Nonlinear Feedback Control of a Five Axes Active Magnetic Bearing

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Abstract: The demands on active magnetic bearings concerning performance as well as costs are high. The proposed five axes active magnetic bearing has a compact design and runs with simplified power electronics. Owing to the nonlinearity of the plant, the use of linear control design methods alone is not suitable for achieving a high operation performance. This paper introduces a novel combined radial and axial position control algorithm. Experimental results demonstrate the effectiveness of the proposed approach.

Keywords: Active Magnetic Bearing, Modeling, Nonlinear Feedback Control, Multi-Axis Magnetic Bearing

Introduction

Due to high demands for energy-efficient drives, one of the major trends for electrical machines of the 21st century is high speed. Thereby, active as well as passive magnetic bearings can be an important element because of zero wear and negligible friction resulting out of their non-contact suspension capability. Mainly, passive magnetic bearing configurations are inexpensive and do not consume power but they suffer from low load capacity, low radial stiffness and the lack of damping [1]. Therefore, they cannot be taken into consideration for many applications. On the other side, active magnetic bearings (AMB) have partly been used in industry and have demonstrated the benefits of active position stabilization compared to a mechanical support [2], [3], [4]. Lots of research work has been done in the field of AMBs to improve dynamics, robustness, reliability and power consumption [5]. However, the system costs including the AMB itself, the sensors and the power amplifiers are still too high to gain ground in the field of mass products or to realize magnetically supported low cost applications.

Today, a high speed motor is typically supported by two radial AMBs and one axial AMB to realize full magnetical suspension (stabilization of five degrees of freedom). Unlike electromagnetic hetropolar bearings, permanent magnet biased homopolar bearings have a unique bias flux which energizes the air gaps while creating low-losses even at high speeds. Furthermore, if we have a closer look at the demands for electronics, the PWM amplifier for a radial AMB has to feature at least three half bridges and for the axial AMB two half bridges. Thus, at least 8 half bridges, each with two power semiconductors, are necessary only for achieving magnetic suspension. In this paper a novel design of a homopolar AMB is presented, which can actively stabilize all five DOF [6], [7]. In principle, this multi-axis active magnetic bearing consists of two conventional radial AMBs, where each holds a three phase winding system as shown in Fig. 1.

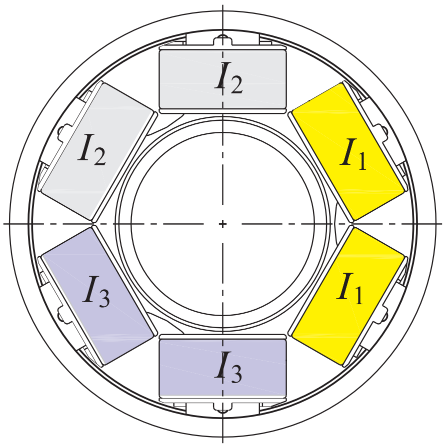

Fig. 1. Principle arrangement

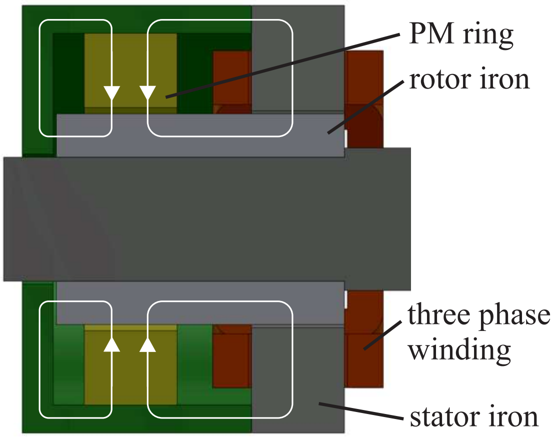

Fig. 2. Permanent magnet bias flux

Additionally, two radially magnetized permanent magnet rings in the housing placed at the shaft ends produce a homopolar pre-magnetization. Fig. 2 shows this PM impressed bias flux, which flows three-dimensionally via the axial air gap as well as via the radial AMB stator teeth. This gives a bias flux not only in radial direction but also in the axial air gap. Active axial forces can then be generated without an extra winding system just by superimposing equal currents in all phase windings. Fig. 3 shows the total setup of the multi-axis AMB. Thereby, some control problems arise because the operating point and the negative stiffness of the radial bearing is dependent on the operating point and the superimposed currents for axial force generation. A detailed description of the AMB design and its functional principle can be found in [7].
Nevertheless, the presented multi-axis AMB has a compact design and simplified power electronics, but due to the coupling between the axes and their nonlinear dependencies, it requires a sophisticated control concept. Therefore, in the following sections a nonlinear control techniques is presented for the proposed multi-axis AMB.

