

Identification of Support Parameters of Magnetic Bearing Based on Model Updating Method

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Abstract

The stiffness and damping of magnetic bearings play an important role in the stability of high speed rotor system. In order to obtain the reliable and accurate support parameters of magnetic bearing, an identification method based on model updating is proposed in this paper. Successful identification of support parameters is based on an accurate system model. The accurate rotor model is first derived by model updating using frequency response function (FRF) data. Then, the identification of support parameters of magnetic bearings is completed by model updating method based on the unbalance response. Unbalance response of magnetic bearing rotor system with assumed initial support parameters can be calculated by the transfer matrix method. Based on an optimization algorithm, the minimum error between the calculated and measured unbalance response is achieved. The support parameters which were used to calculate the unbalance response are the identified stiffness and damping of magnetic bearings. Finally, the effectiveness and accuracy of the identification method are verified by simulation.

1 Introduction

Active magnetic bearings (AMBs), as a new type of non-contact bearing, have been used in an rapidly growing number of applications such as engines, compressors, and pumps [1]. Bearings clearly are obviously crucial components in rotating machines and a good understanding of their dynamic behavior is important for the prediction of the machine's properties.

Because there are multiple factors affecting the dynamic behavior of AMBs, it is difficult to estimate the support parameters by theoretical models. Accurate support parameters can be acquired by identification methods based on experimental data.

There are the several experimental methods to get AMBs support parameters: 1) The AMBs parameters can be obtained by the measurement of electromagnetic force or disturbing force and the acquisition of rotor displacement [2, 3, 4]. 2) Based on the linearization formula of electromagnetic force, the force-current coefficient and force-displacement coefficient notably affect AMBs support characteristics. These two coefficients can be measured from the test, and then the equivalent stiffness and equivalent damping can be calculated based on them [5, 6]. 3) The rotating system is excited by known forces. Then the corresponding response can be measured. With input and output data, the AMBs parameters can be identified based on the model of magnetic bearing rotor system [7, 8]. By the comparison of these methods, the identification method is not only widely applicable but also able to obtain the support parameters in operating condition. Thus, an identification method of AMBs support parameters based on model updating is proposed in this paper.

The procedures of identification are introduced as follows: First, model updating is conducted using FRF data to obtain an accurate rotor model. Then, the assumed initial support parameters of AMBs are set to the updated model. The unbalance response of system at a certain rotating speed can be calculated by transfer matrix method. Moreover, the unbalance response can be also measured from experiments. Finally, the error between the calculated and measured unbalance response could be reduced constantly by optimization algorithm. The parameters which

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make the minimum errors are the identified stiffness and damping of AMBs. The process of identification is shown in Figure 1.

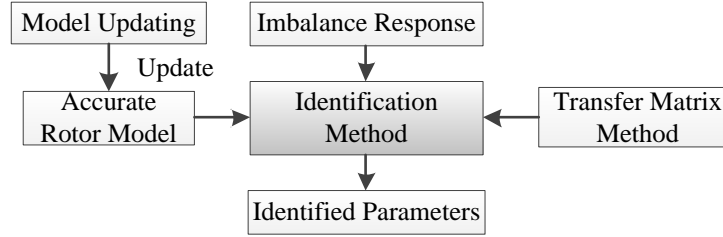


Figure 1: Process of identification method based on model updating

In the paper, Chapter 2 talks about the modeling updating for rotor. Chapter 3 introduces the identification method based on model updating. Chapter 4 shows the verification results of the identification method. Chapter 5 concludes this paper.

2 Model Updating for Rotor Using FRF Data

Rotor is an important component in magnetic bearing rotor system. The simplification in modeling reduces the accuracy of predicting structural dynamic response, so it is necessary to update the rotor model. Moreover, the main structural response of magnetic bearing rotor system involved in this paper is unbalance response. The unbalance response and the frequency response of a structure share some similar features. In order to obtain a rotor model whose unbalance response matches actual structural response, model updating using FRF data is conducted. The main process of model updating using FRF data is as follows:

- 1) Select the proper updating parameters, make an initial finite element method (FEM) model and analyze;
- 2) Establish residual evaluation function between calculated and measured data;
- 3) Obtain new updating parameters by optimization algorithm;
- 4) Return to step 1 until residual evaluation function satisfies the convergence criteria.

