Unbalance Control of Magnetic Bearings by Multi-Degree of Freedom Internal Model Control

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Abstract

This paper suggests a method to control an unbalance vibration of the magnetic bearing by multi-degree of freedom internal model controller which is called Model Bridge Control. This control method can compensate for each control problem, e.g. robust stability and uncertainty compensation, responses, disturbance rejection. By focusing on a disturbance compensator, this paper put forward a control system which rejects agitation due to unbalance. In this paper, a controller design method using a disturbance compensation characteristic of model bridge control is shown. The controller gets possible to control an unbalance forced vibration with the specific frequency by using a disturbance model. The effectiveness of the proposed method has been clarified by simulation.

1 Introduction

One of the features of active magnetic bearings is the ability to control unbalances, and a large number of studies for unbalance compensation have been proposed [1]. Since "generalized notch filters [2]" can be used also within the rigid body critical speeds, it is a very effective method. However, it requires measuring the closed-loop sensitivity function after designing the stabilized controller particularly.
On the other hand, a lot of methods of the control to solve various control specifications at the same time have been suggested e.g. H_∞ control [3][4] and LMI [5]. However, since it is difficult to correspond individually to a control specification, a technique is required to design a good controller. To solve this problem, Model Bridge Control (MBC) which is based on internal model control [6] is proposed [7][8]. MBC is a modification of the generalized stabilizer and has the control structure that adjustable models bridge over the gaps between the uncertainty and the high robust stability, and between the external signals and the desired outputs individually. We aim to construct a robust MIMO controller by MBC in the future.
In this paper, a controller with easy realization for unbalances at the rigid body critical speeds is proposed. This control system is model bridge controller which has the structure that incorporated a stabilizing compensator, a disturbance compensator. However, the uncertainty compensator is not used in this paper.
It is notable that the disturbance compensator enables control of any disturbances by incorporating its Laplace transform model. In this study, the sine wave disturbance model with the same frequency as the rigid body critical speed is included in it.
The performance of the suggested control strategy is well tested via computer simulation. Assuming that the control object is a two degree of freedom active magnet bearing system, decentralized controllers are designed for each degree. The stabilizing compensator is designed so that the rigid body critical speed of this system becomes 25rps, and a sin wave model with 25Hz is incorporated in a disturbance compensator. When a rotating speed is raised from 0rps, both displacement and control voltage are reduced at the time of critical speed (25rps) passage. The simulation results show that the proposed control method makes it possible to pass the rigid body critical speed safely, to control housing vibrations, and to avoid amplifier saturation.

2 Modeling

The configuration of a vertical type rotor system supported by magnetic bearings is shown in Figure 1. The nomenclature in this paper is shown in Table 1.

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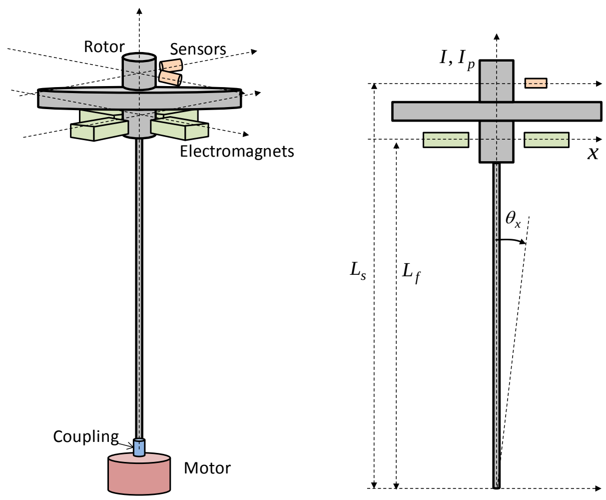


Figure 1: Two degree of freedom model

I	I_p	I_0	R	m
0.0874 [kg/m^2]	9.33×10^{-4} [kg/m^2]	2.0 [A]	2.0 [Ω]	0.01 [kg]
F_0	L	L_f	L_s	ε
4.42×10^{2} [N]	5.0×10^{-3} [H]	0.22 [m]	0.27 [m]	0.05 [m]

Table 1: Nomenclature

A state –space description of this system becomes the following.