Mathematical Model

Clarke Transformation: One of the main properties of the multi-axis AMB is that each current of the six phase winding

$$\mathbf{i} = \begin{bmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \end{bmatrix}^T \tag{1}$$

can generate a radial force and an axial force simultaneously. To reduce the complexity of the power converter the windings are star connected, which enforces

$$\mathbf{1}^T\mathbf{i} = 0\ . \tag{2}$$

Thus, a power amplifier featuring 6 half-bridges is sufficient, which decrease the overall system costs dramatically because standard motor control electronic components can be applied. Now, the phase currents (1) are mapped for decoupling concerning a physical background. Using a transformation

$$\mathbf{i} = \mathbf{V}\bar{\mathbf{i}} \tag{3}$$

with

$$\mathbf{V} = \sqrt{\frac{2}{3}} \begin{bmatrix} \mathbf{V}_c & \mathbf{0} & \frac{1}{2}\mathbf{I} \\ \mathbf{0} & \mathbf{V}_c & -\frac{1}{2}\mathbf{I} \end{bmatrix} \quad , \quad \mathbf{V}_c = \begin{bmatrix} 1 & -0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^T \tag{4}$$

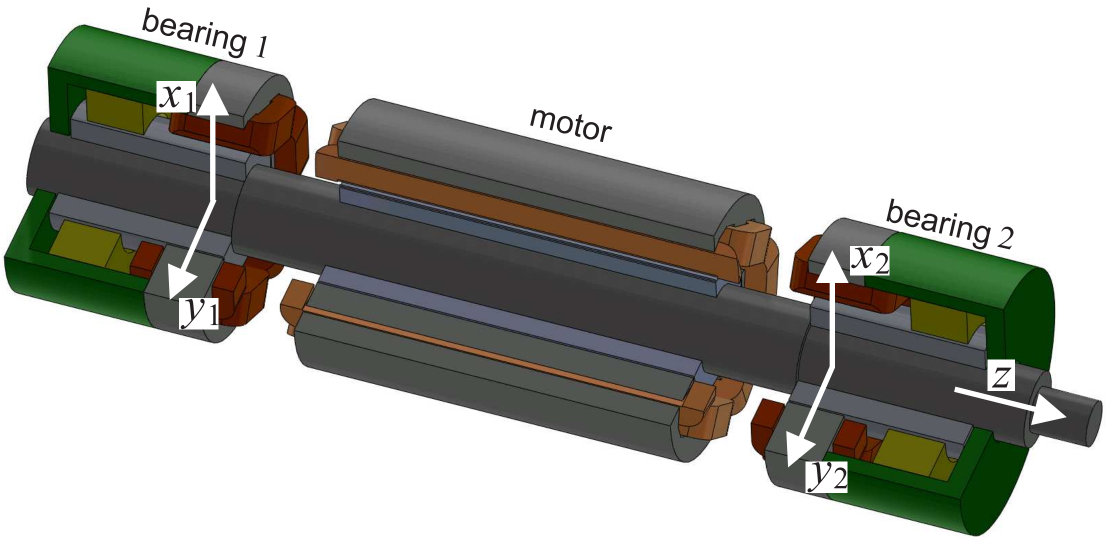

Fig. 3. Arrangement of the AMB parts respectively coordinate systems

and $\mathbf{I}$ as the identity matrix, where

$$\bar{\mathbf{i}} = \begin{bmatrix} i_{D1} & i_{Q1} & i_{D2} & i_{Q2} & i_z \end{bmatrix}^T = \begin{bmatrix} \mathbf{i}_{r_1} \\ \mathbf{i}_{r2} \\ i_z \end{bmatrix} \tag{5}$$

