Fast approximate calculation of the radial and tilt stiffness of magnetic bearings using magnetostatic 2D finite element analysis

Gerald Jungmayr*, Edmund Marth, Martin Panholzer, Wolfgang Amrhein
Johannes Kepler University Linz
Shareholder of the ACCM GmbH
Institute of Electrical Drives and Power Electronics
Linz, Austria

Guenther Weidenholzer
Linz Center of Mechatronics
Shareholder of the ACCM GmbH
Linz, Austria

Abstract

In order to calculate the dynamic behavior of magnetically levitated rotors (eigenfrequencies, rotor deflections...), the knowledge of the radial and tilt stiffness of the active and passive magnetic bearings is necessary. An optimization of the stability against external disturbances and internal excitations (unbalance, magnetic tolerances) requires a large number of calculations of these magnetic bearing parts. Therefore, a short calculation time of the reluctance forces is crucial. As an original rotational symmetry is lost in case of a radially deflected or tilted rotor, the tilt stiffness and the radial stiffness of magnetic bearing parts are usually calculated with a 3D-FE software. These time-consuming calculations prevent fast optimizations of the rotor dynamics. This paper shows a method for the approximate calculation of the radial and tilt stiffness of rotationally symmetric magnetostatic problems by using a 2D-FE program. The accuracy of the approximation method will be verified by comparing the results with the exact analytical solution of a pure permanent magnetic configuration.

1 Introduction

For magnetic bearing systems using passively stabilized degrees of freedom, the optimization of the rotordynamics is essential in the design process. Therefore, all stiffness and damping coefficients of the components (passive magnetic bearings, active magnetic bearings, motor, viscoelastic and eddy current dampers) must be known.

Passive magnetic bearings [9] are an effective way to support some degrees of freedom of a rotor. In [5] Lang showed how to analytically calculate the radial and tilt stiffness by numerically solving elliptical integrals, if there is no iron in the magnetic circuit. Due to today's performance of standard computers, this method is well suited for optimizations and delivers results within one second. Furthermore, the results are exact if the relative permeability of the magnet is equal to one. If the magnetic flux is guided with magnetizable material (iron, ferritic stainless steel) a finite element program has to be used for the computation of stiffness values. Due to the lack of damping of permanent magnetic bearings, additional measures are required to gain sufficient stability against disturbances and to successfully pass the eigenfrequencies during run up. For applications with a limited temperature range, one possibility is to use a viscoelastically

*Contact Author Information: gerald.jungmayr@jku.at, Johannes Kepler University of Linz, Altenbergerstrasse 69, 4040 Linz, Austria, phone: +43 732 2468 6430

supported stator in order to damp the vibrations of the rotor [4, 6]. An alternative is to make use of eddy currents to damp rotor vibrations [1].

Due to Earnshaws theorem, a radially passively supported rotor is unstable in axial direction. If the axial degree of freedom is stabilized by an active magnetic bearing featuring reluctance forces (e.g. as shown in figure 1), the passive stiffness values of this component have to be calculated for the rotordynamic model. Furthermore, these stiffness values are a function of the coil current. Additionally, the negative radial stiffness of permanent magnet synchronous motors (PMSM) has to be included in the design.

This paper proposes a method for the estimation of the radial and tilt stiffness, which are initially 3D problems, by using a 2D-FE program. This allows a much faster optimization of magnetic bearing parts.

2 Calculation method

In this section, approximation formulas for the tilt stiffness $k_{\varphi\varphi}$ and the radial stiffness k_r of axisymmetric components are derived. Figure 1 shows an exemplary embodiment of an active magnetic bearing. The figures 1 and 2 are used to explain the approach, nevertheless the derived formulas are generally applicable for configurations with vanishing magnetic flux in circumferential direction

$$\begin{pmatrix} B_r \\ B_\varphi \\ B_z \end{pmatrix} \approx \begin{pmatrix} B_r \\ 0 \\ B_z \end{pmatrix}. \tag{1}$$

For example, the method can be applied to passive magnetic bearings using ferromagnetic material for the guidance of the magnetic flux. In such cases the mentioned analytical solution is not applicable.

