# Analysis of Unbalanced Force for Flywheel Energy Storage System Based on Active Magnetic Bearing Considering Rotor Eccentricity Effects

**Chen Junfeng<sup>1,a</sup>, Liu Kun<sup>1,b</sup>, Xiao Kai<sup>1,c</sup>, Wang Haoze<sup>1,d</sup>, Ruan Yigao<sup>1,e</sup>** <sup>1</sup>Institute of Aerospace and Material Engineering, National University of Defense Technology ,Changsha, P.R.China, <sup>a</sup>cjf\_hn0227@yahoo.com, <sup>b</sup>liukun@nudt.edu.cn, <sup>c</sup>xiaokai@nudt.edu.cn, <sup>d</sup>wanghaoze@vahoo.com, <sup>e</sup>ruanyigao@vahoo.com

**Abstract:** Flywheel energy storage system(FESS) based on active magnetic bearing(AMB) can be used to generate impulse electric power. The unbalanced forces considering rotor eccentricity need to be calculateded, especially when the discharging current is high. This paper investigates the unbalanced forces in FESS considering rotor eccentricity. The analytic solution for the magnetic flux density distribution in the airgap region is used to calculate the unbalanced Lorentz forces and magnetic forces, which are compared with that of finite element method(FEM). The results show that the unbalanced forces should be calculated for FESS to guarantee the flywheel safe whirling and provide reference to the control system design of AMB.

Keywords: Unbalanced Force, Rotor Eccentricity, Flux Density, Flywheel Energy Storage System, Active Magnetic Bearing

## Introduction

The use of flywheel energy storage system(FESS) as "mechanical batteries" has several significant benefits over the conventional battery systems, it has higher specific energy and specific power, and can be charged/discharged rapidly. Moreover, these mechanical battery systems can generate highpeak/impulse electric power [1]. In FESS based on active magnetic bearing(AMB), mechanical mass unbalance and external forces induce the rotor eccentricity, and the magnetic field distribution in the air gap region of motor/generator causes unbalanced distribution of forces which again adversely affect the whirl of FESS [2].

Thus the rotor eccentricity may cause magnetic and dynamic problems with additional vibration, noise, and torque pulsation [3]. Especially when the AMB flywheel discharges high power, the rotor eccentricity may cause disastrous results to FESS.

The investigation of magnetic force and vibration for a rotor-bearing system has been addressed by many researchers. In [3], an analytic solution is proposed for the calculation of magnetic field induced by rotor eccentricity in permanent magnet motors. Kim[2] investigates the magnetic force of permanent motors with radial rotor eccentricity using perturbation method. Pennacchi [4] provides a method aimed to calculate the unbalanced magnetic pull force and validates the model in the experiment. But they did not discuss the Lorentz force in the motor. Howerer, the unbalanced forces of FESS need to be thoroughly considered, especially when the generator is applied for high pulsed power.

This paper presents the unbalanced force analysis of permanent magnetic generator used for high power releasing in FESS( Fig. 1.), and compares the composements of the force in different conditions for system design consideration.



Fig. 1 Model of FESS

# Description

The FESS has a external rotor structure, the armature windings without slots are attached on the surface of the stator in concentric form. Fig. 2 shows a geometric configuration of permanent magnet generator with rotor eccentricity to be analyzed. For the analysis of electromagnetic forces acting on rotors, the following assumption are made:

- a) Relative permeability of the iron is infinity.
- b) Saturation of the iron is ignored.
- c) The surface of the stator is smooth.



Fig. 2. Geometric configuration of the generator

# Solution

The unbalanced forces acting on the rotor consist of unbalanced magnetic forces induced by permanent magnet and unbalanced Lorentz forces when the generator windings discharge electric energy.

The  $X - Y(r - \theta)$  coordinate system is fixed at the center of the stator, whereas the

 $x - y(\xi - \phi)$  coordinate system is attached to the center of the rotor. The relation of these coordinates is as follows:

$$r = \xi = \varepsilon \cos(\theta - \phi) + O(\varepsilon^2) \tag{1}$$

where  $\varepsilon, \phi$  is the eccentricity of rotor and the angle of eccentricity.

