system at this specific frequency can be calculated by

$$\underline{G}(j\omega_k) = \frac{\underline{Y}(j\omega_k)}{\underline{U}(j\omega_k)}. \quad (1)$$

The obtained complex value of the frequency response represents the identified process-model for this specific frequency and can be divided into magnitude and phase, which represent the transfer-factors of the system. To investigate several frequencies at once, multiple sinusodial signals are superimposed for the stimulation of the specific discrete frequencies.

3 Transfer-Factor Estimation Algorithms

Aiming at the estimation of the transfer-factors of the system at specific frequencies, the two parametric methods RLS and LMS are used for the estimation of the magnitude and phase parameters of the input and output signal for the system at a specific frequency. A signal model consisting of a sine and a cosine signal is used to model the inputs and responses at the frequencies of interest ω_k individually:

$$x(t) = Acos(\omega_k t) + Bsin(\omega_k t). \tag{2}$$

Based on the parameters A and B, that are to be estimated from this signal model, the complex value of the Fourier-coefficients of the signals at this frequency are calculated:

$$\underline{X}(j\omega_k) = B + jA. \tag{3}$$

By identifying the parameters A and B for the input and the output signal, the Fourier-coefficients $\underline{U}(j\omega_k)$ and $\underline{Y}(j\omega_k)$ are determined. In contrast to these parametric methods, the non-parametric Goertzel-algorithm is used to determine the Fourier-transforms $\underline{U}(j\omega_k)$ and $\underline{Y}(j\omega_k)$ of both signals directly, since it is designed to calculate discrete Fourier-transforms (DFT) at a specific frequency [15]. Inserting the Fourier-transforms into (1) leads to the value of the frequency response $\underline{G}(j\omega_k)$ at the investigated frequency independently from the selection of a parametric or non-parametric appraoch.

3.1 Recursice Least Squares Algorithm

In order to estimate the parameters of the signal model (2) based on measured signals, the latter one is described as the product of the data vector ψ^T and the estimated parameter vector θ [14]:

$$\hat{x}[i] = \psi^T[i]\Theta = \begin{bmatrix} cos(\omega_k t[i]) & sin(\omega_k t[i]) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}. \tag{4}$$

With this, an offline estimation of the parameters $\hat{\Theta}$ can be determined based on the least squares algorithm [14], which is extended into the recursive least squares algorithm [16] to enable online identification by

$$\hat{\Theta}[i+1] = \hat{\Theta}[i] + \gamma[i]\left[y[i+1] - \psi^T[i+1]\hat{\Theta}[i]\right]. \tag{5}$$

In this, the factor $\gamma[i]$ and the covariance matrix $P[i+1]$ are determined by

$$\gamma[i] = \frac{1}{\psi^T[i+1]P[i]\psi[i+1] + \lambda} P[i]\psi[i+1] \tag{6}$$

and

$$P[i+1] = \left[I - \gamma[i]\psi^T[i+1]\right]P[i]\frac{1}{\lambda}. \tag{7}$$

where I represents a unit matrix. The forgetting factor λ is used to weight older signal values lower than newer ones to adjust the algorithm to parameter changes more quickly [14]. According to [16], the initial values are chosen to be $\hat{\Theta}[0] = 0$ and $P[0] = \alpha I$, where I represents the unity matrix and α is a high number asuming large variances.