Planar modeling of the problems fulfilling equation 1 can be one possibility to derive the approximate radial stiffness using a 2D finite element analysis. The error made by neglecting the rotational symmetry of the field will be small (see section 3.3). However, for an active magnetic bearing as shown in figure 1, the volume of the back iron increases with the radial distance r. Therefore, a planar model is not suited for the calculation of the actual saturation in the back iron. In order to solve these kinds of problems, a 3D-FE software is typically used where the rotor is moved (tilted) in several steps for the calculation of the radial force (the tilting torque).

2.1 Approximation of the tilt stiffness

If the rotor is tilted by a small angle β (see figure 1), the axial deflection Δz of a magnet cross section at the circumferential angle φ is

$$\Delta z \approx -r_m \cdot \beta \cdot \cos(\varphi), \tag{2}$$

where r_m denotes the mean magnet radius. The tilting torque can be estimated by

$$M_\beta \approx \int_0^{2\pi} \underbrace{r_m \cdot \cos(\varphi)}_{\text{lever arm}} \cdot \frac{F_z(\Delta z)}{2\pi} \cdot d\varphi. \tag{3}$$

By means of the axial stiffness k_z the force of a purely axial displacement can be written as $F_z(\Delta z) \approx -k_z \cdot \Delta z$.

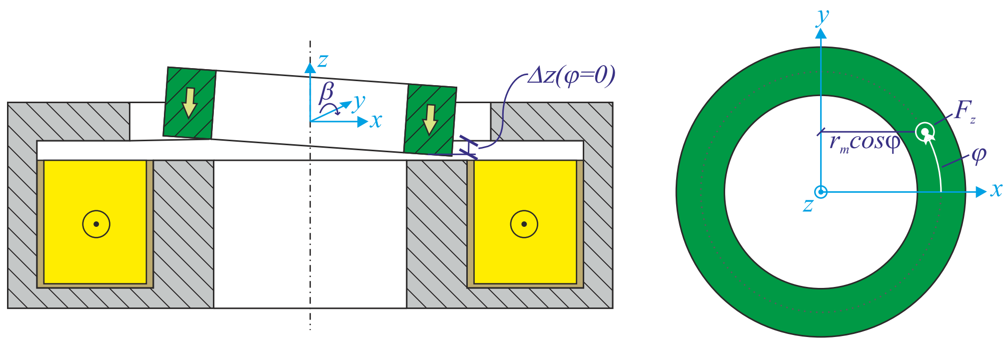

Figure 1: Exemplary illustration of an active magnetic bearing used for the derivation of the tilt stiffness (left: cross section, right: magnet, top view)

The derivation of equation (3) with respect to β leads to an approximation formula for the tilt stiffness

$$k_{\varphi\varphi} \approx \frac{1}{2} r_m^2 \cdot k_z. \tag{4}$$

Therefore, the tilt stiffness can be calculated by using the axial stiffness if the tilting of the magnet cross section is neglected.

2.2 Approximation of the radial stiffness

Although it is not as obvious, also the radial stiffness can be estimated by 2D-FE-calculations. If the rotor is moved by r in direction of the x-axis, the radial deflection of a magnet cross section at the circumferential angle φ is

$$\Delta r = r \cdot \cos(\varphi), \tag{5}$$

as marked in figure 2. Using an axisymmetric analysis in a 2D-FE program means that in the case of $\Delta r = -r$ a smaller magnet is simulated (larger radial air gap), while $\Delta r = +r$ leads to a larger magnet (smaller radial air gap). Therefore, the variation done in the finite element software scales the size of the rotor from a little bit smaller to a little bit bigger dimensions. As the integral of the mechanical stress over the circumference is always zero in rotationally symmetric problems, the radial force will still be zero. However, if the magnetic coenergy W_{mag}^{co*} is calculated for different rotor dimensions (different Δr), an approximate calculation of the radial stiffness can be done. The results gained from the finite element analysis can be expressed as a Taylor series

$$W_{mag}^{co*}(\Delta r) = f_0 + f_1 \Delta r + f_2 \Delta r^2 + f_3 \Delta r^3 + \dots . \tag{6}$$

By using this intermediate result, the magnetic coenergy W_{mag}^{co} of a rotor moved by r in direction of the x-axis can be approximated

$$W_{mag}^{co}(r) \approx \int_0^{2\pi} \frac{W_{mag}^{co*}(r \cdot \cos(\varphi))}{2\pi} d\varphi. \tag{7}$$

The second derivative of the magnetic coenergy delivers the radial stiffness [8]

$$k_r = \left. \frac{d^2 W_{mag}^{co}(r)}{dr^2} \right|_{r=0} = f_2. \tag{8}$$

Therefore, the term proportional to the Δr^2 in the Taylor series expansion of W_{mag}^{co*} is equal to the radial stiffness k_r.

