Dynamic Analysis of the Permanent Magnetic Bearing-Rotor System

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Abstract: The permanent magnetic bearings have many advantages: simple structure, free-friction, low-cost, etc. They can make up various magnetic bearing systems with mechanical bearings or electromagnetic bearings combined, thus have broad prospects. In the paper, the permanent magnet bearings - rigid rotor dynamics model is established. The stiffness matrix is built based on the equivalent electricity model and the preliminary dynamic analysis of permanent magnet bearing-rotor system is drawn. It laid a good foundation for the further researches on permanent magnetic bearing-rotor system.

Keywords: Permanent Magnetic Bearing, Dynamics, Equivalent Current, Stiffness Matrix

Introduction

Permanent magnetic bearings work with shaft suspended by using the magnetic forces generated by permanent magnetic materials. Compared with electro-magnetic bearings and superconducting magnetic bearings, they have advantages of simple structure, small volume, light weight, high reliability, low price, low energy, etc[1].

By Earnshaw Law, we know permanent magnetic bearings can't realize stable suspension in all degrees of freedom [2]. So in the permanent magnetic system, other supported approaches should be introduced at least in one degree of freedom. All forms of magnetic bearing system can be made when combined with electro-magnetic bearings, machinery bearings and superconductor magnetic bearings. They have broad prospects in the high-tech fields like energy transportation, aerospace, wind power, machine-building industry as well as the robot.

Works have been done by the researchers at home and abroad on the mechanical characteristics of permanent magnet bearings by using different calculation methods to derive their formulas [3-7]. But the researches on the dynamic characteristics of permanent magnet bearing-rotor system are insufficient. This paper establishes the dynamics model of permanent magnet bearings - rigid rotor and derives the stiffness matrix of magnetic bearing system. It provides the preliminary dynamic analysis on rigid rotor and lays the foundation for further study of dynamic characteristics of permanent magnet bearing-rotor system.

Dynamic Modeling of Permanent Magnetic Bearings - Rigid Rotor

The so-called rigid rotor refers to the rotor which speed is lower than the first critical speed in the rigid supporting and lower than the third-order critical speed of the rotor in the elastic supporting. Such rotors are very common in engineering.

Equation of Motion of Rigid Rotor System. Dynamics model of the rigid rotor is shown in

Fig.1. Suppose the length of rotor is l, the distance between rotor mass center O_c and left and right end is l_a and l_b, the equation $\alpha=l_a/l=(l-l_b)/l$ exists and displacements of the rotor at left and right supporting bearings from the static equilibrium position accordingly are x_a, x_b and y_a, y_b, thus the displacement of rotor mass center O_c can be expressed as follows:

$$\begin{cases} x_c = (1-\alpha)x_a + \alpha x_b \\ y_c = (1-\alpha)y_a + \alpha y_b \end{cases} \quad (1)$$

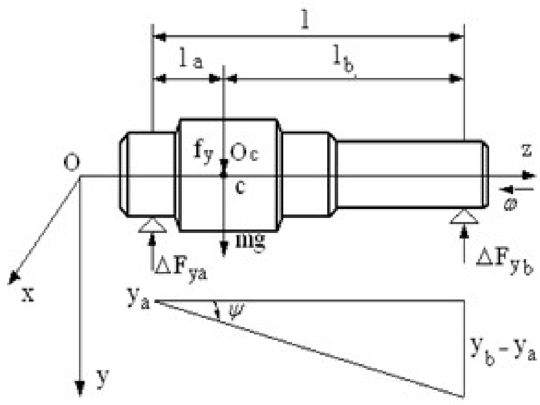

Fig. 1 Model of the rigid rotor

The motion differential equation of generalized coordinates is obtained from the Lagrange equations for the rotor in the bearings.

$$M_B\ddot{q}_B + G_B\dot{q}_B = Q_o + E_oC_s \quad (2)$$

With

$$M_B = \begin{bmatrix} \frac{l_b}{l}m & 0 & \frac{l_a}{l}m & 0 \\ 0 & \frac{l_b}{l}m & 0 & \frac{l_a}{l}m \\ -\frac{1}{l}J_{oy} & 0 & \frac{1}{l}J_{oy} & 0 \\ 0 & -\frac{1}{l}J_{ox} & 0 & \frac{1}{l}J_{ox} \end{bmatrix},$$

$$G_B = \frac{J_{oz}\omega}{l}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}, \quad E_o = \omega^2\begin{bmatrix} me_x & me_y \\ me_y & -me_x \\ J_{oxz} & J_{oyz} \\ J_{oyz} & -J_{oxz} \end{bmatrix}, \quad C_s = \begin{Bmatrix} \cos\omega t \\ \sin\omega t \end{Bmatrix},$$

$$Q_o = \begin{Bmatrix} Q_{ox} \\ Q_{oy} \\ Q_{o\varphi} \\ Q_{o\psi} \end{Bmatrix} = -\begin{Bmatrix} \Delta F_{xa} + \Delta F_{xb} \\ \Delta F_{ya} + \Delta F_{yb} \\ -\Delta F_{xa}l_a + \Delta F_{xb}l_b + \Delta M_{ya} + \Delta M_{yb} \\ -\Delta F_{ya}l_a + \Delta F_{yb}l_b + \Delta M_{xa} + \Delta M_{xb} \end{Bmatrix}$$

Where m is rotor quality and J_{ox} and J_{oy} equatorial moments of inertia of the rotor, J_{oz} polar moment of inertia of the rotor with J_{0xz} and J_{0yz} its product of inertia of the rotor, and ω its angular velocity.