denotes the new currents in the orthogonal systems, one can set up the equation of motion in the form

$$\begin{aligned} m_r \ddot{\mathbf{x}}_{r1} &= \left(k_x + \widetilde{k}_x\right) \mathbf{x}_{r1} + \left(k_i + \widetilde{k}_i\right) \mathbf{i}_{r_1} \\ m_r \ddot{\mathbf{x}}_{r2} &= \left(k_x - \widetilde{k}_x\right) \mathbf{x}_{r2} + \left(k_i - \widetilde{k}_i\right) \mathbf{i}_{r2} \\ m_r \ddot{z}_r &= k_z z + k_{zi} i_z \end{aligned} \tag{6}$$

with

$$\mathbf{x}_{r1} = \begin{bmatrix} x_1 & y_1 \end{bmatrix}^T, \quad \mathbf{x}_{r2} = \begin{bmatrix} x_2 & y_2 \end{bmatrix}^T \tag{7}$$

and

$$\begin{aligned} \widetilde{k}_x &= \widetilde{k}_{xz} z + \widetilde{k}_{xi} i_z \\ \widetilde{k}_i &= \widetilde{k}_{iz} z + \widetilde{k}_{ii} i_z \quad . \end{aligned} \tag{8}$$

Here, x_1 and y_1 as well as x_2 and y_2 indicate the radial coordinates of the two bearing systems and z is the axial displacement of the shaft. The mass of the shaft is referred to as m_r. The negative stiffness values are given by k_x, $\widetilde{k}_x$, k_z and k_i, $\widetilde{k}_i$, k_{zi} denote the current to force relationships. In (6) and (8), only the main parts were taken into consideration and some quadratic dependencies are neglected. Measurements could verify the validity of this simplification.

The multi-axis AMB is driven by a PWM voltage source inverter. Hence, the real control inputs are the terminal voltages $\mathbf{u}$ and thus, the current dynamics in the orthogonal systems is determined by

$$\begin{aligned} \mathbf{u}_{r_1} &= R\mathbf{i}_{r_1} + L_r \frac{d\mathbf{i}_{r_1}}{dt} \\ \mathbf{u}_{r_2} &= R\mathbf{i}_{r_2} + L_r \frac{d\mathbf{i}_{r_2}}{dt} \\ u_z &= R i_z + L_z \frac{di_z}{dt} \end{aligned} \tag{9}$$

with

$$\mathbf{u} = \mathbf{V}\bar{\mathbf{u}} \ . \tag{10}$$

Nonlinear state space representation: Since the mathematical model of the axial part of (6) and (9) is linear and has characteristics equivalent to a conventional AMB [3], the axial part will not be considered further. Moreover, the dynamics of $\mathbf{x}_{r1}$ and $\mathbf{x}_{r2}$ basically show the same behavior. Thus, only the radial deflection $\mathbf{x}_{r1}$ of bearing 1 will further be taken into account.
Combining (6) and (9), the dynamic system of $\mathbf{x}_{r1}$ can be directly expressed in the nonlinear state space representation

$$\dot{\mathbf{x}} = \begin{bmatrix} -RL_r^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ m_r^{-1}\left(k_i + \widetilde{k}_i\right) & m_r^{-1}\left(k_x + \widetilde{k}_x\right) & \mathbf{0} \end{bmatrix} \mathbf{x} + \begin{bmatrix} L_r^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{u}_{r_1} \quad \text{with} \quad \mathbf{x} = \begin{bmatrix} \mathbf{i}_{r_1} \\ \mathbf{x}_{r1} \\ \mathbf{v}_{r1} \end{bmatrix} \tag{11}$$

and the velocity vector $\mathbf{v}_{r1} = \begin{bmatrix} \dot{x}_1 & \dot{y}_1 \end{bmatrix}^T$.

