Experimental Analysis of Frequency Response Function Estimation Methods for Active Magnetic Bearing Rotor System

Hynynen Katja M[1,a], Jastrzebski Rafal P[1,b]
Smirnov Alexander[1,c]
[1]Lappeenranta University of Technology, 53851 Lappeenranta, Finland
[a]katja.hynynen@lut.fi, [b]rafal.jastrzebski@lut.fi, [c]alexander.smirnov@lut.fi

Abstract: System identification can be used in automated retuning of an active magnetic bearing (AMB) rotor system during commissioning. In the case of good signal-to-noise ratio (SNR), the identification measurements are more time effective using multisine excitation than stepped sine excitation. A multisine signal contains selected frequencies in the desired frequency range. Since in multisine excitation the signal distributes the power over F frequencies, the SNR of the measurement deteriorates. The SNR is improved by averaging the measured frequency response function (FRF) over several measurements. The purpose of this study is to investigate the suitability of different frequency response function estimators for an AMB rotor system identification. The results show that the most accurate estimator for this setup with non-synchronous measurements and the excitation used, is a joint-input-output estimator.

Keywords: System Identification, Frequency Response Functions, Closed-Loop Identification

Introduction

Advanced multivariable control methods such as H_∞ control and μ-synthesis are useful methods for highly complex multiple input, multiple output (MIMO) systems like active magnetic bearings (AMBs). These control methods require an accurate model of the entire system. The rotor is modeled using finite element modeling (FEM) and experimental modal analysis (EMA). Also the linearized model of magnetic bearings is found to be accurate enough in normal conditions when the system is running near to the operating point. Besides the rotor and magnetic bearings, the overall system contains dynamics of joints, seals, bearing journals and possibly couplings between radial and axial bearings. Making an engineering model for that dynamic is difficult while their effects are not well known. However, they can be modeled using system identification.

The AMB rotor system identification has been investigated among others in [1-3]. In those studies, a stepped sine excitation that contains a series of measurements with a single sine excitation signal, has been used. An advantage of the single sine excitation is that all the signal power is concentrated in one single frequency at the time, and thus the *signal-to-noise* (SNR) ratio of the measurement is maximized. A disadvantage of the stepped sine excitation is, that the time required for the identification measurements, may be significant since for every frequencies, at least one period of the sine has to be measured, and after each frequency step there is a waiting time until the transient is stabilized. The time efficiency of the identification measurements can be increased using a multisine excitation instead of the stepped sine excitation. A random phase multisine signal can be written as

$$r(t) = \sum_{k=1}^{N_f} A_k \cos(2\pi f_k t + \phi_k) \quad (1)$$

where A_k and ϕ_k are the amplitudes and phases of each frequency component f_k. The phases ϕ_k are uniformly distributed. If the SNR is good, the time effectivity of the identification measurement is better than using stepped sine excitation. In case of very poor SNR, the measurements with both excitations require approximately the same measurement time [4]. When using a multisine excitation that distributes the power over several frequencies, the SNR is increased averaging over multiple measurement blocks. The suitability of the different methods have been investigated in [5-8].

In this study, the suitability of the different *frequency response function* (FRF) estimators for AMB rotor systems is investigated. The study shows that the best results are obtained using a joint-input-output estimator that is especially meant for a closed-loop identification.

Frequency response function estimators

The simplest method for a nonparametric identification of single input, single output (SISO) systems is an *empirical transfer function estimate* (ETFE). For multiple input, multiple output (MIMO) systems, an *empirical frequency response matrix* (EFRM) estimate

$$\hat{\mathbf{G}}_{\mathrm{ERFM}}(\omega_k) = \mathbf{Y}(k)\mathbf{U}^{-1}(k) \tag{2}$$

is used instead, where $\mathbf{Y}(k)$ and $\mathbf{U}(k)$ are the output and input matrices

$$\mathbf{Y}(k) = \begin{bmatrix} | & | & & | \\ \mathbf{Y}^{(1)}(k) & \mathbf{Y}^{(2)}(k) & \dots & \mathbf{Y}^{(n_u)}(k) \\ | & | & & | \end{bmatrix}, \tag{3}$$

$$\mathbf{U}(k) = \begin{bmatrix} | & | & & | \\ \mathbf{U}^{(1)}(k) & \mathbf{U}^{(2)}(k) & \dots & \mathbf{U}^{(n_u)}(k) \\ | & | & & | \end{bmatrix}. \tag{4}$$

The output and input vectors $\mathbf{Y}^{(i)}(k)$ and $\mathbf{U}^{(i)}(k)$ contain the discrete Fourier transforms (DFTs) of all the output and input signals of the separate experiments. In order to distinguish the influence of each input on each output in a MIMO identification, at least as many distinct experiments with different sets of excitations must be made as there are system inputs.