2.1 Definition of Residual Evaluation Function

The definition of residual evaluation function is an important procedure in model updating. Based on the method proposed in the literature [7], the model assurance criterion (MAC) of resonance mode shape is used to match structural mode and that is the way to find out the calculated modal parameter of each level by FEM and the corresponding experimental modal parameter. Because the amplitude of resonance point is affected excessively by the uncertain error, the amplitude vectors are not included in residual evaluation function. Meanwhile, the frequency and amplitude of half-power bandwidth have smaller experimental error and FRF of half-power bandwidth contains some important information. So the amplitude and frequency of half-power point are selected for calculation of residual evaluation function. In order to ensure the matching of FRF data is away from peaks, the amplitude of frequency trisection between adjacent modal frequencies is added to the calculation. The residual evaluation function is defined as follows

$$R = \left\| \{\omega_E\} - \{\omega_A\} \right\|_2^2 + \left\| \log \{H_E(\omega)\} - \log \{H_A(\omega)\} \right\|_2^2 \quad (1)$$

where $\{\omega_E\}, \{\omega_A\}$ are measured and analyzed frequency vectors of selected points respectively. The selected points contain resonance points and half-power points. $\{H_E(\omega)\}, \{H_A(\omega)\}$ respectively denote measured and analyzed amplitude vectors of selected points. The selected points contain half-power points and frequency trisection points.

The analyzed frequency and amplitude of the selected points vary in the process of model updating. With the initial updating parameters, the analyzed and measured FRF data is shown in Figure 2. The selected points for residual evaluation function are also shown in the figure. The initial values of selected points are shown in Table 1.

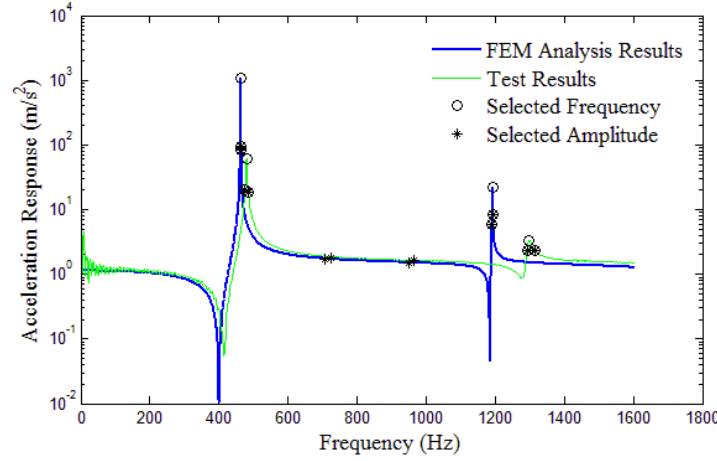


Figure 2: The selected updating points for residual evaluation function

Table 1: Initial Value of Frequency and Amplitude Vectors

Measured Frequency Vector (Hz)	Analyzed Frequency Vector (Hz)	Measured Amplitude Vector (m/s^2)	Analyzed Amplitude Vector (m/s^2)
475	462.29	20.4117	92.9537
480	463.29	18.8303	81.5166
485	464.29	1.8198	1.7311
1292	1190.75	1.6017	1.5129
1298	1191.75	2.3313	5.8044
1314	1193.75	2.3617	8.2525

2.2 Model Updating for Material Parameters of Rotor

Beam elements and lumped mass elements are usually applied to rotor modeling. For the rotor in magnetic bearing rotor system, the plain shafts are simplified into beam elements while sleeves and silicon-steel sheet rings are simplified into lumped mass elements. Because the increase of rotor bending stiffness is not considered in simplification, some error exists between analyzed and experimental results. The material parameters (elastic modulus of material) of the shafts which fit with silicon-steel sheet rings are selected as updating parameters, such as E_1 (in AMBs position) and E_2 (in motor position). The selected updating parameters are shown in Figure 3.

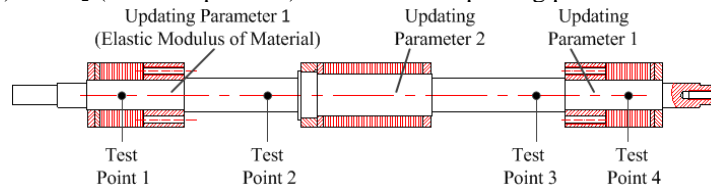


Figure 3: The selection of updating parameters and the location of test points

Model updating for the rotor starts with the matching of modal analysis results and experimental modal data. The experimental facilities used in modal testing include a dynamic signal analyzer, a charge amplifier, a force-hammer and an acceleration sensor. Due to the simple structural response of free-free rotor, the modal analysis results and experimental modal data can be matched manually.