$$\dot{X}_f = A_f X_f + B_f u + D_f d \tag{1}$$

$$Y = C_f X_f \tag{2}$$

Where

$$x_f = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ i_x \\ i_y \end{bmatrix}, \quad A_f = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 4\dfrac{\beta}{\alpha} & 0 & 0 & -\dfrac{\gamma}{\alpha}\omega & 2\dfrac{\delta}{\alpha} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dfrac{\gamma}{\alpha}\omega & 4\dfrac{\beta}{\alpha} & 0 & 0 & 2\dfrac{\delta}{\alpha} \\ 0 & 0 & 0 & 0 & -\dfrac{R}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{R}{L} \end{bmatrix}, \quad B_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ G/L & 0 \\ 0 & G/L \end{bmatrix}, \quad D_f = \begin{bmatrix} 0 & 0 \\ 1/\alpha & 0 \\ 0 & 0 \\ 0 & 1/\alpha \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_f = \begin{bmatrix} L_f/L_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_f/L_s & 0 & 0 & 0 \end{bmatrix}, \quad u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \quad d = \begin{bmatrix} m\varepsilon\omega^2 \cos\omega t \\ m\varepsilon\omega^2 \sin\omega t \end{bmatrix}$$

$$\alpha = \frac{I}{L_s}, \quad \beta = L_f \frac{F_0}{X_0}, \quad \gamma = \frac{I_p}{L_s}, \quad \delta = L_f \frac{F_0}{I_0}$$

The proposed controller is designed using the following equation which ignores gyroscopic effects.

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u + D_g d \\ y &= C_g x_g \end{aligned} \tag{3}$$

where

$$x_g = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}, \quad A_g = \begin{bmatrix} 0 & 1 & 0 \\ 4\dfrac{\beta}{\alpha} & 0 & 2\dfrac{\delta}{\alpha} \\ 0 & 0 & -\dfrac{R}{L} \end{bmatrix}, \quad B_g = \begin{bmatrix} 0 \\ 0 \\ G/L \end{bmatrix}, C_g = \begin{bmatrix} L_f/L_s & 0 & 0 \end{bmatrix}$$

3 Controller design

3.1 Basic concept (Mode Bridge Control)

The controlled system is be controllable, observable, invertible and its transfer function is denoted by $G_p(s) = (1+\Delta(s))G(s) \in R^{m\times m}(s)$ with uncertainty. Consider the control system as shown in Figure 2, where $G_M(s) = M(s)G(s) \in R^{m\times m}(s)$ is the adjustable model of $(1+\Delta(s))G(s)$, and $M(s) \in R(s)$ is the uncertainty compensator. It is assumed that $G_p(s)$ and $G_M(s)$ do not contain any zeros on the imaginary axis for internal stability of the resulting control system and the number of the unstable poles of $G_M(s)$ is equal to the number of the unstable poles of $G_p(s)$. The state space form of $G_p(s)$ is denoted by

$$G_M(s) = C(sI - A)^{-1}B \tag{4}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{m\times n}$, (A, B) is controllable and (C, A) is observable. The $F \in R^{m\times n}$ stabilizes $A - BF$ and $K \in R^{n\times m}$ stabilizes $A - KC$. The $Q_a \in R^{m\times m}(s)$ and $Q_b \in R^{m\times m}(s)$ are stable parameters. The control system as shown Figure 2 is said to be the model bridge control (MBC), because the gaps between the uncertainty and robust stability, between the reference inputs and desired responses and between the disturbances and adequate rejection are bridged by adjustable models individually [7][8]. Since the model bridge control is equivalent to the generalized stabilizer [9] for $Q(s) = Q_a(s)Q_b(s)$ and decoupling of the state feedback and the observer holds, the model bridge control systems is stable.