The corresponding dimensionless form of Eq. 2 is

$$\bar{M}_B\ddot{\bar{q}}_B + \bar{G}_B\dot{\bar{q}}_B = \bar{Q}_o + \bar{E}_o\bar{C}_s \quad (3)$$

Dimensionless free vibration equation is

$$\bar{M}_B\ddot{\bar{q}}_B + \bar{G}_B\dot{\bar{q}}_B = \bar{Q}_o \quad (4)$$

Free vibration equation is

$$\bar{M}\bar{q}''_B + \bar{G}_B\bar{q}'_B + \bar{K}q_B = 0 \quad (5)$$

Equation of State. Eq. 5 represents system for free vibration differential equation. The following identity equation is added to calculate eigenvalues and eigenvectors of Eq. 5.

$$\overline{M}\overline{q}'_B - \overline{M}\overline{q}'_B = 0 \tag{6}$$

The equation of system for free vibration in the state space is obtained by combing Eq. 5 with Eq. 6:

$$\begin{bmatrix} \overline{G} & \overline{M} \\ \overline{M} & 0 \end{bmatrix} \begin{Bmatrix} \overline{q}'_B \\ \overline{q}''_B \end{Bmatrix} = \begin{bmatrix} -\overline{K} & 0 \\ 0 & \overline{M} \end{bmatrix} \begin{Bmatrix} \overline{q}_B \\ \overline{q}'_B \end{Bmatrix} \tag{7}$$

And $A = \begin{bmatrix} -\overline{K} & 0 \\ 0 & \overline{M} \end{bmatrix}$, $B = \begin{bmatrix} \overline{G} & \overline{M} \\ \overline{M} & 0 \end{bmatrix}$, $Y = \begin{Bmatrix} \overline{q}_B \\ \overline{q}'_B \end{Bmatrix}$, Eq. 7 becomes

$$BY' = AY \tag{8}$$

Consequently, Eigenvalues and eigenvectors of the system can be solved through Eq. 8.

Solution of Stiffness of Permanent Magnetic Bearing System

Analytical Model of Permanent Magnet Bearing. In order to facilitate the establishment of an analytical model, we take a dynamic magnetic ring and the adjacent static magnetic one as the target for study and establish cylindrical coordinate system(r, φ, z) as shown in Fig. 2. Where center coordinates of magnetic ring 1 is (0, z_1), center coordinates of magnetic ring 2 (0, z_2). Two magnetic parameters are of the same size with external diameter R, height b and thickness a.

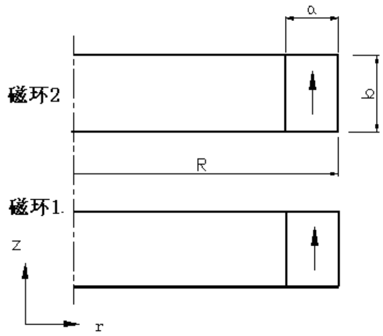

Fig.2 Size of Axially Magnetized Permanent Magnetic Radial Bearing

Stiffness Matrix of Permanent Bearings. In suspension system, as the rotor rotates around axis z, regardless of degree of freedom around axis z, the stiffness matrix is 5×5 matrix[8].

$$K_{PM} = \begin{Bmatrix} K_{xx} & K_{xy} & K_{xz} & K_{x\alpha} & K_{x\beta} \\ K_{yx} & K_{yy} & K_{yz} & K_{y\alpha} & K_{y\beta} \\ K_{zx} & K_{zy} & K_{zz} & K_{z\alpha} & K_{z\beta} \\ K_{\alpha x} & K_{\alpha y} & K_{\alpha z} & K_{\alpha\alpha} & K_{\alpha\beta} \\ K_{\beta x} & K_{\beta y} & K_{\beta z} & K_{\beta\alpha} & K_{\beta\beta} \end{Bmatrix} \tag{9}$$

In cylindrical coordinates, the magnets are the cylinder, thus $K_{xx}=K_{yy}=K_r$, $K_{\alpha\alpha}=K_{\beta\beta}=K_\varphi$; Magnetic field is a conservative field, so the stiffness matrix is symmetric matrix, that is, $K_{ij}=K_{ji}$.