## a) Magnetic flux density distribution

The analytic solutions for the magnetic flux density distribution of in the airgap region can be found from previous work [3] and given as

$$B_{r}(r,\theta) = \sum_{n=1,3,5\cdots}^{\infty} (np)A_{0} \left(r^{np-1} + R_{s}^{2np}r^{-np-1}\right) \cos[np(\theta - \omega t)] + \\ \varepsilon \sum_{n=1,3,5\cdots}^{\infty} \left(np \frac{A_{0}}{2} \left\{(2 - np)r^{np-2} + npR_{s}^{2np}r^{-np-2}\right\} - \mu_{0}(np - 1)\left(W_{n}r^{np-2} - X_{n}r^{-np}\right)\right) \cos[(np - 1)\theta - np\omega t + \phi] + \\ \varepsilon \sum_{n=1,3,5\cdots}^{\infty} \left(np \frac{A_{0}}{2} \left\{-npr^{np-2} + (np + 2)R_{s}^{2np}r^{-np-2}\right\} - \mu_{0}(np + 1)\left(Y_{n}r^{np-2} - Z_{n}r^{-np}\right)\right) \cos[(np + 1)\theta - np\omega t - \phi] \\ B_{\theta}(r,\theta) = \sum_{n=1,3,5\cdots}^{\infty} (np)A_{0} \left(r^{np-1} - R_{s}^{2np}r^{-np-1}\right) \sin[np(\theta - \omega t)] + \\ \varepsilon \sum_{n=1,3,5\cdots}^{\infty} \left(np \frac{A_{0}}{2} \left\{(np - 2)r^{np-2} + npR_{s}^{2np}r^{-np-2}\right\} + \mu_{0}(np - 1)\left(W_{n}r^{np-2} + X_{n}r^{-np}\right)\right) \sin[(np - 1)\theta - np\omega t + \phi] + \\ \varepsilon \sum_{n=1,3,5\cdots}^{\infty} \left(np \frac{A_{0}}{2} \left\{(np r^{np-2} + (np + 2)R_{s}^{2np}r^{-np-2}\right\} + \mu_{0}(np + 1)\left(Y_{n}r^{np} + Z_{n}r^{-np-2}\right)\right) \sin[(np + 1)\theta - np\omega t - \phi]$$
(3)

Where,

$$\begin{split} A_{0} &= \frac{\mu_{0}M_{n}}{\mu_{r}} \frac{R_{m}^{-(np-1)}}{(np)^{2} - 1} \begin{cases} \frac{(np-1)R_{m}^{2np} + 2R_{r}^{np+1}R_{m}^{np-1} - (np+1)R_{r}^{2np}}{\frac{\mu_{r} + 1}{\mu_{r}}[R_{s}^{2np} - R_{r}^{2np}] - \frac{\mu_{r} - 1}{\mu_{r}}[R_{m}^{2np} - R_{s}^{2np}(R_{r} / R_{m})^{2np}]} \end{cases} \\ W_{n} &= \frac{-npA_{0}R_{s}^{2np-2}\{(1+\mu_{r})R_{m}^{2np-2} - (1-\mu_{r})R_{r}^{2np-2}\}}{(1+\mu_{r})(R_{r}^{2np-2} - R_{s}^{2np-2})R_{m}^{2np-2} - (1-\mu_{r})(R_{m}^{4np-4} - R_{r}^{2np-2}R_{s}^{2np-2})} \end{cases} \\ X_{n} &= \frac{-npA_{0}R_{m}^{2np-2}R_{s}^{2np-2}\{(1-\mu_{r})R_{m}^{2np-2} - (1-\mu_{r})(R_{m}^{4np-4} - R_{r}^{2np-2}R_{s}^{2np-2})}{(1+\mu_{r})(R_{r}^{2np-2} - R_{s}^{2np-2})R_{m}^{2np-2} - (1-\mu_{r})(R_{m}^{4np-4} - R_{r}^{2np-2}R_{s}^{2np-2})} \end{cases}$$

$$Y_{n} = \frac{-npA_{0}R_{s}^{2np}\{(1+\mu_{r})R_{m}^{2np+2}-(1-\mu_{r})R_{r}^{2np+2}\}}{(1+\mu_{r})(R_{r}^{2np+2}-R_{s}^{2np+2})R_{m}^{2np+2}-(1-\mu_{r})(R_{m}^{4np+4}-R_{r}^{2np+2}R_{s}^{2np+2})}$$