In order to obtain the frequency response function, four test points are set on the rotor and these four points are shown in Figure 3. The FRF data of test point 1 and 4 under the excitation of test point 1 and the FRF data of test point 2 under the excitation of test point 3 can be collected from the experiments. The FRF data of test point 1 under excitation of test point 1 is used for model updating while the others are used for the verification of updated results. The experimental facilities are listed in Figure 4.



Figure 4: Photograph of the experimental facilities

In model updating using FRF data, the initial value of updating parameters are $E_1=206$ GPa and $E_2=206$ GPa. After model updating based on global optimization algorithm, two updated parameters of shafts are $E_1=372$ GPa and $E_2=236$ GPa. After model updating, the analyzed and measured FRF of different test points are shown in Figure 5, 6, 7. As shown in the figures, the analyzed data based on updated model matches well with the measured data, so the updated model is accurate enough to predict the structural response. The mathematical model of rotor is always the same no matter whether the structural response is calculated by FEM or transfer matrix method. Therefore, by transfer matrix method, the updated rotor model can also predict the exact structural response.

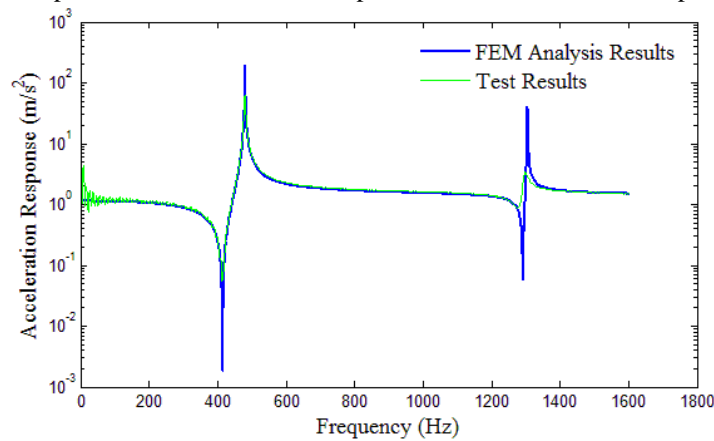


Figure 5: The analyzed and measured FRF data of test point 1 under the excitation of test point 1

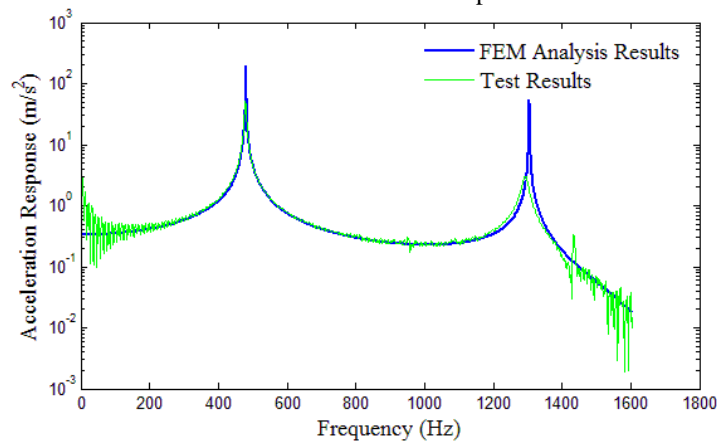


Figure 6: The analyzed and measured FRF data of test point 4 under the excitation of test point 1

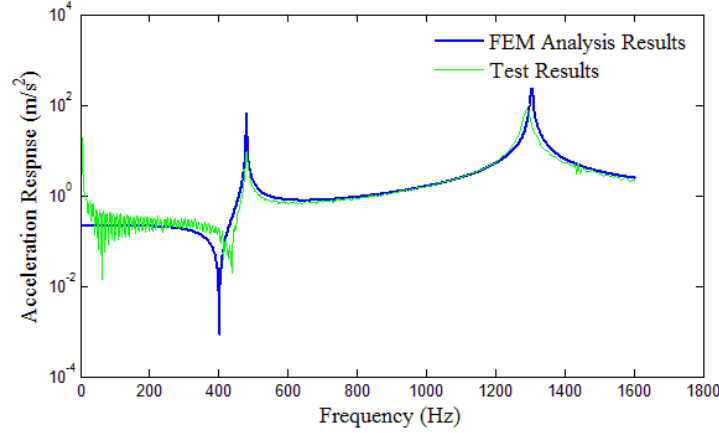


Figure 7: The analyzed and measured FRF data of test point 2 under the excitation of test point 3

3 Identification of AMBs Support Parameters Based on Model Updating

Normally, the identification of support parameters can be achieved by using the impulse, step change in force, random, unbalance excitation techniques. Methods using unbalance forces are adopted in wide application because no sophisticated equipment is required for excitation. Therefore, the identification of AMBs support parameters is based on the unbalance response.