3.2 Least Mean Squares Algorithm

In analogy to the RLS method, the signal is transcribed to the dot product of a parameter vector Θ and a reference signal vector $\psi[i]$ for the LMS algorithm:

$$\hat{x}[i] = \psi^T[i]\hat{\Theta} = \begin{bmatrix} cos(\omega_k t) & sin(\omega_k t) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}. \tag{8}$$

Leading to the estimation equation

$$\hat{\Theta}[i+1] = \hat{\Theta}[i] + \mu e[i]\psi[i] \tag{9}$$

given in [17]. In this, $e[i]$ represents the estimation error $x[i] - \hat{x}[i] = x[i] - \psi^T[i]\Theta[i]$. In order to achieve a good convergence of the estimation, the adaptation step size μ should be chosen in the range of

$$0 < \mu < \frac{1}{\psi^T[i]\psi[i]} = \mu_{max} \tag{10}$$

theoretically [17]. For practical implementation a step size in the range of $\frac{1}{10} .. \frac{1}{100}\mu_{max}$ has shown to be advantageous [18]. The initial parameter vector $\Theta[0]$ is here again chosen to be a zero vector as for the RLS algorithm.

3.3 Goertzel-Algorithm

Like the fast Fourier-transform FFT, the Goertzel-algorithm [15] or Goertzel-filter is a method to calculate DFTs very effectively. While the FFT is designed for the evaluation of a complete spectrum, the Goertzel-algorithm aims at calculating the DFT of specific frequency components. As shown in [19], the Goertzel-algorithm is deduced from the definition of the DFT

$$\underline{X}[k] = \sum_{i=0}^{N-1} x[i]\underline{W}_N^{ki}. \tag{11}$$

Here $\underline{W}_N = e^{-j\frac{2\pi}{N}}$ is the rotation factor of the DFT. Based on this, the transfer function of the Goertzel-algorithm can be represented as an infinite impulse response filter by

$$\begin{aligned} v[i] &= x_f[i] + av[i-1] - v[i-2], \\ \underline{y}_k[i] &= v[i] - \underline{W}_N^k v[i-1]. \end{aligned} \tag{12}$$

The initial conditions of $v[i]$ are chosen to be $v[i-1] = v[i-2] = 0$. While the first equation is evaluated continuously, the result of the second equation is determined every Nth step, because only then the correct value of the DFT $\underline{X}[k]$ is resulting from the filter. Hence, the block length N describes the number of steps that are requirred to update the result of the Goertzel-algorithm.

4 Simulation

In this simulation study the algorithms are assessed regarding their convergence behaviour and updating rate as well as computational and implementation effort. For that purpose, a synthetical discrete second order delay element is used as the investigated system. The parameters of this system are a time constant $T = 31.62\,\mathrm{s}^{-1}$, a damping factor $d = 0.32$ and a gain $K = 1000$ with a sampling time of $T_S = 0.02\,\mathrm{s}$. Sinosodial signals with and without noise as well as multi-sine signals are used as system input. The sinusoidal signal is applied with a frequency of 10 Hz and a magnitude of 3. The sinosodial signal with noise shows a signal-to-noise ratio of 0.3. For the multi-sine signal, three sinusoidal oscillations with frequencies of 5 Hz, 10 Hz and 30 Hz and magnitudes of 3, 2 and 1 and phase shifts of 30°, 45° and 60° are superimposed.

4.1 Convergence Behaviour and Updating Rate

To investigate the convergence behaviour of the methods, a sinusoidal signal at 10 Hz with noise is chosen.

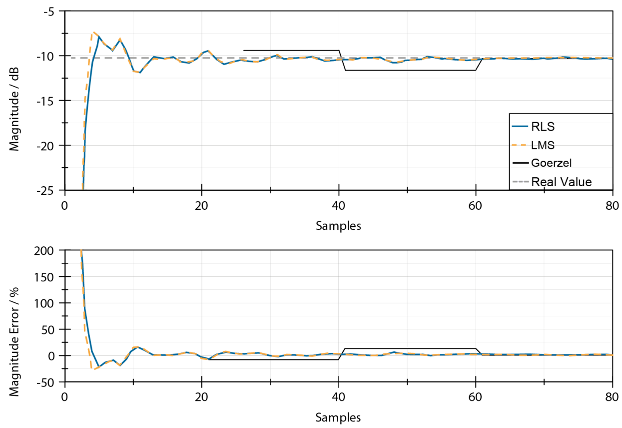

Figure 2: Magnitude estimation in the simulation

The estimation of magnitude and the corresponding estimation error are shown in Figure 2. It becomes distinct, that the values are converging to the real value for the parametric methods, since the estimation errors reduces with an increasing number of sampling points. In case of the Goertzel-algorithm, the estimated values are close to the real values in an early stage of the identification process, but a higher number of samples is required before the estimation error is reduced to zero. For the phase, comparable behaviour can be observed considering the results shown in Figure 3.