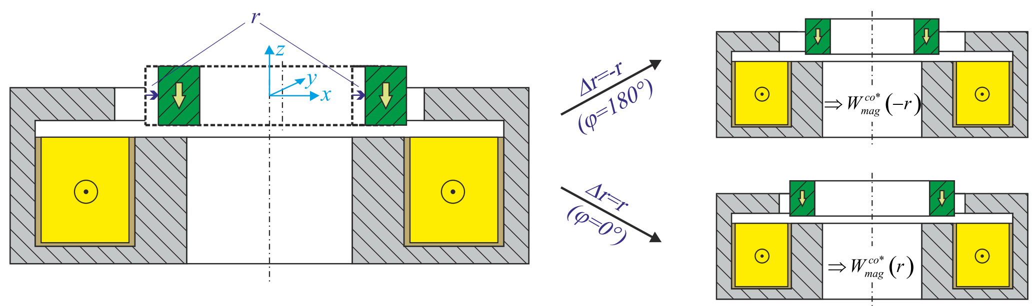

Figure 2: Exemplary illustration of an active magnetic bearing used for the derivation of the radial stiffness. In the rotationally symmetric 2D finite element analysis the magnet is varied from smaller ($\Delta r < 0$) to bigger dimensions ($\Delta r > 0$).

3 Proof of concept by means of permanent magnetic bearings

3.1 Geometry of permanent magnetic bearings

The axial force F_z, the axial stiffness k_z, the radial stiffness k_r, the tilt stiffness $k_{\varphi\varphi}$ and the couple stiffness $k_{r\varphi}$ of permanent magnetic bearings can be exactly calculated if the relative permeability of the magnet is equal to one and if there is no other magnetically conductive material affecting the field of the magnets [5]. Therefore, this magnetostatic problem can be used to estimate the error of finite element discretization in the software FEMM [3] and the error of the approximation methods described in the sections 2.1 and 2.2.

Figure 3 specifies the geometry of two attractive permanent magnetic radial bearings used for the verification. While for parameter set I the diameter ratio d_{ri}/d_{ro} is near to one, the parameter set II describes a configuration with significant radial extent ($\frac{d_{ri}}{d_{ro}} \ll 1$).

3.2 Verification of the finite element analysis accuracy using the axial force

Two different boundary conditions are used in the finite element calculations:

1. Closed region: Vector potential $A = 0$ on the boundary.

2. “Open boundary”: Using the Kelvin transformation. See [2] for a review of different finite element boundary techniques.

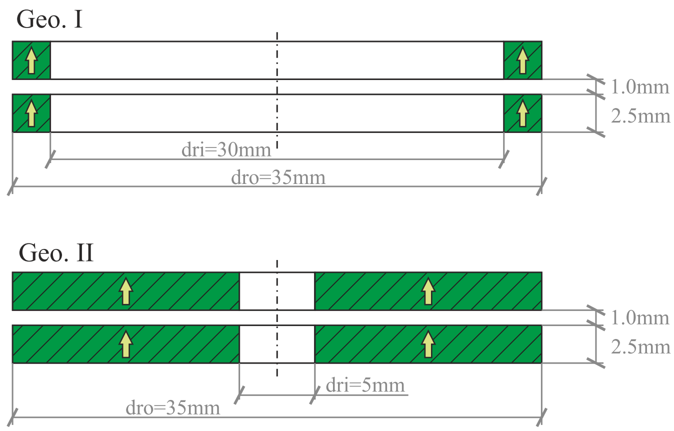

Figure 3: Two attractive permanent magnetic ring bearing geometries used for the verification

	Analytical result	Method	Steps	Error FE calculation	
	Geo. I			Geo. I	Geo. II
F_z	-40.714 N	Maxwell force	1	0.02%	-1.45%
		Maxwell force	11	0.01%	-1.44%
		Coenergy	11	-0.32%	-1.57%
k_z	-16 645 N/m	Maxwell force	11	1.64%	1.91%
		Coenergy	11	3.42%	-1.21%
k_r	8323 N/m	Planar, Maxwell force	11	0.72%	2.17%
		Coenergy, equation 8	11	0.72%	-1.25%
$k_{\varphi\varphi}$	-2.177 Nm/rad	equation 4	-	2.53%	-60.8%

Table 1: Relative error of the finite element calculation/approximation for geometry I and II. Parameters: open boundary with 40mm diameter, mesh element size 0.05mm.