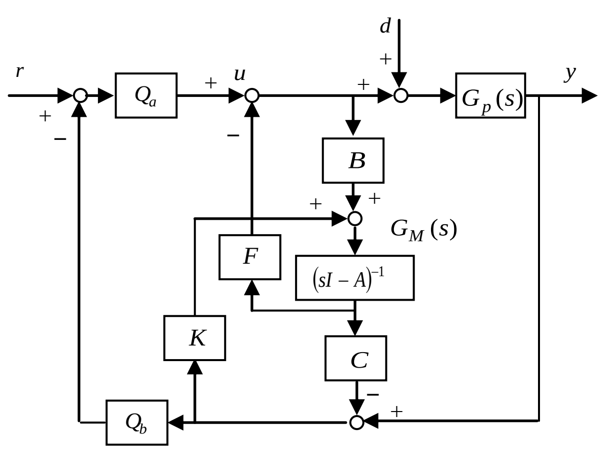


Figure 2: A state-space representation of the presented controller

For $G_M(s)=G_p(s)$, the outputs y to reference r and to the disturbance d in Figure 2 are given by

$$y(s)=G_M(s)P(s)r(s) \tag{5}$$

$$y(s)=[I-G_M(s)P(s)N(s)]G_M(s)d(s) \tag{6}$$

respectively, where

$$P(s)=\left[I+F(sI-A)^{-1}B\right]^{-1}Q_a(s)=\left[I-F(sI-A+BF)^{-1}B\right]Q_a(s) \tag{7}$$

$$\begin{aligned} N(s)&=\left[Q_a{}^{-1}(s)F(sI-A)^{-1}K+Q_b(s)\right]\left[I+C(sI-A)^{-1}K\right]^{-1} \\ &=Q_a{}^{-1}(s)F(sI-A+KC)^{-1}K+Q_b(s)\left[I+C(sI-A+KC)^{-1}K\right] \end{aligned} \tag{8}$$

are adjustable models that bridge the gaps between the reference input and the desired output responses, and between the disturbance and the its rejection individually. It follows from Equation (5) and (7) that

$$y(s)=C(sI-A+BF)^{-1}BQ_a(s)r(s) \tag{9}$$

The transient responses can be adjusted by F in $P(s)$. The zero steady state error is accomplished by choosing $Q_a(s)$ such that

$$C(s_iI-A+BF)^{-1}BQ_a(s_i)=I \tag{10}$$

is satisfied for poles $s_i(i=1,\cdots,\eta)$ of the reference input $r(s)$. The output to the disturbance is given by

$$y(s)=\left[I+C(sI-A+BF)^{-1}\{K-BQ_a(s)Q_b(s)\}\right]C(sI-A+KC)^{-1}Bd(s) \tag{11}$$

from Equation (3) and (5). The transient responses can adjusted by K in $N(s)$ .The zero steady state error is obtained by setting $Q_b(s)$ to satisfy

$$I+C(s_iI-A+BF)^{-1}\{K-BQ_a(s_i)Q_b(s_i)\}=0 \tag{12}$$

for poles $s_i(i=1,\cdots,\zeta)$ of $d(s)$.

In order to design model bridge control system for $G_M(s)$, it is assumed that (a) $G_M(s)$ can be decoupled by state feedback [10] and (b) the unstable zeros are row zeroes to yield

$$G_M(s)=G_I(s)G_o(s), \quad G_I(s)=\begin{bmatrix} G_{I1}(s) & & 0 \\ & \ddots & \\ 0 & & G_{\mathrm{Im}}(s) \end{bmatrix} \tag{13}$$

where $G_{Ii}(s)$ is inner with unstable row zeros and $G_o(s)$ does not contain any unstable zeros.

3.2 Controller for unbalance control

In this study, it is assumed that (i)the reference input $r(s)=1/s$, (ii) $G_p(s)=G_m(s)$ (without uncertainty) and the uncertainty compensator $M(s)=1$, (iii)the decentralized controller is designed based on Equation (3). Therefore, Equation (10) turns as follow

$$G_M(s) = C_g\left(sI - A_g\right)^{-1} B_g \tag{14}$$

This system does not contain any unstable zeros. The design procedures are as follow.
Step1: Design of F and $Q_a(s)$ The output to the reference input is given by Equation. (9) F and $Q_a(s)$ are designed to satisfy $G_M(s_i)P(s_i) = 1$ for poles $s_i(i = 1, \cdots, \eta)$ of $r(s)$. The relative degree of Equation.(14) is defined as v. Let

$$\begin{aligned} &C_g A_g{}^j B_g = 0, \quad j = 0,1,\cdots v-2 \\ &C_g A_g{}^{v-1} B_g \neq 0 \end{aligned} \tag{15}$$