$$Z_{n} = \frac{-npA_{0}R_{m}^{2np}R_{s}^{2np}\{(1-\mu_{r})R_{m}^{2np+2}-(1+\mu_{r})R_{r}^{2np+2}\}}{(1+\mu_{r})(R_{r}^{2np+2}-R_{s}^{2np+2})R_{m}^{2np+2}-(1-\mu_{r})(R_{m}^{4np+4}-R_{r}^{2np+2}R_{s}^{2np+2})}$$

$$M_{n} = 2\left(\frac{B_{r}}{\mu_{0}}\right)\alpha_{p}\frac{\sin\frac{n\pi\alpha_{p}}{2}}{\frac{n\pi\alpha_{p}}{2}}$$

## b) Unbalanced magnetic forces

According to assumption (a), the unbalanced magnetic forces is the radial traction acting on the rotor which is determined using Maxwell stress tensor, and the analytic solutions for unbalanced magnetic forces can be found from previous work [2], and given as

$$F_{x,mag} = \frac{\pi R_s L_s \varepsilon}{2\mu_0} \sum_n (A_n B_n + A_n C_n) \cos \phi$$

$$F_{y,mag} = \frac{\pi R_s L_s \varepsilon}{2\mu_0} \sum_n (A_n B_n + A_n C_n) \sin \phi$$
(4)

Where,

$$\begin{aligned} A_n &= (np)A_0(r^{np-1} + R_s^{2np}r^{-np-1}) \\ B_n &= \left( np\frac{A_0}{2} \{ (2-np)r^{np-2} + npR_s^{2np}r^{-np-2} \} - \mu_0(np-1)(W_nr^{np-2} - X_nr^{-np}) \right) \\ C_n &= \left( np\frac{A_0}{2} \{ -npr^{np-2} + (np+2)R_s^{2np}r^{-np-2} \} - \mu_0(np+1)(Y_nr^{np} - Z_nr^{-np-2}) \right) \end{aligned}$$

# c) Unbalanced Lorentz forces

The unbalanced Lorentz forces comprise  $f_{lorz}^{(t)}$  which is induced by the radial magnetic flux density  $B_r$  and  $f_{lorz}^{(r)}$  which is induced by the tangent magnetic flux density  $B_{\theta}$ . The unbalanced Lorentz forces for a single amature winding can be written as:

$$f_{lorz}^{(t)} = B_r I L^{(k)}$$

$$f_{lorz}^{(r)} = B_t I L^{(k)}$$
(5)

Using Cartesian coordinates, the corresponding forces are given as:

$$f_{x,lorz}^{(t)} = B_r I L^{(k)} \sin \theta$$

$$f_{y,lorz}^{(t)} = B_r I L^{(k)} \cos \theta$$

$$f_{x,lorz}^{(r)} = B_t I L^{(k)} \sin \theta$$

$$f_{y,lorz}^{(r)} = B_t I L^{(k)} \cos \theta$$
(6)

Where k is the amature winding number,  $\theta$  is the position angle of the amature winding respect to x-direction.

The unbalanced Lorentz forces acting on the rotor center can be determined by summating the corresponding forces over the amature windings and are given as:

$$F_{x,lorz}^{(t)} = \sum_{n} B_{r} I L^{(n)} \sin \theta$$

$$F_{y,lorz}^{(t)} = \sum_{n} B_{r} I L^{(n)} \cos \theta$$

$$F_{x,lorz}^{(r)} = \sum_{n} B_{t} I L^{(n)} \sin \theta$$

$$F_{y,lorz}^{(r)} = \sum_{n} B_{t} I L^{(n)} \cos \theta$$
(7)