For the geometry I, figure 4(a) reveals the relative error of the axial force F_z using a closed region in the 2D finite element analysis, while figure 4(b) shows the results of the open boundary calculations. The finite element results are calculated in FEMM. From figure 4 it is evident, that a small mesh size is necessary for a good accuracy. The closed region calculations with sufficiently large boundary diameter (for this problem $\geq$ 50mm) are equally well suited as the open boundary computations. In case of the open boundary, the amount of the relative error stays below 1 percent for the finest mesh size.

3.3 Verification of the approximation methods

Using the knowledge gained from figure 4, the calculation of all stiffness values (table 1) is done with an open boundary of 40mm diameter and a mesh size of 0.05mm. The last two rows, k_r and $k_{\varphi\varphi}$, are derived as described in sections 2.2 and 2.1. The radial stiffness k_r was additionally calculated by a planar setup, all other problem definitions were set to axisymmetric.

The table states the number of finite element calculations (column "Steps") used to calculate the force/stiffness. The method "Maxwell force" denotes that the forces were calculated directly

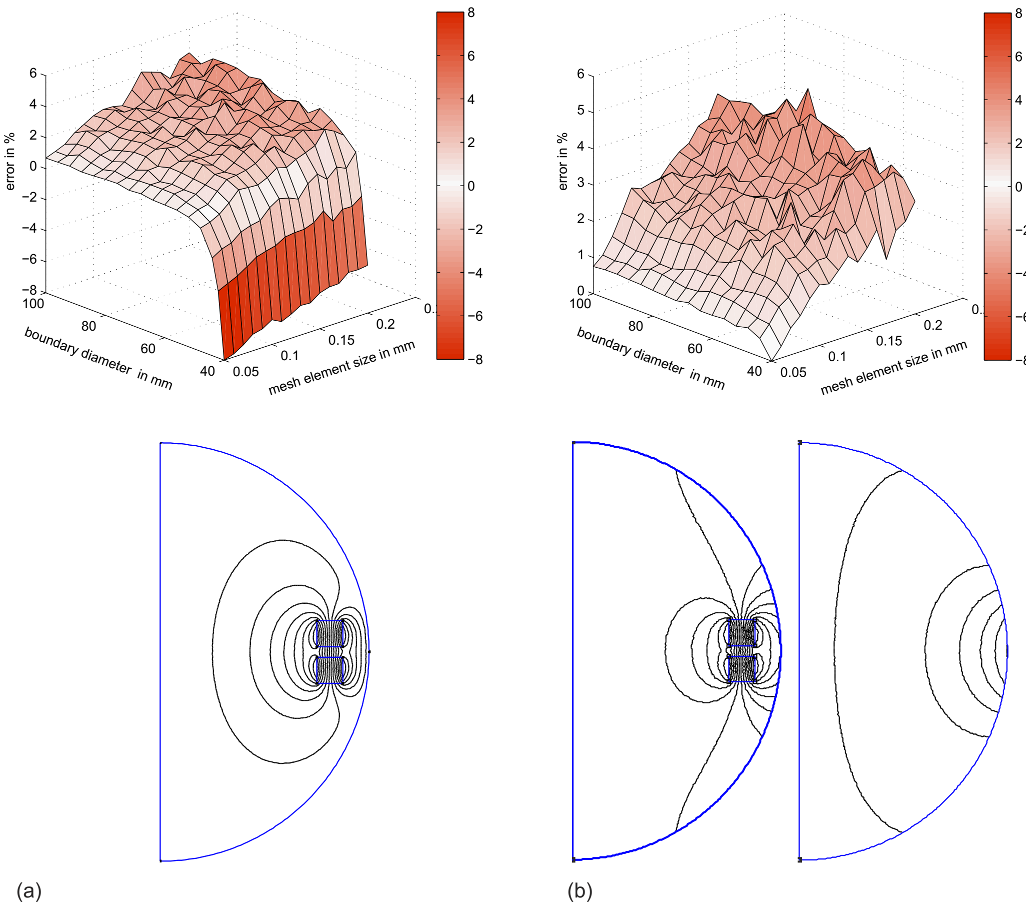

Figure 4: Relative error in the axial force of geometry I for a closed region (a) and for an open boundary (b).