Identification of support parameters is the process of continuous reduction of errors between calculated and measured unbalance response. The unbalance response can be calculated by FEM or transfer matrix method based on the updated rotor model. Calculating by transfer matrix method is faster, easier for programming and less memory required because the orders of transfer matrix do not increase as the degrees of freedom increase. Moreover, the identification proposed in this paper is essentially an iterative process and it requires a great amount of computation. Thus, transfer matrix method is preferred for calculating unbalance response. The updated rotor model from the last section, support parameters and unbalance are necessary for the calculation of unbalance response by transfer matrix method.

3.1 Fundamental Theory of Calculating Unbalance Response by Transfer Matrix Method

In the process of calculation with transfer matrix method, rotor is simplified into a concentrating model consisting of disks, elastic axis and elastic bearings. It is divided into N parts, and each part is numbered $1, 2, \dots, N$ from the left to the right and each section is numbered $1, 2, \dots, N, N+1$. The state vectors of every section are given as

$$\begin{aligned}
 Z_2 &= T_1 Z_1 \\
 &\vdots \\
 Z_i &= T_{i-1} Z_{i-1} = T_{i-1} T_{i-2} \cdots T_1 Z_1 = A_{i-1} Z_1 \\
 &\vdots \\
 Z_{N+1} &= A_N Z_1
 \end{aligned} \tag{2}$$

where Z_i and T_i are the state vector of i^{th} section and the transfer matrix of i^{th} shaft segment respectively. The equations (2) show the relation between the state vector of each section and the state vector of initial section. It illustrates the elements of each section's state vector can be obtained by the linear combination of the elements of initial section's state vector.

For all calculating situations, boundary conditions of the rotor can be obtained directly from the structure of bearing rotor system. These conditions are the supplementary equations for the transfer functions and they can simplify transfer functions to homogeneous equations.

Because the item ω^2 exists among most elements in transfer matrix, these elements will be large numbers when the number of nodes N and rotating speed ω are large. Therefore, calculating the difference between two similar large numbers is necessary in the process of computing mode shapes or unbalance response by traditional transfer matrix method. It causes the decline of calculating accuracy and even leads to the numerical instability of calculated results. In the simulation, these errors show up as the rapid increase of amplitude at rotor's one end [9].

Riccati transfer matrix method, as an improved algorithm, transforms the two-point boundary value problem of original differential equations to one-point initial value problem through Riccati transformation. It improves the numerical stability effectively. With such an advantage, unbalance response is solved by Riccati transfer matrix method in identification.

3.2 Principle of Identifying Stiffness and Damping Based on Model Updating Method

By transfer matrix method, the unbalance response at a specific rotating speed can be calculated after the initial support parameters are set to the model of magnetic bearing rotor system. In the same condition, the measured unbalance response can be also obtained from the experiments. By the comparison of calculated data with measured displacement response, the support parameters which make the errors decrease can be achieved constantly by optimization algorithm. The identified support parameters at certain speed are those minimizing the errors. Because the derivative of optimization objective function is difficult to be calculated, the simplex method is used to find new parameters [10]. Identifying the dynamic support parameters of magnetic bearing rotor system based on model updating method is an iterative process. The identifying process is shown in Figure 8.