then

$$F = \left(a_v C_g A_g{}^{v-1} B_g\right)^{-1} \psi \tag{16}$$

$$Q_a(s) = \left(a_v C_g A_g{}^{v-1} B_g\right)^{-1} Q_r(s) \tag{17}$$

for arbitrary $\tau > 0$, where

$$(1 + \tau s)^v = a_v s^v + a_{v-1} s^{v-1} + \cdots + a_1 s + 1 \tag{18}$$

$$\psi = a_v C_g A_g{}^v + a_{v-1} C_g A_g{}^{v-1} + \cdots + a_1 C_g A_g + C_g \tag{19}$$

$$Q_r = \frac{p_1 + p_2 s + \cdots + p_\eta s^{\eta-1}}{(1 + \tau s)^{\eta-1}} \tag{20}$$

where the coefficients are determined to satisfy

$$G_I(s_i) \frac{1}{(1 + \tau s_i)^v} Q_r(s_i) = 1 \tag{21}$$

for poles $s_i(i = 1, \cdots, \eta)$ of $r(s)$. Then Equation (9) becomes

$$y(s) = \frac{1}{(1 + \tau s)^\eta} Q_r(s) G_I(s) r(s) \tag{22}$$

The output can follow to the reference input with pre-assigned transient response and the zero steady state error.
Step2: Design of K and $Q_b(s)$ The output to the disturbance is given by Equation (11). The transient response is adjusted by K. Compute the stabilizing solution $Y = Y^T \geq 0$ of the Riccati equation

$$Y\left(A_g{}^T + \delta I\right) + \left(A_g + \delta I\right)Y - YC_g{}^T C_g Y = 0 \tag{23}$$

for $\delta \geq 0$ such that does not contain any eigenvalues at the imaginary axis. Let

$$K = YC_g{}^T \tag{24}$$

Then the roots of $A_g - KC_g$ lie in the left half-plane. The transient response of Equation (11) can be adjusted by δ. The $Q_b(s)$ is designed to yield zero steady state error. Let

$$Q_b(s) = \frac{q_1 + q_2 s + \cdots + q_{\xi-1} s^{\xi-1}}{(1+\tilde{\tau}s)^{\xi-1}} \quad where\,\tilde{\tau} > 0 \tag{25}$$

and compute the coefficients q_k to satisfy Equation (12) for $s_i(i = 1,\cdots,\xi)$ which are poles of $r(s)$ and $d(s)$.

3.3 Design example

$v = 3$ is given by Equation (14). By $\tau = 0.01$, Equation (18) and (19) are as follows.

$$(1+0.01s)^3 = a_3 s^3 + a_2 s^2 + a_1 s + 1 = 1.0\times10^{-6} s^3 + 3.0\times10^{-4} s^2 + 3.0\times10^{-2} s + 1 \tag{26}$$

$$\psi = \begin{bmatrix} 4.89\times10^1 & 1.21\times10^6 & -1.31\times10^8 \end{bmatrix} \tag{27}$$

From the assumption (i), $\eta = 1$, furthermore, Equation (20) becomes $Q_r = 1$. Therefore, F and $Q_a(s)$ are as follows.

$$F = \begin{bmatrix} 1.20\times10^3 & 4.05\times10^0 & -5.00\times10^{-2} \end{bmatrix} \tag{28}$$

$$Q_a(s) = 2.04\times10^0 \tag{29}$$

Compute the stabilizing solution of Equation (23) for $\delta = 10$, and substitute the result for Y in Equation (24), this equation becomes

$$K = \begin{bmatrix} 3.83\times10^3 & 5.93\times10^6 & 4.26\times10^{-10} \end{bmatrix}^T \tag{30}$$

Because the resonance frequency of a system stabilized with Equation (28) and (29) is 25Hz, the disturbance d is determined as follows.

