Influence of Viscoelastic Elements on Magnetically Passive Stabilized Degrees of Freedom

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Abstract: Magnetic bearing technology is one of the most promising future technologies in the area of electrical drive systems. Due to the contact-free rotor suspension numerous advantages arise. The utilization of passive magnetic bearings is a very effective way of realizing magnetic levitation. Thereby, systems with a minimal constructive complexity can be obtained. However, such constructions have a main drawback: the damping of the passively stabilized degrees of freedom is negligibly small. External excitations or resonant vibrations may cause considerable damage to the bearing system. This study describes an effective way to damp vibrations by means of viscoelastic elements. The mathematical formulation of the overall dynamic system is derived and the analytic results are verified by measurements.

Keywords: Passive Magnetic Bearings, Permanent Magnetic Bearings, Damping, Viscoelastic Materials

Introduction

Thanks to the steady improvement of power electronic components, digital signal processing and sensor technologies, the field of magnetic levitation is a contemporary issue with high potential. But at the time, a rapid spreading of magnetic bearing systems and bearingless drives is slowed down mainly by their relatively high complexity and – as a consequence – their high costs. The great expenses mainly arise from the actively stabilized degrees of freedom (DOFs) where signal processing, power electronics, sensors and actuators are needed. To avoid some of this effort passive permanent magnetic bearings can be used. In this manner, bearing systems with only one actively controlled degree of freedom can be realized. The main drawback of passively stabilized DOFs is their lack of damping. Only the viscosity of the surrounding medium or eddy currents may lead to damping effects. Unfortunately, these are negligibly small in most cases. One way to overcome this problem is to introduce additional active damping elements in order to control the vibrations. In that case, however, the benefits of the passive stabilization are widely eliminated by the efforts necessary to realize the active damping elements. Furthermore, passive damping concepts utilizing eddy currents have been investigated by the scientific community. However, eddy current dampers often need elaborate mechanic constructions with yet moderate damping capabilities, especially at low rotational speeds. The approach followed in the course of this paper deals with the utilization of passive damping elements made of viscoelastic materials. Main advantages of such damping devices are the high adaptability to the magnetic bearing system, their big damping ratios and the possibility to realize as simple structures such as rings with rectangular or circular cross section. Therefore, magnetic bearing systems with high robustness but very low mechanical complexity and therefore low costs can be realized.

The Investigated System

The investigated magnetic bearing concept is shown in Fig. 1. It consists of a cylindric rotor which is stabilized by two passive permanent magnetic radial bearings, placed with a certain axial distance

to each other. Thereby the radial and the tilt displacements of the rotor are stabilized passively. In axial direction the rotor position is controlled by an active magnetic bearing (AMB). A motor unit is arranged between the passive radial bearings.
Since the damping of the passively stabilized DOFs is almost negligible such a system is instable against external disturbances. To overcome this problem the whole system, i.e. the stator, is mounted to the fixed housing by means of viscoelastic elements.

The Dynamic Behavior: In order to describe the dynamic behavior of this system some assumptions are made. First, no acceleration or deceleration of the rotor is applied, thus the dynamics is studied at stationary operation points with constant angular speed Ω. Second, the active axial bearing is presumed to be ideal, i.e. the relative axial position of the rotor to the stator is kept constant. If no excitation in axial direction is applied, this assumption is valid and this in turn means that also the axial position of the viscoelastic mounted stator is constant. For the case of an external axial excitation Jungmayr has investigated the influence of additional damping elements on the behavior of the closed loop control system in axial direction [1].
Considering the excitation of forced vibrations, two effects can be observed. The first one is mass unbalance, occurring when the rotor's principal axis of inertia does not coincide with the geometric axis of the rotor. The second effect is an eccentricity of the magnetic axis relative to the geometric axis of the bearing system. The magnetic axis is always defined by the radial rotor position, where the sum of the magnetic forces is zero. Therefore, it is not fixed but moves along an orbit as the rotational angle φ changes (considering low rotational speed). Reasons for an eccentricity of the magnetic axis are inhomogeneous magnetization along the circumference as well as geometric positioning or fabrication tolerances. As shown in [2], both effects lead to resonance effects at the same rotational speed. Since unbalance forces act in radial direction and small inclinations of the rotor are assumed, only a negligible part of the described excitation affects the axial dynamics. Thus, the forced vibrations only concern the radial and tilt DOFs and the assumption of neglecting axial excitation is valid.

