Realization of Parallel-Connected Double Magnetic Suspension

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Abstract

A double magnetic suspension system with parallel-connected coils is actualized in this work. The double magnetic suspension system includes a pair of subsystems, each of them has an electromagnet and a floator, and controls them by a single power amplifier solely. Double suspension systems are classified into two types based on the connection of the double coils: series connection and parallel connection. The feasibility of double magnetic suspension with series-connected coils has been already demonstrated in the previous works. This work focuses on double magnetic suspension with parallel-connected coils. The conditions for the system to be controllable are presented for its mathematical model. The feasibility of double magnetic suspension is demonstrated experimentally.

1 Introduction

A basic active magnetic suspension system consists of the following components:

- object to be suspended (floator)
- electromagnet to produce suspension force
- sensor to detect the displacement of the floator
- controller
- power amplifier to feed current to the winding of the electromagnet

This system is inherently unstable in the normal direction. Stable action can be achieved by sensing the position of the rotor and controlling the force fields to prevent the floator from departing from its desired position. This suspension technique has been used in various industrial fields. However, the full potential of industrial applications has not been acheived yet. To achieve hardware savings, self-sensing magnetic suspension has been proposed and studied extensively as one of the most promising methods of reducing the cost of hardware [1-4].

As a quite different approach of cost reduction, we have proposed parallel magnetic suspension in which multiple floators are controlled with a single power amplifier [5]. According to the output of the power amplifier, parallel magnetic suspension systems are classified into two types: current-controlled and voltage-controlled. A current-controlled parallel magnetic suspension has been already realized [6, 7]. We focus on voltage-controlled parallel suspension.

Voltage-controlled parallel magnetic suspension systems are classified into two types based on the connection of the multiple coils: series connection and parallel connection. The feasibility of double magnetic suspension with series-connected coils has been already demonstrated in the previous work [8]. This paper focuses on a voltage-controlled parallel magnetic suspension system with parallel-connected coils (parallel connection). The controllability of this system was discussed based on its mathematical model [5]. In this study, an experimental apparatus is fabricated, which consists of two magnetic suspension subsystems. Stable levitation is realized by applying state feedback control to the fabricated experimental apparatus.

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2 Parallel Magnetic Suspension

2.1 Concept

The concept of parallel magnetic suspension is illustrated by Figure 1 in which the multiplicity (number of floator and electromagnet) is three; three floators and electromagnets are controlled with a single power amplifier. In principle, the multiplicity can be set arbitrary [5]. The connection of coils is of two types: (a) series and (b) parallel. We focus on parallel-connected system whose multiplicity is two, which is referred to as double magnetic suspension system, in this paper.



Figure 1: Parallel Magntic Suspension Systems

2.2 Mathematical Model

2.2.1 Single Suspension System

Figure 2 shows a single-degree-of-freedom model of the magnetic suspension system used for the analysis. An electromagnet is used to generate and control the suspension force. The floator is assumed to move only in the vertical direction. The variable x indicates the displacement of the floator from the equilibrium position where the attractive force generated by the permanent magnet balances the gravitational force. The linearized equation of motion in the neighbourhood of the equilibrium point is given by

$$m\ddot{\mathbf{x}}(t) = k_s \mathbf{x}(t) + k_i i(t), \tag{1}$$

where *m*: mass of floator, k_s : gap-force coefficient of hybrid magnet, k_i : current-force coefficient of hybrid magnet, and *i*: current flowing through the coil.

The electrical circuit equation associated with the electromagnet is given by

$$L\frac{di}{dt} + Ri + k_b \dot{x} = v(t), \tag{2}$$

where L: inductance of coil, R: resistance of coil, k_b : velocity-voltage coefficient, and v: voltage applied to the coil. When a power amplifier with voltage output is used, the voltage applied to the coil is treated as a control input while the coil current is treated as a state variable. This is called a voltage-controlled magnetic suspension system, and the state equation is given by

$$\dot{\boldsymbol{x}}_{\boldsymbol{v}}(t) = \boldsymbol{A}_{\boldsymbol{v}} \boldsymbol{x}_{\boldsymbol{v}}(t) + \boldsymbol{b}_{\boldsymbol{v}} \boldsymbol{v}(t), \tag{3}$$