$$d(s) = \frac{2\pi\times25}{s^2 + (2\pi\times25)^2} \tag{31}$$

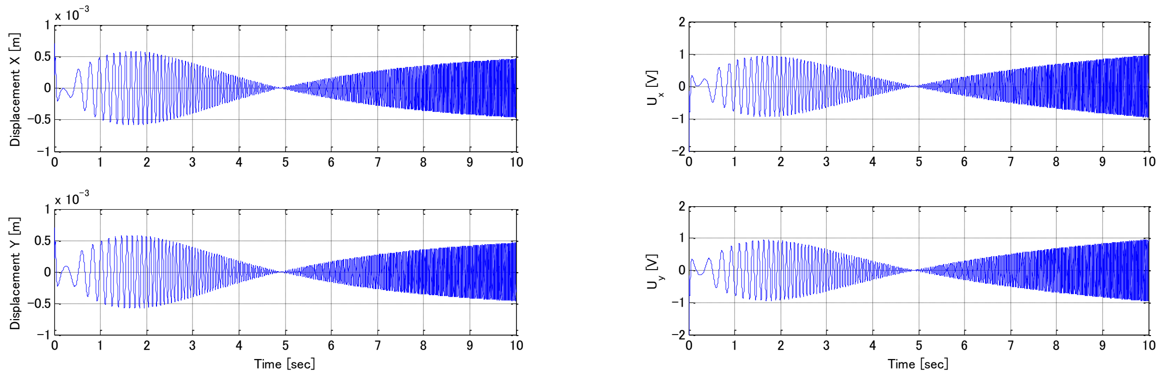
Substitute $\tilde{\tau} = 0.1$ in Equation (25), compute the coefficients q_k to satisfy Equation (12) for $s = 0, \pm 2\pi\times25i\ (\xi = 3)$ which are poles of $r(s)$ and $d(s)$. As a result, Equation (25) becomes as follows

$$Q_b(s) = \frac{9.06\times10^1 s^2 + 3.71\times10^2 s + 9.03\times10^3}{(0.1s+1)^2} \tag{32}$$

4 Simulation results

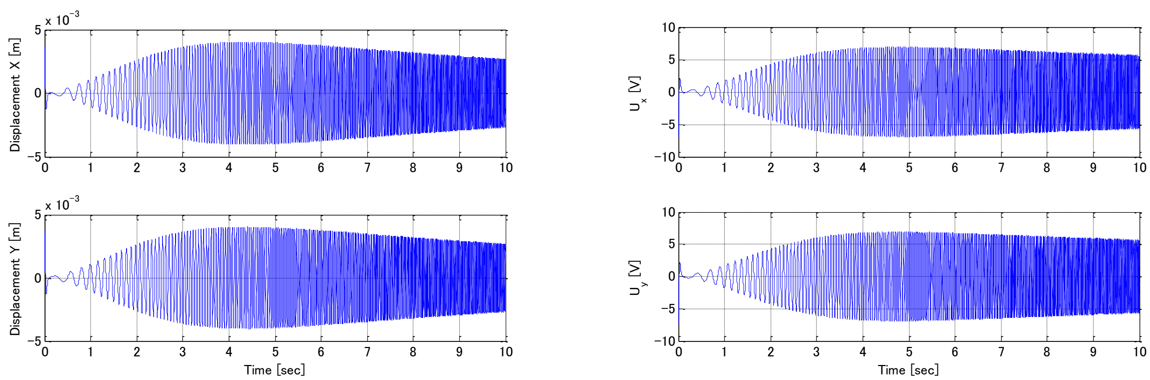
The simulation results in the condition which raise a rotational speed in 5rps/sec from 0rps is evaluated. The results using the proposal controller is shown in Figure 3. The frequency which the disturbance compensator operates is passed at 5sec. It has been shown from this result that the displacements and the input signals are controlled very small by a controller.

For comparison, the result without the disturbance compensator is shown in Figure 4. Because of an imbalanced influence, the amplitude of displacement and input have been become large. From these results, the validity of the disturbance compensator has been confirmed.



(a) Displacement (b) Control inputs

Figure 3: Unbalance responses using Equation (32)



(b) Displacement (b) Control inputs

Figure 4: Unbalance responses without the disturbance compensator ($Q_b = 1$)

3 Conclusion

In this paper, an active magnetic bearing controller for stabilization and unbalance disturbance rejection has been proposed. The controller is constructed as a multi-degree of freedom controller based on internal model control. The disturbance compensator for unbalance is designed to control a sine wave disturbance which is same frequency as a critical speed. The validity of the disturbance compensator has been confirmed by simulation results.
It had been clear that the proposal technique can be applied to control a MIMO system, and to compensate several sine wave disturbances from which frequency differs [8]. The future object of this study will be to realize MIMO controller, and to control parallel and conical modes.

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