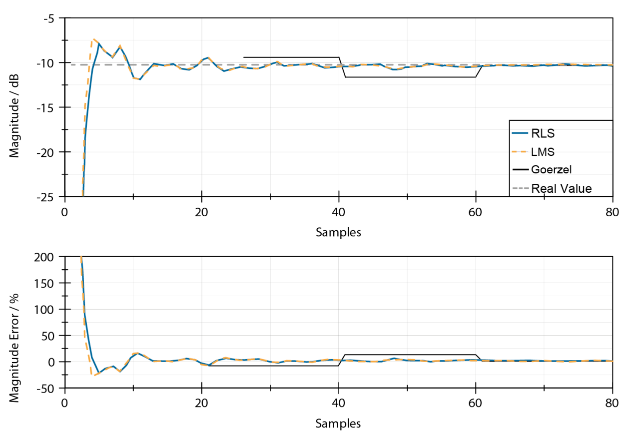

Figure 3: Phase estimation in the simulation

As shown analytically in Section 3.3, the Goertzel-algorithm calculates a new DFT value after the block length of N samples in the simulation. In contrast to this, the parametric methods determine a new value in every sample and thus show a significantly higher updating rate.

4.2 Computational and Implementation Effort

For the investigation of the computational effort required by the methods, the system was stimulated with different signals for a duration of 2000 samples - corresponding to a simulation time of 40 s. The values of the system's frequency response at 10 Hz are estimated using the proposed methods with $\lambda = 1$ for the RLS, $\mu = 0.001$ for LMS and $N = 100$ for the Goertzel-algorithm. In the simulation, the computing times for the online estimation are acquired independently and averaged over several trials for every algorithm. The results summarized in Table 1 are used as a measure and thus as the assessment criterion for the computational effort. As the comparison shows, the Goertzel-algorithm outperforms both parametric methods in computational effort. With the chosen settings the required computing time is approximately 20 times less than the one of LMS and about 100 times lower than the one of RLS.

Table 1: Required computing time t_{alg} in ms

Input-signal	RLS	LMS	Goertzel
Sinosodial w/o noise	298	62	3
Sinosodial w/ noise	294	62	3
Multi-sine	295	62	3

Regarding the implementation effort of the methods, their program flow charts are analyzed. As shown in Figure 4, both parametric methods require more implementation effort than the Goertzel-algorithm, as a higher number of steps is required for the estimation. In comparison to the RLS algorithm, the LMS needs less effort, since the "Estimation of output and error determination" is easier to realize than the determination of γ and P as well as the implementation of the actual parameter estimation step. For the Goertzel-algorithm a digital filter has to be programmed. Here, most attention has to be incorporated for the segmentation of the input signal, as this is crucial for the accuracy of the recieved results.

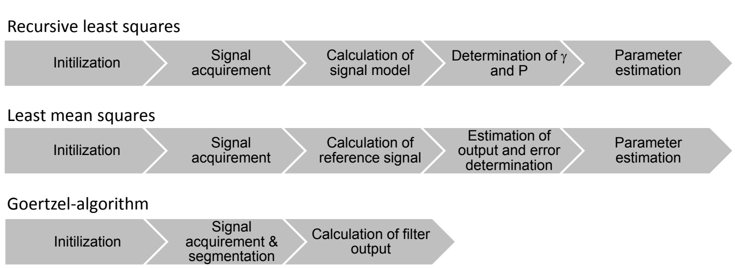

Figure 4: Condensed program flow charts of the algorithms

5 Experiments

For the experimental evaluation, the fault diagnosis concept and the three estimation methods are implemented on the real time controller of a centrifugal pump test rig in magnetic bearing. This system and the results from the experiments are presented subsequently.