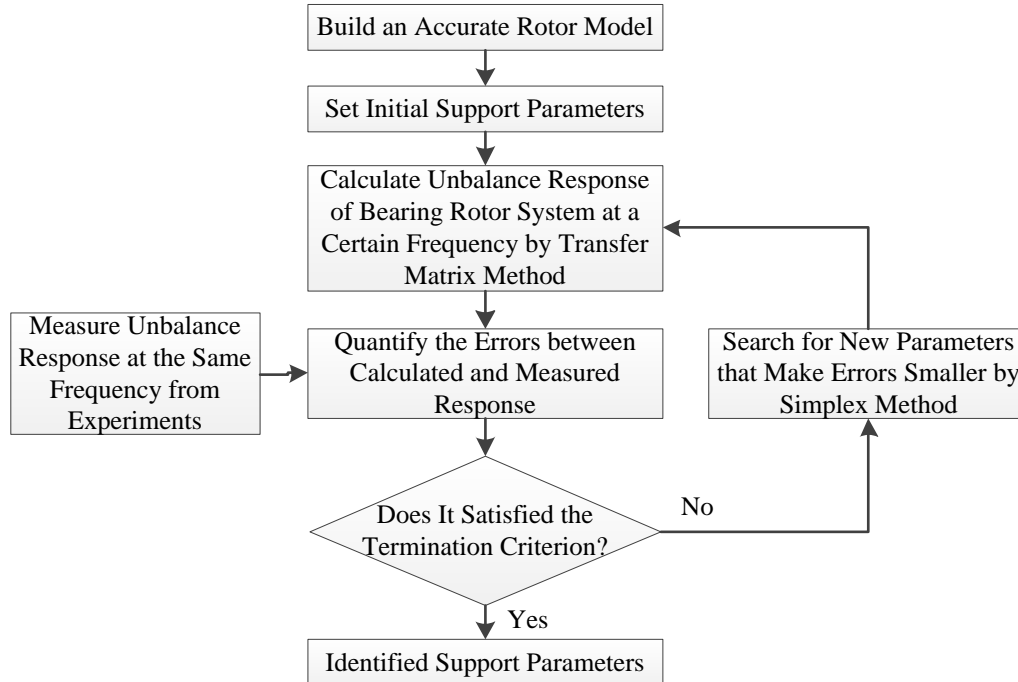


Figure 8: The procedures of identifying support parameters

Quantifying the difference between measured and calculated response is one of the most important procedures. According to Yang's paper [11] and the features of calculated and measured data, the residual evaluation function is defined as follows

$$Q(p) = \sum_{i=1}^m \left\{ K_A(i) [\log(A(i) / \text{abs}(R(i)))]^2 + K_P(i) [\log(P(i) / \text{arg}(R(i)))]^2 \right\} \quad (3)$$

where $A(i)$, $P(i)$ are the measured amplitude and phase when the system operates at a certain speed, $\text{abs}(R(i))$, $\text{arg}(R(i))$ are the amplitude and phase calculated by transfer matrix method at the same speed, p is the vector of support parameters, m is the number of data included in the residual evaluation function, $K_A(i)$, $K_P(i)$ are the weighting factors. In different frequency band, different weighting factors are required. The amplitude's weighting

factors $K_A(i)$ should be larger in low frequencies, while the phase's weighting factors $K_P(i)$ are larger in high frequencies. Then the procedure of identifying dynamic support parameters can be converted to an optimization problem

$$\begin{aligned} & \text{Min}Q(p) \\ & \text{s.t. } VLB \leq p \leq VUB \end{aligned} \tag{4}$$

where VLB, VUB represent the upper and lower bound of support parameters. The optimization problem is solved by the simplex method.

4 Simulation and Verification of the Identification Method

In order to verify effectiveness and validity of the identification method, simulations and verification on the magnetic bearing rotor system are conducted.

The finite element model of rotor's composite structure is established in Nastran by Timoshenko Beam elements and lumped mass elements. The rotor model contains 50 beam elements and 12 lumped mass elements. In order to simulate the unbalance of rotator and the bearings of system, unbalance elements and grounded bush elements are also applied to the model, respectively. Parameters of magnetic bearing rotor system are shown in Table 2. Finite element model is shown in Figure 9.

Table 2: Parameters of magnetic bearing rotor system

Length of Rotor	436 mm	Diameter of Shaft in Bearing Position	39.8 mm
Assumed Stiffness of Left Bearing	35000 N/m	Assumed Stiffness of Right Bearing	5000000 N/m
Assumed Damping of Left Bearing	250 N s/m	Assumed Damping of Right Bearing	450 N s/m
Unbalance in Left Bearing Position	0.8856e-5 kg m	Unbalance in Right Bearing Position	1.1355e-5 kg m



Figure 9: The finite element model of the magnetic bearing rotor system

When the assumed support parameters are set to the model, the amplitude and phase of displacement unbalance response at a certain rotating speed can be calculated by rotor dynamic solver in Nastran. The frequency band of calculation ranges from 10 Hz to 280 Hz and 55 sample points are included. The unbalance response from the simulation is treated as the measured data. The initial support parameters $K_1=K_2=3e6$ MN/m, $C_1=C_2=500$ N s/m are set to the optimization algorithm of the identification method. K_1, K_2 are the support stiffness of the left and right bearings and C_1, C_2 are the support dampings. The support parameters at each calculated rotating speed can be obtained when the minimum of $Q(p)$ is reached. The identified and the assumed parameters are shown in Figure 10.