Feedback Linearization

Nonlinear Change of Coordinates: Since explaining the derivation of the nonlinear state transformation is beyond the scope of this paper, only the basic idea will be presented. For more details regarding feedback linearization of nonlinear systems, please refer to [8], [9] and [10].
Changing the coordinates of a general plant described by a system of nonlinear differential equations

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{12}$$

can only be done, if the state transformation

$$\mathbf{z} = \mathbf{\Phi}(\mathbf{x}) \ , \tag{13}$$

with $\mathbf{z}$ as the new state vector, satisfies the relation

$$\mathbf{x} = \mathbf{\Phi}^{-1}(\mathbf{z}) \ . \tag{14}$$

Differentiating (13) with respect to time yields

$$\dot{\mathbf{z}} = \left.\frac{\partial \mathbf{\Phi}(\mathbf{x})}{\partial \mathbf{x}} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}\right|_{\mathbf{x}=\mathbf{\Phi}^{-1}(\mathbf{z})} \ . \tag{15}$$

Then, the following new coordinates

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix} = \begin{bmatrix} \left(1 + k_i^{-1}\widetilde{k}_i\right)\mathbf{i}_{r_1} + k_i^{-1}\widetilde{k}_x \mathbf{x}_{r1} \\ \mathbf{x}_{r1} \\ \mathbf{v}_{r1} \end{bmatrix} \tag{16}$$

are used, where $\mathbf{z}_1$ allows direct access to the radial force generation independent of the axial position z and the current component i_z.

Static Feedback Control Law: The nonlinear change of states applying (15) leads to a system that is still not linear, but all nonlinearities disappear in the lower four state equations. Choosing a static state feedback law of the form

$$\mathbf{u}_{r_1} = \left(1 + k_i^{-1}\widetilde{k}_i\right)^{-1} \left(\mathbf{v}_1 - Rk_i^{-1}\widetilde{k}_x\mathbf{z}_2 - L_r k_i^{-1}\frac{d\widetilde{k}_i}{dt}\frac{k_i\mathbf{z}_1 - \widetilde{k}_x\mathbf{z}_2}{k_i + \widetilde{k}_i} - L_r k_i^{-1}\frac{d\left(\widetilde{k}_x\mathbf{z}_2\right)}{dt}\right) \tag{17}$$

the nonlinear system can be transformed into a linear one. Here, $\mathbf{u}_{r_1}$ and $\mathbf{v}_1$ specify the control inputs of the original and the new system respectively. Thus, the resulting closed loop system including the transformation (16) and the state feedback (17) becomes linear and can be written as

$$\dot{\mathbf{z}} = \begin{bmatrix} -RL_r^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ m_r^{-1}k_i & m_r^{-1}k_x & \mathbf{0} \end{bmatrix} \mathbf{x} + \begin{bmatrix} L_r^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{v}_1 \ . \tag{18}$$

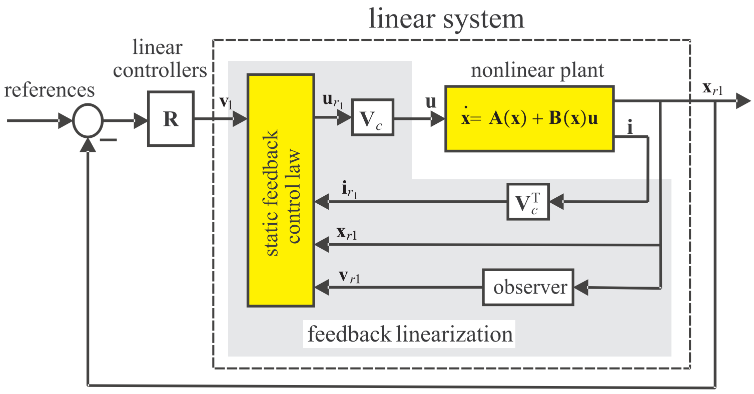

Fig. 4. Block diagram of the proposed feedback linearization

Controller Implementation: The proposed nonlinear control scheme is outlined in Fig. 4. Thanks to the feedback linearization, the main controllers for both position and speed can be defined separately by means of any single input single output (SISO) design method for linear time invariant (LTI) systems. However, due to the decoupled configuration, cascaded PID controllers are usually applied in practice. For the calculation of (17), all state variables have to be known. To keep system costs low, state observers based on a mathematical model can also be employed.