5.1 Test Rig

The test rig used for the experimental investigations is based on a common one-level centrifugal pump. As depicted in Figure 5 the radial rolling bearings and the axial bearings of this pump are replaced by active magnetic bearings and located on both sides of the pump housing. The magnetic forces are applied in the middle of the bearings while the displacements are measured on the sides of the bearings that are facing the pump.

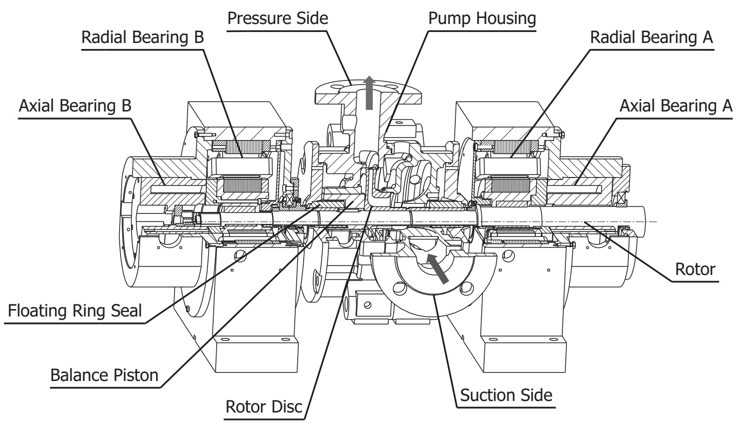

Figure 5: Sketch of the test rig

Since the lowest three natural frequencies of the rotor and the magnetic bearings lie at 285 Hz, 640 Hz and 1015 Hz, the rotor shows flexible behavior within the frequency range up to 800 Hz, that is considered for fault diagnosis. Hence, the deviations of the rotor's flexible transfer behavior caused by fault states can be used for the diagnosis.

5.2 Experimental Investgations

In the experimental investigations, the system is operating in reference state and the sinosodial input signal at 440 Hz is continuously applied to the test rig's radial magnetic bearings for $50,000$ samples corresponding to a period of 10 s. Based on the acquired sensor data, the values of the frequency response of the process at the selected frequencies is identified with the three presented methods. For the experiments the parameters of the estimation algorithms are chosen to be $\lambda = 1$, $\mu = 0.0005$ and $N = 200$.

As in the simulation, the two parametric methods show to converge against the true values for magnitude and phase (-109.7 dB and -219.6 deg) as depicted in Figure 6. It further becomes distinct, that the values estimated by the RLS algorithm converges slightly slower to the real value than the ones determined with the LMS algorithm. Below 600 samples the phase estimated by the RLS algorithm shows a change of 360 deg. As this is caused by an delayed correction of the phase in the post processing of the RLS results, it does not affect the estimation negativly in general.

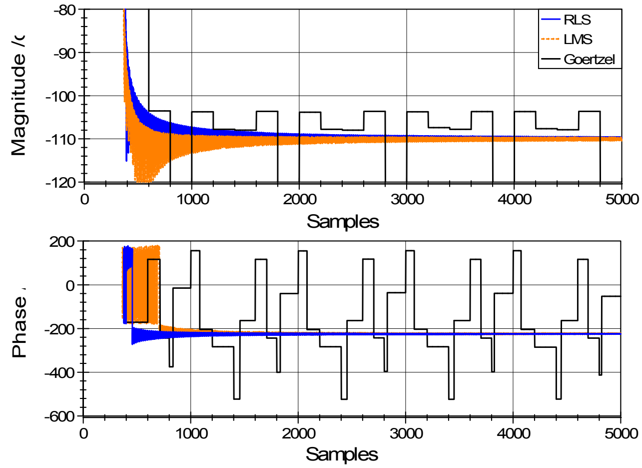

Figure 6: Experimental comparison of the algorithms

Due to the high computational effort of the RLS algorithm, a reduction of the samples used for the identification with a decimation factor of 4 is necessary to avoid delays in real-time system. With the chosen parameters, only poor results are achieved with the Goertzel-algorithm, since both estimated values - magnitude and phase - show distinct deviations from their true value. These results show to be due to the chosen block length of $N = 200$.