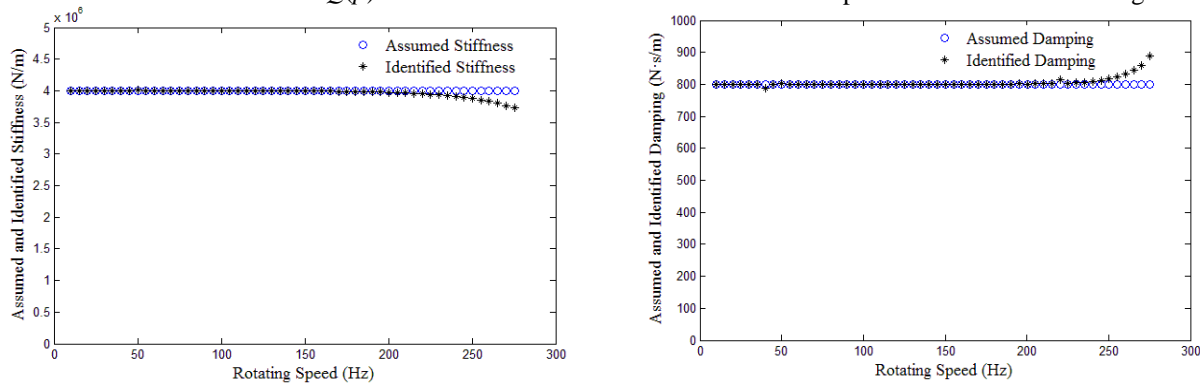


Figure 10: The identified and assumed stiffness and damping of the AMBs

As shown in figure 10, the identified stiffness and damping of AMBs are consisted of the assumed data at the rotating speed from 10 Hz to 280 Hz. The maximum error between identified and assumed support parameters is no less than 10 percent. The unbalance response of the system with the initial and identified support parameters could be calculated by the transfer matrix method. The unbalance response from simulation and the unbalance response of the system are shown in Figure 11.

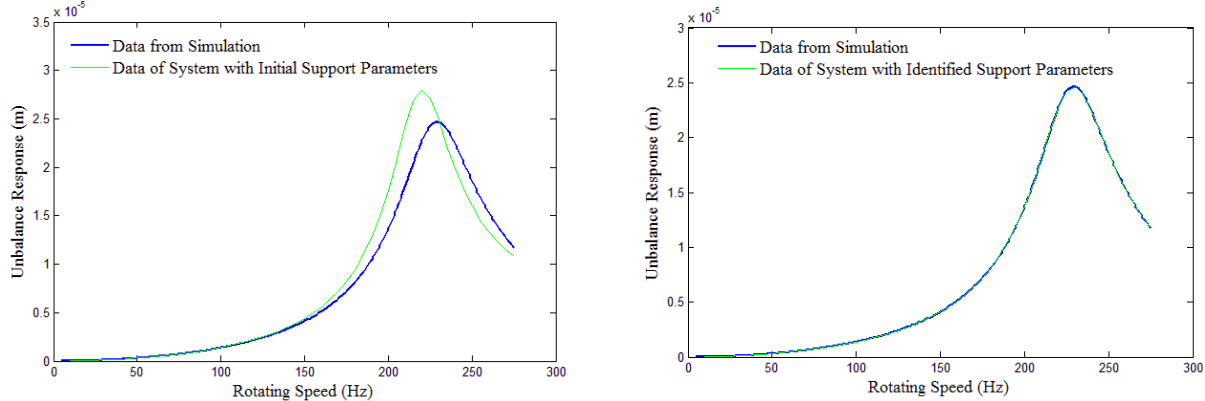


Figure 11: The unbalance response of the system before (left) and after (right) identification

5 Conclusion

A method for identification of AMBs support parameters is presented. By the model updating using FRF data, an accurate rotor model which can predict exact structural response can be obtained. Through simulation and verification, the support parameters can be obtained effectively by the identification method based on model updating. The proposed method is applicable for different magnetic bearing rotor system and it offers a reliable way to obtain the support parameters of AMBs.

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