Experimental Results

The prototype of the five axes active magnetic bearing is shown in Fig. 5. The produceable radial force is 25 N per bearing, the total axial force amounts to 50 N.

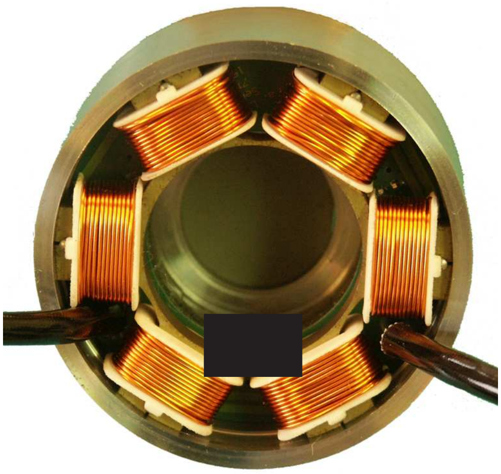

Fig. 5. Prototype of the five axes active magnetic bearing

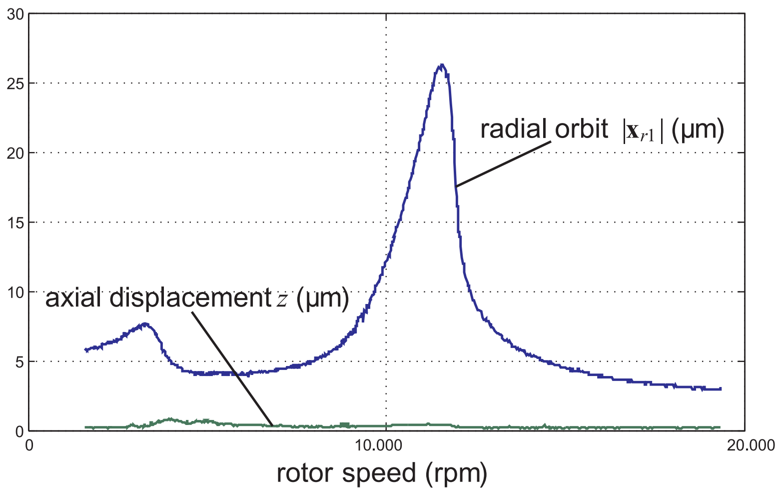

Fig. 6. Run-up of the rotor

As far as the realization and implementation in hardware is concerned, a TMS320F2811 controller by Texas Instruments is employed. This conventional 16 bit fixpoint digital signal processor has 256 kB on-board flash memory and a performance up to 150 MIPS. The software implementation was realized in ANSI C-code to make modifications or extensions to the code as convenient as possible. The sampling time is set to 100 μsec.

For the implementation of (17), the phase currents are measured by shunts and eddy current sensors integrated in the housing detect the axial and radial rotor positions. All remaining states of (17) are observed using mathematical models.

In addition, to verify the mathematical model and to obtain all required parameters the multi-axis AMB was mounted on the cross table of a test bench equipped with a load cell and measurements were carried out [7].

Fig. 6 shows the axial deflection and the orbit $\mathbf{x}_{r1} = \sqrt{x_1^2 + y_1^2}$ at one radial bearing while the rotor is speeding up to 20.000rpm. One can see, that there is almost no coupling between z and $\mathbf{x}_{r1}$. Due to a missing unbalance compensation, the radial orbit shows the peak at the resonance frequency, typical for magnetic bearings.

Summary

This paper deals with a novel nonlinear control scheme for a multi-axis AMB. Using a change of coordinates in combination with static state feedback, a linear system in new states with new control inputs is obtained. Thereby, it should be pointed out, that the system only shows linearity in the transformed states. To confirm the efficiency of the presented multi-axis AMB, the proposed control law has been tested on a prototype. It can be shown that even in the range of resonance frequencies, no coupling between radial and axial deflections occurs. The presented multi-axis AMB in combination with nonlinear control shows high potential for many applications. Due to the compact design and the reduced power electronics complexity, it is not only reserved for academic use, but also applicable in industry.

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