The ETFE and the ERFM estimates give correct FRFs for linear systems with periodic noiseless data. In practice, the estimates can be improved by averaging over several measurements [5]. If the excitation signal is measured, a *joint-input-output (JIO) estimator*

$$\hat{\mathbf{G}}_{\mathrm{JIO}}(\omega_k) = \left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{Y}^{(i)}(k)\mathbf{R}^{(i)}(k)\right)\left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{U}^{(i)}(k)\mathbf{R}^{(i)\mathrm{H}}(k)\right)^{-1} \tag{5}$$

gives an asymptotically unbiased FRF for the closed-loop identification. $\mathbf{R}$ is the excitation matrix and P is the number of blocks used in the averaging. Assume that the same, periodic excitation is applied in every block used for averaging, and the measurements are synchronized, which means that the blocks used in averaging are single periods from a multiple period measurement. Under these assumptions, the JIO estimator reduces to an *errors-in-variables (EIV) estimator*

$$\hat{\mathbf{G}}_{\mathrm{EIV}}(\omega_k)=\left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{Y}^{(i)}(k)\right)\left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{U}^{(i)}(k)\right)^{-1}. \tag{6}$$

The EIV estimator gives the asymptotically best linear approximation for the FRF both for open-loop and closed-loop measurements, if the above assumptions hold [6].

When using random excitation signals, such as Gaussian noise or a *pseudorandom binary sequence* (PRBS), the asymptotically best linear approximation is obtained using an *H1 estimator*

$$\hat{\mathbf{G}}_{\mathrm{H1}}(\omega_k)=\left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{Y}^{(i)}(k)\mathbf{U}^{(i)}(k)\right)\left(\frac{1}{P}\sum_{i=1}^{P}\mathbf{U}^{(i)}(k)\mathbf{U}^{(i)\mathrm{H}}(k)\right)^{-1} \tag{7}$$

where $(\cdot)^{\mathrm{H}}$ denotes a complex conjugate transpose. The H1-estimator is also suitable when using periodic excitation if the measurements are not synchronous. The problem of the H1-estimator is that it only considers disturbances in the output of the system. In the case of noisy input measurements, it leads to a rather large bias error. Also in closed-loop measurements, the H1-estimator gives a biased estimate [7]. Another method that suits for non-synchronous MIMO measurements, and have smaller bias error in the closed-loop measurements comparing to the H1 estimator, is an *arithmetic mean* (ARI) estimator

$$\hat{\mathbf{G}}_{\mathrm{ARI}}(\omega_k)=\frac{1}{P}\sum_{i=1}^{P}\hat{\mathbf{G}}^{(i)}_{\mathrm{ERFM}}(k). \tag{8}$$

A disadvantage of the ARI estimator is that if some of the blocks have poor SNR, the estimator may deteriorate [8]. The averaging used in the ARI estimator of Eq. (8) can be extended to contain nonlinear averaging methods. A suitable nonlinear averaging method for MIMO systems is, for example, a *logarithmic mean* (LOG) estimator

$$\hat{\mathbf{G}}_{\mathrm{LOG}}(\omega_k)=\exp\left(\frac{1}{P}\sum_{i=1}^{P}\log\left(\hat{\mathbf{G}}^{(i)}_{\mathrm{ERFM}}(k)\right)\right) \tag{9}$$

that is especially robust to outliers [5,8].

Experimental results

The identified plant consists of a non-rotating rotor suspended by two radial active magnetic bearings. In the case of a non-rotating rotor, it is sufficient to consider it as a two-input, two-output system.

The excitation signal $r(t)$ used in the identification measurements is a random phase multisine containing every 4th frequency in the grid 2...1200 Hz. Only the odd multiples of the base frequency 2 Hz are selected in order to avoid even harmonics created by a nonlinear system. The two inputs of the system are excited using an orthogonal random multisine signals for what the excitation matrix is written as

$$\mathbf{R}_2(k) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} r(k). \tag{10}$$

In the first experiment, both inputs are excited with the same signal $r(t)$, and in the second experiment the excitation signal of the second input is inverted [9], [10]. 50 blocks of length 1.5 sec are measured for both experiments and the transients are removed so that the period length of each block is 0.5 sec. The control currents i_c, rotor displacements x, and excitation signals r in each radial bearings A and B in x plane are measured.