where

$$\boldsymbol{x}_{\boldsymbol{v}} = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}, \ \boldsymbol{A}_{\boldsymbol{v}} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & 0 & a_{23} \\ 0 & -a_{32} & -a_{33} \end{bmatrix}, \ \boldsymbol{b}_{\boldsymbol{v}} = \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix}, \ a_{21} = \frac{k_s}{m}, \ a_{23} = \frac{k_i}{m}, \ a_{32} = \frac{k_b}{L}, \ a_{33} = \frac{R}{L}, \ b_0 = \frac{1}{L}.$$

When the magnetic circuit is ideal, that is, there is no leakage flux, no saturation and no hysteresis, the following relation holds [2].

$$a_{21} = a_{23}a_{32} \tag{4}$$



Figure 2: Basic Model of Single Magnetic Suspension System

2.2.2 Double Suspension System with Parallel Connection

Below, matrix, vector, variables and parameters related to the *k*th subsystem consisting of a floator and the corresponding electromagnet are denoted by the parenthetic superscript *k*, e.g., $A^{(k)}$, $x^{(k)}$, and $a^{(k)}$. The state equation of the double parallel magnetic suspension system with parallel connection is represented as follows.

$$\dot{\mathbf{x}}_{vp2}(t) = \mathbf{A}_{vp2} \mathbf{x}_{vp2}(t) + \mathbf{b}_{vp2} \mathbf{v}(t),$$
(5)

where

$$\boldsymbol{x}_{vp2} = \begin{bmatrix} \boldsymbol{x}_{v}^{(1)} \\ \boldsymbol{x}_{v}^{(2)} \end{bmatrix}, \ \boldsymbol{A}_{vp2} = \begin{bmatrix} \boldsymbol{A}_{v}^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_{v}^{(2)} \end{bmatrix}, \ \boldsymbol{b}_{vp2} = \begin{bmatrix} \boldsymbol{b}_{v}^{(1)} \\ \boldsymbol{b}_{v}^{(2)} \end{bmatrix}.$$

2.3 Controllability

The controllability matrix C_{vp2} of the system represented by Eq.(4) is

$$\boldsymbol{\mathcal{C}}_{vp2} = \begin{bmatrix} \boldsymbol{\mathcal{C}}_{v}^{(1)} & (\boldsymbol{A}_{v}^{(1)})^{3} \boldsymbol{\mathcal{C}}_{v}^{(1)} \\ \boldsymbol{\mathcal{C}}_{v}^{(2)} & (\boldsymbol{A}_{v}^{(2)})^{3} \boldsymbol{\mathcal{C}}_{v}^{(2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{C}}_{v}^{(1)} & 0 \\ 0 & \boldsymbol{\mathcal{C}}_{v}^{(2)} \end{bmatrix} \begin{bmatrix} I & (\boldsymbol{\mathcal{C}}_{v}^{(1)})^{-1} (\boldsymbol{A}_{v}^{(1)})^{3} \boldsymbol{\mathcal{C}}_{v}^{(1)} \\ I & (\boldsymbol{\mathcal{C}}_{v}^{(2)})^{-1} (\boldsymbol{A}_{v}^{(2)})^{3} \boldsymbol{\mathcal{C}}_{v}^{(2)} \end{bmatrix},$$
(6)

where

$$\boldsymbol{C}_{v}^{(k)} = \begin{bmatrix} \boldsymbol{b}_{v}^{(k)} & \boldsymbol{A}_{v}^{(k)} \boldsymbol{b}_{v}^{(k)} & (\boldsymbol{A}_{v}^{(k)})^{2} \boldsymbol{b}_{v}^{(k)} \end{bmatrix}.$$

The determinant of this matrix becomes

$$\left|\boldsymbol{\mathcal{C}}_{vp2}\right| = \left|\boldsymbol{\mathcal{C}}_{v}^{(1)}\right\| \left|\boldsymbol{\mathcal{C}}_{v}^{(2)}\right\| \left(\left(\boldsymbol{\mathcal{C}}_{v}^{(1)}\right)^{-1} \left(\boldsymbol{A}_{v}^{(1)}\right)^{3} \boldsymbol{\mathcal{C}}_{v}^{(1)}\right)^{3} - \left(\left(\boldsymbol{\mathcal{C}}_{v}^{(2)}\right)^{-1} \left(\boldsymbol{A}_{v}^{(2)}\right)^{3} \boldsymbol{\mathcal{C}}_{v}^{(2)}\right)^{3}\right|.$$
(7)