#### Results

For a 4-pole permanent magnet genetor with geometric parameters as in Table 1, the magnetic field is analyzed using the perturbation method. The analytic radial magnetic flux density of the rotor is shown in Fig. 3 with the eccentricity ratio (eccentricity/airgap length) of 0.11 and compared with FEM results, and the results of two methods coincide well with each other. The subtracted flux density distributions of  $(B_{z=0.1mm} - B_{z=0})$  along the surface of the stator is also shown in Fig. 4. The unbalanced forces are induced by the differences of flux density distribution along the airgap. It can be seen from Fig. 4(a) and Fig. 4(b) that the subtracted values of flux density in radial direction and in tangent direction are very small. However, considering of the high current (I = 20kA) discharged by FESS and the geometric configuration (L = 0.4m) of the armature winding, it can be calculated that the unbalanced Lorentz forces are 286N, unbalanced magnetic forces are 72N, and composition of forces reach to 341N. The unbalanced forces will disturb the rotation of the flywheel, and it should be taken as an important design reference for the control system design of AMB in FESS. In Fig. 5, flux density distributions with 0.1mm eccentricity are shown in FEM-simulation.

Fig. 6 shows the tangent Lorentz forces acting on each amature winding. It shows that tangent Lorentz forces are so big that the adhesive technics of slotless windings need to be considered to avoid windings breaking off from the stator.

It can be seen from Fig. 7 and Fig. 8 that the unbalanced Lorentz forces happen to be nearly

linear with current and eccentricity of airgap. In Fig. 9, it shows that the unbalanced magnetic forces are also nearly linear with eccentricity of airgap. Moreover, in the high current condition when generator is discharging impulse electric power, the unbalanced Lorentz forces are the dominating part in unbalanced forces compared with unbalanced magnetic forces, thus they are inneglectable in the circumstance.

Parameter	Symbol	Value(unit)
Pole number	2p	4
Pole-arc/pole-pitch ratio	$\alpha_{_p}$	1.0
Airgap length	G	10.45(mm)
Radial length of magnets	$H_{m}$	13.00(mm)
Stator Radius	$R_s$	55.00(mm)
Magnet remanence	$B_r$	1.33(T)
Relative permeability	$\mu_r$	1.15

Table 1 Geometric parameters of the analyzed generator



Fig. 3 Radial flux density distributions with 0.1mm eccentricity.

The Twelfth International Symposium on Magnetic Bearings (ISMB 12) Wuhan, China, August 22-25, 2010



Fig. 4 Subtracted flux density distributions of  $(B_{\varepsilon=0.1mm} - B_{\varepsilon=0})$ 



Fig. 5 Flux density distributions with 0.1mm eccentricity in FEM simulation.



Fig. 6 Tangent Lorentz forces acting on amature windings



Fig. 7 Unbalanced Lorentz forces versus current



Fig. 8 Unbalanced Lorentz forces versus eccentricity of airgap



Fig. 9 Unbalanced magnetic forces versus eccentricity of airgap

# Conclusion

The paper investigates the unbalanced forces induced by rotor eccentricity in FESS based on AMB. The analytic results compared with FEM results in the unbalanced forces calculation are proved credible. It is verified from the calculation results that the unbalanced forces should be considered for FESS, especially when the discharging current is high. In addition, the results of unbalanced force calculation can be used for the control system design of AMB in FESS.

## References

- Hyungjoo Yoon: Space Attitude and Power control Using Variable Speed Control Moment Gyros[D]. Graduate School of Georgia Institute of Technology. 2004
- [2] Tae-Jong Kim, Sang-Moon Hwang, and No-Gill Park. Analysis of Vibration for Permanent Magnet Motors Considering Mechanical and Magnetic Coupling Effects. IEEE TRANSACTIONS ON MAGNETICS, VOL. 36, NO. 4, JULY 2000 1346–1350
- [3] U. Kim and D. K. Lieu. Magnetic field calculation in permanent magnet motors with rotor eccentricity: Without slotting effect. IEEE Transactions on Magnetics, vol. 34, no. 4, pp. 2243–2252, July 1998.
- [4] Paolo Pennacchi. Computational model for calculating the dynamical behaviour of generators caused by unbalanced magnetic pull and experimental validation. Journal of Sound and Vibration 312 (2008) 332-353

## Appendix :

#### Address:

Dr. Junfeng Chen, Team 6, College 1, National University of Defense Technology,No.137,Yanwachi,Street,Changsha,410073,Hunan, P.R.China

Tel: +86 731 84573186 Fax: +86 731 84512301 E-mail address: <u>cjf\_hn0227@yahoo.com</u>