The frequency response functions of the system are calculated using the JIO, the EIV, the H1, the ARI, and the LOG estimators averaged over 50 blocks. The FRFs are shown in Fig. 1. For comparison, an engineering model obtained with FEM, complemented with linearized actuator model and current controller dynamics, is presented.

It can be seen from Fig. 1 that parts of the FRFs are not identifiable. For example, the third flexible mode is not visible in G_{11} and G_{12}. That is because the third flexible mode passes through the sensor A. Additionally, the second mode passes through the actuator A and sensor B for what the transmission zero of the second mode is also not identifiable from G_{11}, G_{12}, and G_{21}.

Fig. 1 shows that in the identifiable frequencies, all the used estimates give quite similar results. When comparing the measured FRFs and the FEM based frequency response, it can be seen that there are some unmodeled dynamics in the true system. Thus, the measured FRFs are not fully comparable with the FEM based result. Since the true system is not known, the FRFs calculated using the JIO estimator are taken as references because this closed-loop method gives very smooth result. The H1 estimator gives almost identical FRFs comparing to the JIO estimate, as well as the ARI estimator with only slight fluctuation. The FRFs calculated with the EIV estimator fluctuate quite much that is a reason of non-synchronous measurements. The LOG gives a biased estimate for the transmission zero of the third eigenmode in G_{22}.

The FRFs of the system estimated with the ARI, the H1, and the JIO using only 10 blocks are shown in Fig. 2. It can be seen that the variance of the JIO increases slightly. The H1 estimator gives a partly biased estimate and the ARI deteriorates.

The different estimators are compared as a function of the number of blocks used for the averaging. Since the true system is not fully known, the JIO estimate that is calculated using 50 blocks, has been chosen as a reference. The differences have been calculated using the cost [6]

$$c_{\log}\left(G^1, G^2\right) = \left(\sum_{i,j,k} \left| \log G_{ij}^{\mathrm{LOG},50}(\omega_k) - \log G_{ij}^{2}(\omega_k) \right|^2 \right). \tag{11}$$