by using the Maxwell stress tensor. The stiffness values were gained by the differential quotient. In both cases, the value at the nominal air gap was calculated by using a least square fit. The method “Coenergy” means that the force/stiffness was calculated by using the derivative of the magnetic coenergy with respect to the axial or radial degree of freedom. Also the magnetic coenergy was fitted by a Taylor series. As an example, the figure 6 illustrates the fitted Taylor series of W_{mag}^{co*} used for the calculation of the radial stiffness k_r.

3.3.1 Radial stiffness

As can be seen from table 1, the radial stiffness k_r was calculated by a planar setup and by the method described in section 2.2 using equations 6 - 8. The table 1 reveals that the results from the planar setup are sufficiently precise for most technical problems. The influence of the curvature is quite low, even for the case ${}^{d_{ri}}/_{d_{ro}} \ll 1$ (geometry II). This outcome is confirmed by the investigations done in [7].

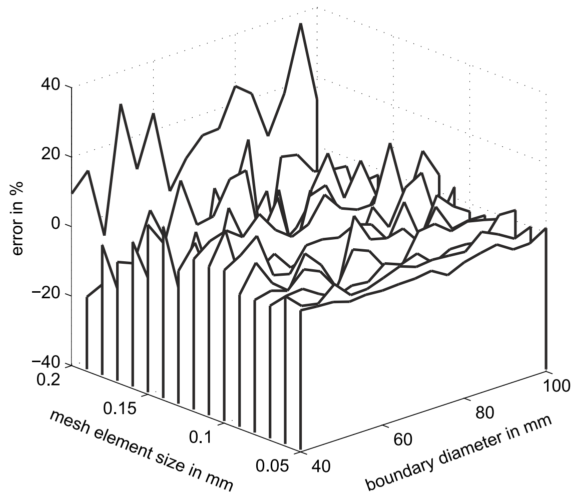

Figure 5: Relative error in the radial stiffness gained from the approximation method using the magnetic coenergy (open boundary calculations, geometry I).

However, the planar modeling is not possible for problems including magnetically conductive elements as shown in figure 2. For this kind of problems, the radial stiffness k_r has to be calculated by the approximation method described in section 2.2. This method was applied to the two permanent magnetic bearing geometries. The relative low error of k_r in table 1 (method "Coenergy, equation 8") indicates, that this approximation method delivers satisfying results. The calculation accuracy of the radial stiffness k_r is at the same level as the calculation accuracy of the axial stiffness k_z, where no methodical error is made.

However, special attention must be paid to the mesh element size. Figure 5 shows that the relative error for k_r can reach up to 40 percent for a rough mesh, but stays below 3 percent for the finest investigated mesh. The calculated values of the magnetic coenergy W_{mag}^{co*} as well as the corresponding Taylor series, gained by using a least square fit, are shown in figure 6. The legend states the calculated radial stiffness k_r and its relative error for 48mm, 72mm and 96mm boundary diameter. As depicted in figure 6(a) the relative error of k_r is quite low for the fine mesh, while figure 6(b) reveals that the curvature of the magnetic coenergy $W_{mag}^{co*}(\Delta r)$ fluctuates in case of the rough mesh. Therefore, also the relative error of k_r gets high, almost 40 percent as mentioned before.

3.3.2 Tilt stiffness

In case of geometry I ($^{d_{ri}}/_{d_{ro}} \approx 1$), the tilt stiffness $k_{\varphi\varphi}$ can be calculated sufficiently exact by the approximation formula 4, see table 1. For geometries with larger radial extent ($^{d_{ri}}/_{d_{ro}} \ll 1$), as geometry II, this simple approximation leads to huge errors. However, for problems without nonlinear materials (as shown in figure 3), better results are possible by dividing the magnet into several rings. Due to the linearity, the individual results of the tilt stiffness $k_{\varphi\varphi}$ can be superposed.