The necessary and sufficient condition of controllability is given by [5]

$$a_{23}^{(k)} \neq 0, \ b_0^{(k)} \neq 0,$$
 (8)

and

$$\left| (\hat{A}_{\nu}^{(1)})^{3} - (\hat{A}_{\nu}^{(2)})^{3} \right| \neq 0,$$
(9)

where

$$\hat{A}_{\nu}^{(k)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{21}^{(k)} a_{33}^{(k)} & a_{32}^{(k)} a_{23}^{(k)} - a_{21}^{(k)} & -a_{33}^{(k)} \end{bmatrix}.$$
(10)

In addition, when the relation given by (4) holds in each subsystem, the condition (9) is simplified as [5]

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$$(a_{21}^{(1)}a_{33}^{(2)} - a_{21}^{(2)}a_{33}^{(1)})^3 - (a_{21}^{(1)} - a_{21}^{(2)})^2 (a_{33}^{(1)} - a_{33}^{(2)}) (a_{33}^{(1)}a_{33}^{(2)})^2 \neq 0,$$
(11)

Equation (8) indicates that each subsystem have to be controllable for the controllability of the parallel connected system. In addition, Equation (11) indicates that the parallel connected becomes controllable in the following conditions.

[Condition 1]

$$a_{21}^{(1)} \neq a_{21}^{(2)}, \ a_{33}^{(1)} = a_{33}^{(2)}.$$
 (12)

[Condition 2]

$$a_{21}^{(1)} = a_{21}^{(2)}, \ a_{33}^{(1)} \neq a_{33}^{(2)}.$$
(13)

Condition 2 implies that the system can be controllable by adjusting the time constant of the electrical circuit, for example, connecting an external resistance to the coil, even if subsystems have the same mechanical dynamics governed by the parameter $a_{21}^{(k)}$. It is to be noted that the double parallel magnetic suspension systems with series-connected coils are uncontrollable when $a_{21}^{(1)} = a_{21}^{(2)}$ [5].

3 Experiment

3.1 Apparatus

An experimental apparatus for double parallel magnetic suspension was constructed. Figure 3(a) shows a schematic view of one suspension subsystem. It was originally fabricated to study zero-power control with a function of switching (subsystem I) [9]. The motion of the floator is constrained to one-degree-of-freedom rotational motion with a set of bearings. It has four arms suspending a suspended object and a counter weight behind the pillar. Figure 3(b) shows the other subsystem with an arm as a floator (subsystem II). The equivalent mass of floator is 2.78kg.



Figure 3: Experimental Apparatues

3.2 Control System

The displacement of each floator is detected by an eddy-current type gap sensor. A power amplifier with voltage output is used to energize the electromagnets. The designed controllers were implemented with a DSP-based digital controller DS1103. The control period is 100μ s. In the voltage-controlled amplifier, the current signals are state variables. The current of each coil is detected with a current sensor and its output is used for the feedback. The feedback gains of the controller are tuned by trial and error.

3.3 Results

Double parallel magnetic suspension achieved with the tuned feedback gains. Figure 4 shows the displacement of



(c) Control Voltage

Figure 4: Behavior when parallel suspension is achieved

each floator and the control voltage. The deviations from the average positions were within $2\mu m$ in both Subsystem I and Subsystem II. This result demonstrates well that double parallel suspension was actually achieved.

4 Conclusion

The feasibility of double magnetic suspension was demonstrated experimentally in the fabricated apparatus with a pair of suspension subsystems. Each subsystem had an electromagnet and a floator whose motion was constrained to one-degree-of-freedom rotation. The electromagnets were connected in parallel and driven by a single power amplifier with the voltage output. The feedback gains of the controller were tuned by trial and error. Double parallel magnetic suspension achieved with the tuned feedback gains.

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