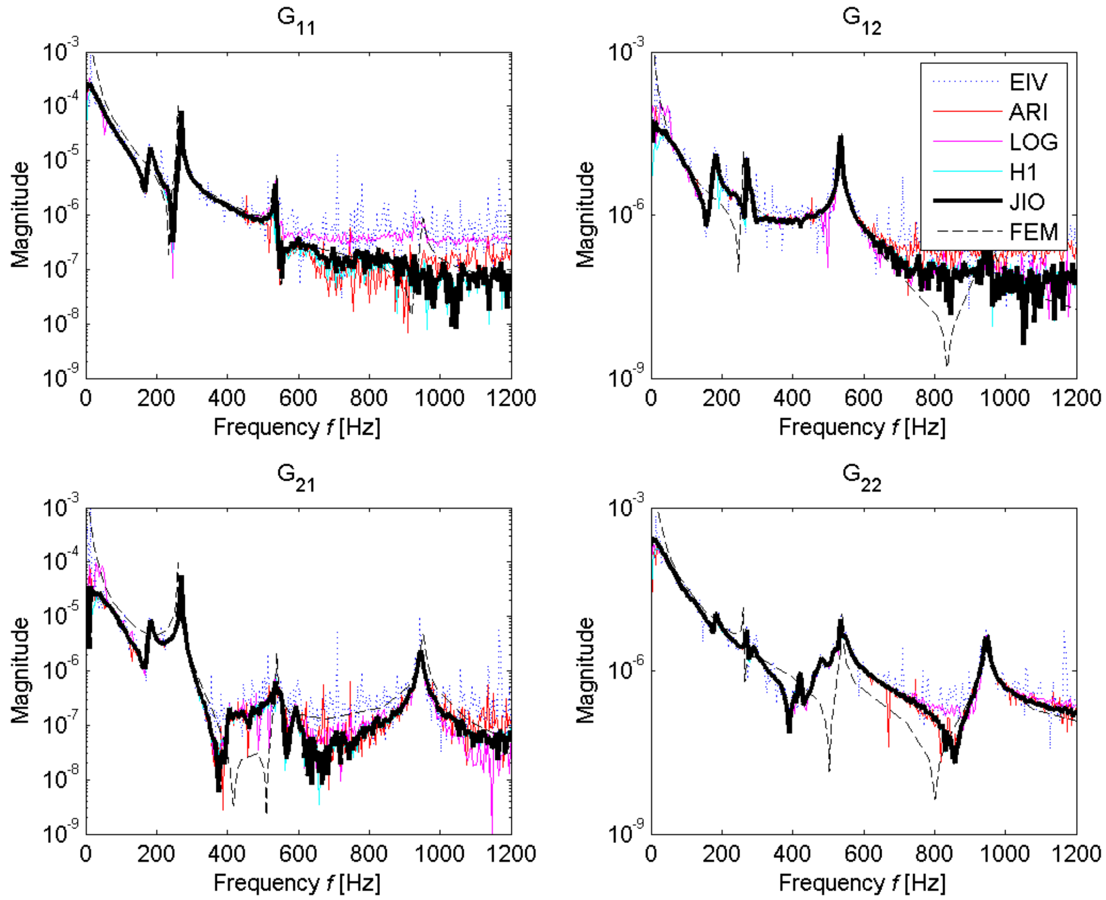

Fig. 1 FRFs of an AMB rotor system from the control currents $i_{c,Ax}$ and $i_{c,Bx}$ to the rotor displacements x_A and x_B calculated using different FRF estimators and 50 blocks for averaging.

When calculating the difference, the non-identifiable frequencies in G_{11}, G_{12}, and G_{21} have been ignored. The comparison is shown in the Fig. 3. With only one block, that is without averaging, all the estimates equal and thus give the same difference. It can be seen from Fig. 3 that the differences of the JIO, the H1, and the ARI estimates compared to the reference decrease when the number of the blocks increases. The differences of the EIV and the LOG do not decrease when increasing the number of the blocks.

The experimental verification show that the JIO, the ARI, and the H1 estimators give relatively good estimates for the investigated setup if sufficient number of blocks for averaging is used. When the number of the blocks decreases, the H1 estimate gives a biased estimate that was expected for closed-loop identification. The ARI estimate deteriorates when the number of the blocks decreases. That was also expected when some blocks have poor SNR.

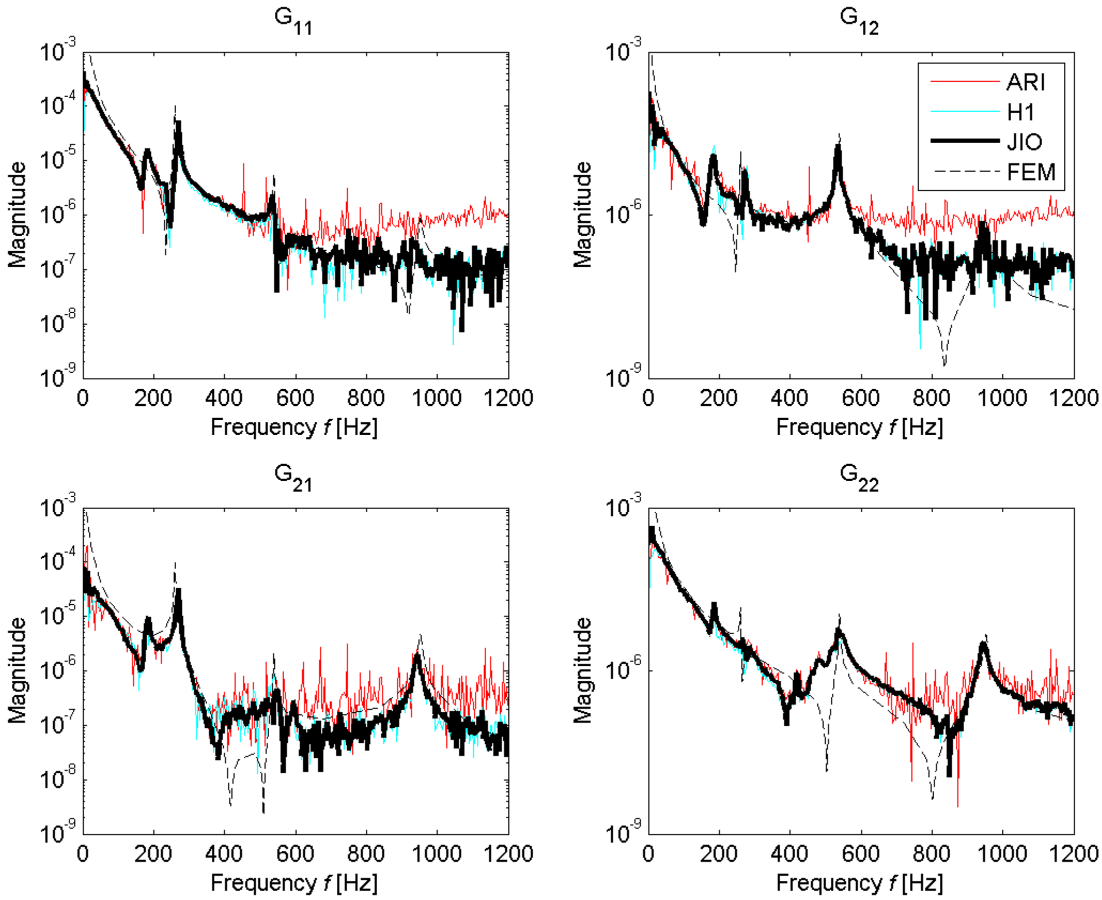

Fig. 2 The FRFs of an AMB rotor system calculated using the ARI, the H1, and the JIO estimators with 10 blocks for averaging.