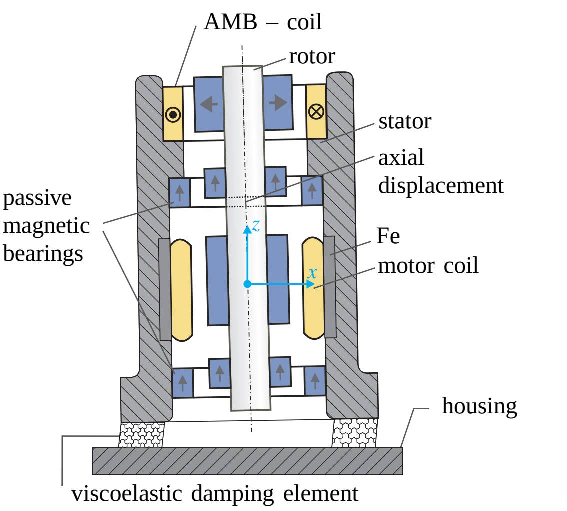

Fig. 1. Principle illustration of the investigated system

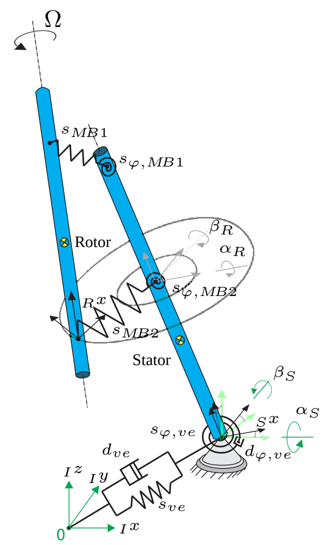

Fig. 2. Schematic figure of the overall dynamic model

The resulting model is pictured in Fig. 2. The viscoelastic elements are represented by the spring – damper combinations for translational and tilting movements with frequency dependent stiffness ($s_{ve}(\omega), s_{\varphi,ve}(\omega)$) and damping values ($d_{ve}(\omega), d_{\varphi,ve}(\omega)$). The passive magnetic bearings are modeled as linear springs and are designated by the $_{MB}$ subscript.
To describe the dynamic behavior of this system eight DOFs remain, combined in the vector of generalized coordinates $\mathbf{q} = [x_S, y_S, \alpha_S, \beta_S, x_R, y_R, \alpha_R, \beta_R]^T$. Thereby α_S and β_S are the first two cardan angles [3] describing the transformation between an inertial coordinate system and

the stator-fixed coordinate system. Further, α_R and β_R are the first two cardan angles describing the transformation between the stator-fixed and the rotor-fixed coordinate system. The states x_S and y_S describe the translational displacement of the stator represented in the inertial frame. The coordinates x_R and y_R denote the *relative* movement between the stator and the rotor and are defined in the stator-fixed system.

Mathematical Formulation: The equations of motion of the previously introduced dynamic system are derived by an energy based method following the Lagrange–II formalism (1).

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{\mathbf{q}}}T\left(\mathbf{q},\dot{\mathbf{q}}\right)\right)-\frac{\partial}{\partial \mathbf{q}}T\left(\mathbf{q},\dot{\mathbf{q}}\right)+\frac{\partial}{\partial \mathbf{q}}V\left(\mathbf{q}\right)=\mathbf{Q}_{NK}^{T}\left(\mathbf{q},\dot{\mathbf{q}},t\right) \tag{1}$$

Therein T, V and $\mathbf{Q}_{NK}$ denote the system's kinetic energy, potential energy and the vector of the generalized non-conservative forces, respectively [3].
With the assumption of small stator and rotor movements the nonlinear equations of motion can be linearized around the position of rest $\mathbf{q}_0 = \mathbf{0}^T$. Furthermore, the rotational symmetry of the assembly can be used for a transformation into complex states. Thereby the system order can be reduced by half [4]. The reduced set of generalized complex coordinates is given by

$$_c\mathbf{q}=[\underline{r}_S,\underline{\varphi}_S,\underline{r}_R,\underline{\varphi}_R]^T \quad \text{with} \quad \underline{r}_{S,R}=x_{S,R}+j\cdot y_{S,R} \quad \text{and} \quad \underline{\varphi}_{S,R}=\beta_{S,R}-j\cdot\alpha_{S,R}\ . \tag{2}$$