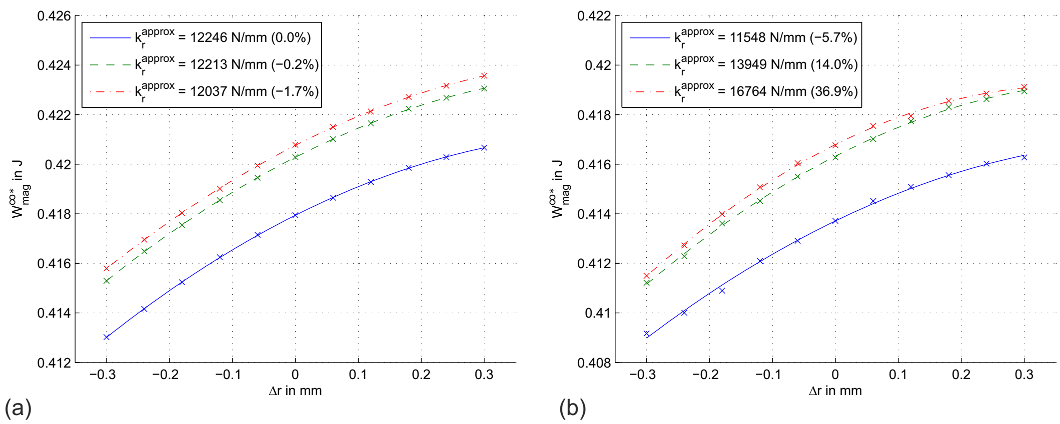

Figure 6: Magnetic coenergy $W_{mag}^{co*}(\Delta r)$ for a fine (a) and for a rough mesh (b). Results for geometry I and an open boundary with 48mm, 72mm and 96mm boundary diameter.

4 Conclusions

This paper considered fast methods for the calculation of radial and tilt stiffness of rotationally symmetric magnetic bearing parts. Concerning the radial stiffness, a planar problem definition can usually only be used for problems without magnetically conducting material. Therefore, the proposed solution method is necessary for axisymmetric configurations with ferromagnetic material used for the guidance of the magnetic flux. Since the stiffness values of pure permanent magnetic ring bearings can be calculated exactly by analytical means, the accuracy of the approximation method was verified using this magnetic bearing type. It could be proven that the proposed approach delivered radial stiffness values which are sufficiently precise for most technical problems. A major influence was shown for the mesh size, while the difference between open and closed boundary results was negligible. It was shown that the tilt stiffness can be approximated by the axial stiffness within certain limits.

Acknowledgment

This work was conducted in the realm of the research program at the Austrian Center of Competence in Mechatronics (ACCM), which is a part of the COMET K2 program of the Austrian government. The authors thank the Austrian and Upper Austrian government for their support.

References

[1] Nicola Amati, Xavier De Lepine, and Andrea Tonoli. Modeling of electrodynamic bearings. *Journal of Vibration and Acoustics*, 2008.

[2] Qiushi Chen and A. Konrad. A review of finite element open boundary techniques for static and quasi-static electromagnetic field problems. *Magnetics, IEEE Transactions on*, 33(1):663 –676, jan 1997.

[3] Meeker David. Finite element method magnetics. *User's manual, Version 4.2*, 2007.

[4] Gerald Jungmayr, Edmund Marth, Wolfgang Amrhein, and Guenther Weidenholzer. Influence of viscoelastic damping elements on magnetically active stabilized degrees of freedom. page 6, August 2010.

[5] M. Lang. Berechnung und optimierung von passiven permanentmagnetischen lagern fuer rotierende maschinen. 2003.

[6] Edmund Marth, Gerald Jungmayr, and Wolfgang Amrhein. Influence of viscoelastic elements on magnetically passive stabilized degrees of freedom. page 6, August 2010.

[7] Edmund Marth, Gerald Jungmayr, Martin Panholzer, and Wolfgang Amrhein. Optimization of stiffness per magnet volume ratio of discrete and continuos halbach type permanent magnetic bearings. 2012.

[8] D.C. White and H.H. Woodson. *Electromechanical energy conversion*. M.I.T. core curriculum program in electrical engineering. Wiley, 1959.

[9] J.-P. Yonnet. Permanent magnet bearings and couplings. *Magnetics, IEEE Transactions on*, 17(1):1169 – 1173, jan 1981.