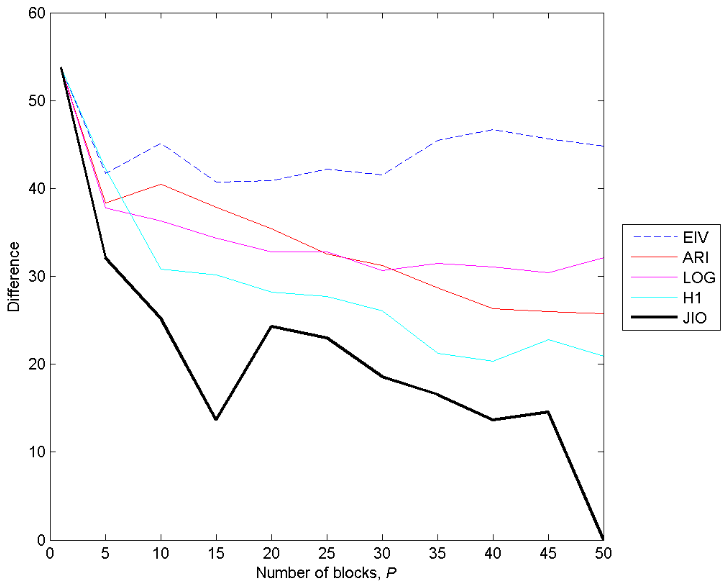

Fig.3 Comparison of different estimators as a function of the number of blocks. The LOG estimate calculated using 50 blocks is used as a reference.

Summary

The suitability of different FRF estimators for an AMB rotor system identification was investigated. The results concluded that for this setup with the orthogonal random multisine excitation and non-synchronous measurements, the JIO estimator gives the best estimate. If the reference signal cannot be measured and thus the JIO estimator is not possible to use, the H1 estimator with sufficient enough blocks should be used.

References

[1] C. Gähler, Rotor Dynamic Testing and Control with Active Magnetic Bearings, Dissertation, ETH Zürich, Switzerland, 1998.
[2] F. Lösch, Identification and Automated Controller Design for Active Magnetic Bearing Systems, Dissertation, ETH Zürich, Switzerland, 2002.
[3] J. T. Sawicki and E. H. Maslen, Toward automated AMB controller tuning: progress in identification and synthesis, Proc. 11th International Symposium on Magnetic Bearings, Nara, Japan, 2008, pp. 68-74.
[4] J. Schoukens, R. M. Pintelon, and Y. J. Rolain, Broadband Versus Stepped Sine FRF Measurements, IEEE Transactions on Instrumentation and Measurement, vol. 49, no. 2 (2000), pp. 275-278.
[5] P. Guillaume, Frequency Response Measurements of Multivariable Systems Using Nonlinear Averaging Techiques, IEEE Transactions on Instrumentation and Measurements, vol. 47, no. 3 (1998), pp. 796-800.
[6] E. Wernholt and S. Moberg, Experimental Comparison of Methods for Multivariable Frequency Response Function Estimation, Report no.: LiTH-ISY-R-2827. (Linköpings universitet, Sweden, 2007).
[7] R. Pintelon and J. Schoukens, Measurement of Frequency Response Functions Using Periodic Excitations, Corrupted by Correlated Input/Output Errors, IEEE Transactions on Instrumentation and Measurement, vol. 50, no. 6 (2001), pp. 1753-1760.
[8] E. Wernholt and S. Gunnarsson, Analysis of methods for multivariable frequency response function estimation in closed loop, Report no.: LiTH-ISY-R-2775 (Linköpings universitet, Sweden, 2007).
[9] T. Dobrowiecki and J. Schoukens, Measuring a linear approximation to weakly nonlinear MIMO systems, Automatica, vol. 43, no. 10 (2007), pp. 1737-1751.
[10] K. Hynynen and R. Jastrzebski, Optimized excitation signals in AMB rotor system identification, Proc. Identification, control and applications, Honolulu, Hawaii, USA, 2009.