Hence, the linearized system can be written as

$$\mathbf{M}\ {_c\ddot{\mathbf{q}}}+(\mathbf{D}(\Omega)+\mathbf{G}(\Omega))\ {_c\dot{\mathbf{q}}}+\mathbf{K}(\Omega)\ {_c\mathbf{q}}=\mathbf{f}(\Omega,t)\ , \tag{3}$$

with the matrices

- $\mathbf{M}$ … Mass matrix; constant
- $\mathbf{D}$ … Damping matrix; Ω–dependent due to viscoelastic material characteristics
- $\mathbf{G}$ … Gyro matrix; Ω–dependent due to gyroscopic effects
- $\mathbf{K}$ … Stiffness matrix; Ω–dependent due to viscoelastic material behavior
- $\mathbf{f}$ … Vector of exciting forces and torques; unbalance forces are proportional to Ω^2 and the direction moves with Ωt in the inertial coordinate system, magnetic tolerances are modeled by an eccentricity of the force-free position of the springs which represent the passive magnetic bearings and also move with Ωt in the inertial frame.

As mentioned, magnetic tolerances and mass unbalance can be considered in (3). Mass unbalance is applied in two planes whereby not only static unbalance but also dynamic unbalance phenomena can be modeled. A magnetic tolerance is considered in each of the passive bearings. With $\mathbf{f} = \mathbf{B}\cdot\mathbf{u}$, $\mathbf{B} \in \mathbb{R}^{4\times4}$, the input vector $\mathbf{u}$ is

$$\mathbf{u}=\begin{bmatrix} u_1\,\mathrm{e}^{j\gamma_{u,1}} \\ u_2\,\mathrm{e}^{j\gamma_{u,2}} \\ e_1\,\mathrm{e}^{j\gamma_{e,1}} \\ e_2\,\mathrm{e}^{j\gamma_{e,2}} \end{bmatrix}\cdot\mathrm{e}^{j\Omega t} \tag{4}$$

where u is the unbalance, e the magnetic tolerance and γ the phase angle of each corresponding disturbance.

Linear Viscoelasticity

Viscoelasticity describes a combined viscous and elastic material behavior, where the viscous part is responsible for the damping and the elastic part represents the stiffness of the material. The properties of such materials are influenced by a lot of quantities as load frequency, temperature, static pre-load or dynamic strain amplitude. However, frequency and temperature influence are most significant.

Phenomenologically, viscoelasticity can be explained by a spring-damper combination as illustrated in Fig. 3. E' is called storage modulus and E'' loss modulus. The loss factor is $\eta = \tan\delta = E''/E'$. For an ideal elastic material the phase between strain $\varepsilon(t)$ and stress $\sigma(t)$ is $\delta = 0\ \mathrm{rad}$, whereas pure viscous behavior shows a phase lag of $\delta = \pi/2$. Viscoelastic materials thus have a phase lag of $0 < \delta < \pi/2$.

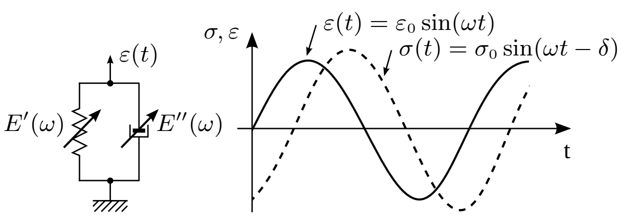

Fig. 3. Voigt-Kelvin model with σ and ε as functions of time

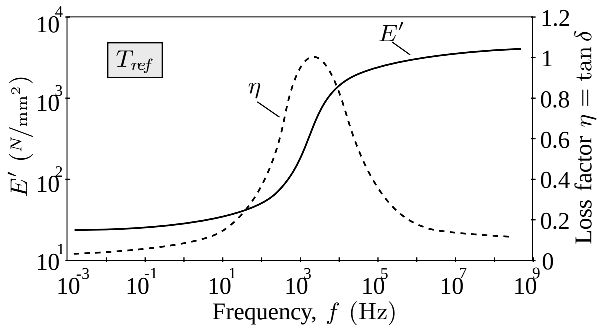

Fig. 4. Qualitative master curve of a viscoelastic material

The typical characteristics of E' and η as a function of the frequency and at a certain temperature T_{ref} are shown in Fig. 4. One can see that the loss factor η has a distinct maximum at the frequency where the storage modulus rises significantly. The material's loss per cycle and volume (w_L) is proportional to the loss modulus E'' and can be derived from the area enclosed by the hysteresis of σ and ε [5], yielding

$$\mathrm{w}_L = \oint \sigma \mathrm{d}\varepsilon = \pi \underbrace{\eta E'}_{E''} \varepsilon_0^2 \ . \qquad (5)$$

For further references regarding viscoelastic materials and the modeling of appropriate damping elements see [5]–[7].