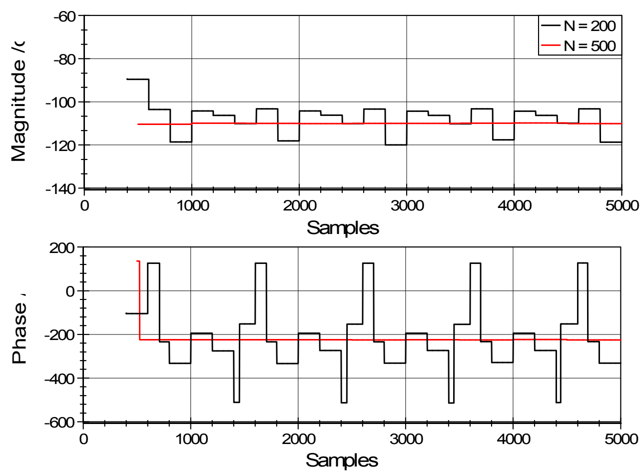

Figure 7: Influence of the block length N in the Goertzel-algorithm

As depicted in Figure 7, the results can be improved significantly by an increase of the block length to $N = 500$. With this block length, fluctuations of the estimated value as well as the estimation error decrease significantly. The consequent results show the quality required for a pratical utilization. Yet, the updating rate of the Goertzel-algorithm is decreasing furthermore compared to the parametric methods.

6 Comparison and Discussion

The simulation study as well as the experimental results show, that RLS and LMS ehibit fast convergence behaviour. As the estimation by the Goertzel-algorithm reaches the correct values, but requires more time than RLS and LMS, it's convergence behaivour is assessed positive. Regarding the updating rate, the two parametric algorithms provide new estimated values at each time step, while the Goertzel-algorithm needs a block length of N time steps to update the estimated values. It's main advantages is the very low required computational effort. In this criterion the Goertzel-algorithm outperforms the RLS and LMS distinctly. Regarding the implementation effort also shows lower requirements than the other methods. Hence, the Goertzel-algorithm is the method of choice for a fast implementation with minimum computational effort. Only if very fast response to sudden changes in the system is required, the parametric methods provide the better tradeoff. Anyhow, all proposed methods are assessed to be suitable for online fault detection on active magnetic bearings, as they generate reliable features for the classifier. The results of the study are briefly concluded in Table 2.

Table 2: Assessment of the algorithms

Criterion	RLS	LMS	Goertzel
Convergence Behaviour	++	++	+
Updating Rate	++	++	0
Computational Effort	--	-	++
Implementation Effort	-	0	+

7 Conclusion

This paper shows the suitability of RLS, LMS and Goertzel-algorithm as feature generators for fault diagnosis on magnetically beared systems. Further, the advantages and disadvantages of these methods are compared: While the RLS and LMS algorithm show very good convergence behaviour and updating rates, the Goertzel-algorithm is beneficial regarding computational and implementation effort. As the Goertzel-algorithm shows this in combination with a good convergence behaviour and an acceptable updating rate, it suits this diagnosis concept best. In contrast to the Goertzel-algorithm, that is optimized for the identification at specific frequencies, the RLS and LMS method are capable to conduct general parameter estimation tasks. Hence, they can be applied for the identification of a broader spectrum of possible features for the diagnosis beyond particular transfer-factors.

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