Verification of the Linearized System

The viscoelastic stiffness and damping relations are used in the linearized system (3) for the calculation of the system response. To verify the theoretically established results a laboratory model was built up.

Measuring Setup: In the measuring setup the *absolute* rotor position, i.e. the combined absolute stator and relative rotor movement, is detected by means of eddy current sensors and an additional measuring target which is fixed on top of the rotor. The measurement gives no explicit information about the movement of the stator. Thus, if the excitation shows the general form $\underline{u}_i = \hat{u}_i \cdot \mathrm{e}^{j\omega t}$ the measured value is

$$\underline{r}_{abs} = \sum_{i=1}^{4} T_{r,u_i}(j\omega) \cdot \underline{u}_i = \hat{r}_{abs} \cdot \mathrm{e}^{j\omega t + \delta} \quad . \qquad (6)$$

The transfer functions T_{r,u_i} can not be calculated directly from the linearized model (3) where the rotor displacement is relative to the stator. To obtain the absolute rotor deflection all states (2) have to be superposed with the proper phase correlation. For measurements, the viscoelastic damping material Sylomer® manufactured by Getzner [8] was used.

Comparison: In order to compare the measurements with the calculations it is crucial to know the exact excitation acting on the system. Therefore, the rotor was balanced in a first step. Nevertheless, displacements due to magnetic tolerances are still present, even if the rotor is perfectly balanced. The eight parameters of the remaining excitation values (4) were identified from a first measurement with the balanced rotor. The result of this identification is shown in Fig. 5.
The interesting resonance phenomena at 35Hz is caused by the combination of mass unbalance and magnetic tolerance and is described theoretically in [9].
As can be seen, the measured characteristics of the combined stator and rotor movements can be matched very well by the derived model.

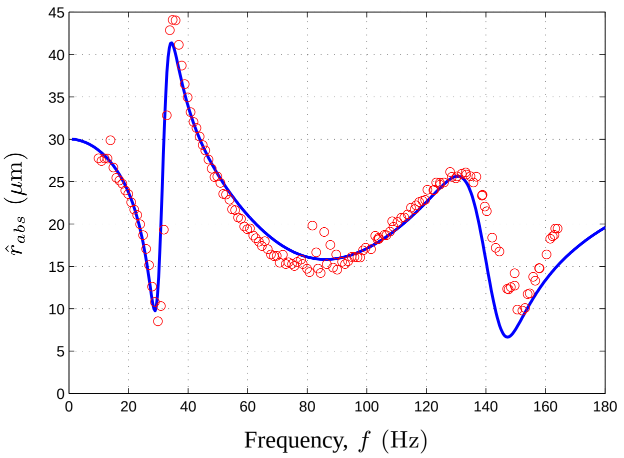

Fig. 5. Measurement results and numerical calculation results of the absolute rotor displacement where the basic excitation (mass unbalance and magnetic tolerance) was identified from the measurements

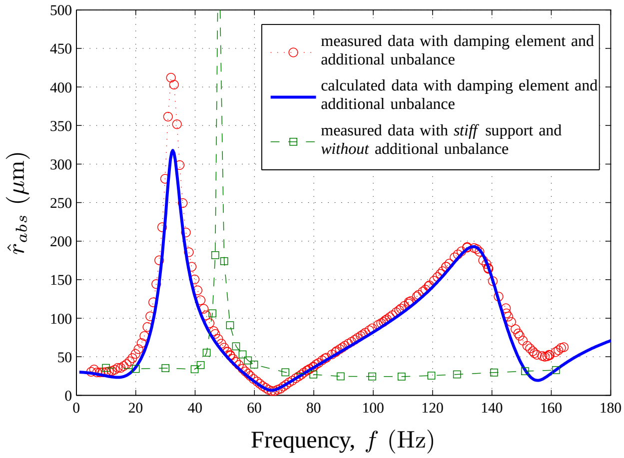

Fig. 6. Measurement results and numerical calculation results with an additional, well defined mass unbalance and stiff case with only basic unbalance as in Fig. 5

In the next step a defined mass unbalance was mounted on the rotor. In Fig. 6 the calculated and measured harmonic response is shown. Again, the calculated values are very close to the measured ones. Especially the resonance frequencies are matched very precisely by the calculation, verifying the used model. The error around 35Hz might be explained by observing the single states of the dynamic system, pictured in Fig. 7. Note, that at the first resonance frequency all states are in phase, thus all their movements contribute to the absolute value $\hat{r}_{abs}$ with their full magnitude. At the same time, the gradients of all phases are very steep. Thus, if a calculated phase has a small error a significant mistake of the superposed movements will be the consequence.
Another interesting behavior is, that at the second resonance frequency, around 135Hz, the rotor and stator movements have opposite phase. Thus the stator and the relative rotor movements partly compensate, leading to smaller values of absolute rotor displacement than at the first resonance occurrence. However, the movement of the stator and especially the relative movement of the rotor are much larger than those at 35Hz. Around 65Hz the absolute rotor displacement is almost zero due to the compensation of stator and relative rotor displacements (Fig. 6).
When looking at the phase characteristics in Fig. 7 it has to be mentioned, that the values at 0Hz are the result of the phase positions γ_i of the inputs $\underline{u}_i$ (see (4) and (6)).
Also shown in Fig. 6 is the case of a missing damping element, i.e. a stiff mounted stator. This measurement was done with *no* additional mass unbalance and therefore has to be compared to the characteristics of Fig. 5, but was drawn in this figure because of the better scaling. The very narrow and high resonance peak of the measurements with stiff supporting confirm the negligible damping properties of the basic assembly. Even aside the resonance frequency a robust operation was hard to obtain, because already minimal disturbances were destabilizing the system.

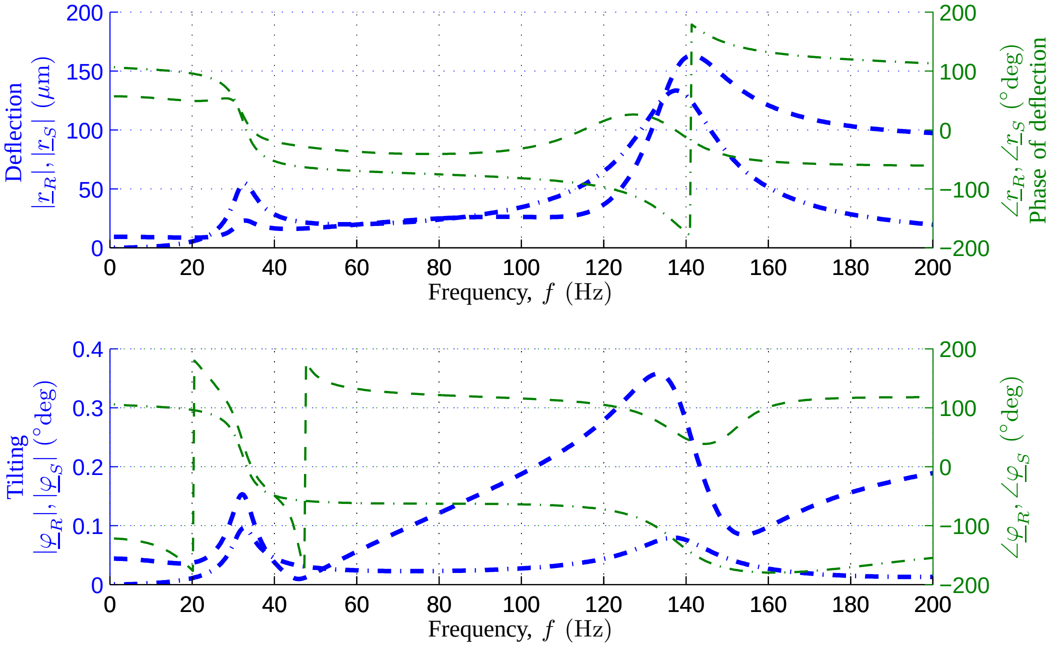

Fig. 7. Magnitude and phase of the complex states (2) for the case of an additional mass unbalance (same as in Fig. 6). Dashed lines are rotor signals, dashdotted lines are stator signals; bold lines are magnitudes, thin lines phase angles

Conclusion

This paper shows that the rotor-dynamic behavior of the presented magnetic bearing system — including viscoelastic damping elements — can be predicted by an analytical model. The correlation of the numerical calculations and the measurements is proven to be very high. Thus the basis for further optimizations is given. Even without an optimized design, the measurements have shown that viscoelastic damping elements are well suited to damp rotor vibrations. Nevertheless, it is advisable to act with caution when utilizing viscoelastic materials. Some critical aspects of viscoelastic materials as longtime stability, temperature influence or large strain amplitudes which are not considered in this paper may have tremendous effects on the rotor-dynamic behavior.

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