When rotor lies in the center position, moving magnetic ring along axis z, the radial force is always zero, so except K_{zz}, line 3 and column 3 are all zero. Moving magnetic ring along axis x, Force F_y and Torque M_x is always zero, so $K_{yx}=K_{\alpha x}=0$; Similarly $K_{\beta\alpha}=K_{x\alpha}=0$.

Therefore, in cylindrical coordinates, stiffness matrix of permanent magnet rotor - bearing system of five degrees of freedom can be reduced to:

$$K_{PM}=\begin{Bmatrix} K_r & 0 & 0 & 0 & K_{\varphi r} \\ 0 & K_r & 0 & K_{\varphi r} & 0 \\ 0 & 0 & K_z & 0 & 0 \\ 0 & K_{\varphi r} & 0 & K_\varphi & 0 \\ K_{\varphi r} & 0 & 0 & 0 & K_\varphi \end{Bmatrix} \tag{10}$$

According to Earnshaw Law [2], the equation $2K_r+K_z=0$ exists, so we just need to consider the value of the three unknown stiffness.

Basic Calculation Equations of Permanent Magnet Bearings. Ring magnets can be regarded as two cylindrical magnets and the magnetic field of cylindrical permanent magnet is equivalently generated by circular currents of the surface shown in Fig. 3.

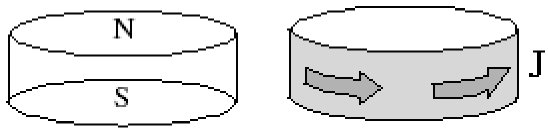

Fig. 3 Equivalent Current Model of Cylindrical Permanent Magnet

By Lorentz formula, force formula is expressed as[9]

$$\vec{F}=\int\left(\vec{J}\times\vec{B}\right)dA \tag{11}$$

Basic formula of magnetic is

$$\vec{B}=\nabla\times\vec{A} \tag{12}$$

$$\vec{A}=-\frac{\mu_0}{4\pi}\int\frac{\nabla\times\vec{M}}{r}dv \tag{13}$$

$$\vec{J}=\nabla\times\vec{M} \tag{14}$$

Where $\vec{B}$ refers to permanent magnet flux density, $\vec{A}$ vector potential, $\vec{J}$ equivalent surface current and $\vec{M}$ magnetization strength.

Axial force and the y-axis torque of cylindrical Magnet can be derived from the above equation. Subscript r、φ、z represents the corresponding vector in the radial, circumferential, and axial components.

$$F_z=2\pi RJ_\varphi[A_\varphi(R,z_2+0.5b)-A_\varphi(R,z_2-0.5b)] \tag{15}$$

$$M_y = -J_\varphi B_z (R\cos^2\varphi_z \sin\varphi_y - z_2 \cos\varphi_z \cos\varphi_y + J_\varphi B_r (R\sin^2\varphi_z \cos\varphi_y + Rz_2 \cos^3\varphi_z \cos\varphi_y + z_2 \cos^2\varphi_z \sin\varphi_y) \tag{16}$$

$$M_y = -J_\varphi B_z \left(R\cos^2\varphi_z \sin\varphi_y - z_2 \cos\varphi_z \cos\varphi_y\right) + J_\varphi B_r \left(R\sin^2\varphi_z \cos\varphi_z + Rz_2 \cos^3\varphi_z \cos\varphi_y + z_2 \cos^2\varphi_z \sin\varphi_y\right) \tag{17}$$

Expression of Stiffness Matrix. Radial stiffness is expressed as follows [10]:

$$K_r = \frac{1}{2}\frac{\partial F_z}{\partial z} = \sum_{i=1,2} -\pi a J_\varphi B_r(R_i, z)\Big|_{z_2-0.5b}^{z_2+0.5b} \quad (R_1 = R, R_2 = R-a) \tag{18}$$

Tilt stiffness:

$$K_\varphi = -\frac{\partial M_y}{\partial \varphi_y} = \sum_{i=1,2} J_\varphi \pi\{-2azA_\varphi(R_i,z) + 2a\Lambda(R_i,z) + a(2R-a)\mu_0\Phi(R_i,z) + 2az(2R-a)B_z(R_i,z) + \left(3aR^2 - az^2 - 3a^2R + a^3\right)B_r(R_i,z)\}\Big|_{z_2-0.5b}^{z_2+0.5b} \quad (R_1 = R, R_2 = R-a) \tag{19}$$

Where μ_0, Φ, Λ respectively stands for vacuum permeability, scalar and vector potential integral of permanent magnet. And coupling stiffness is

$$K_{\varphi r} = -\frac{\partial F_x}{\partial \varphi_y} = -J_\varphi \pi[2aA_\varphi(R_i,z) + a(2R-a)B_z(R_i,z) + aB_r(R_i,z)z]\Big|_{z_2-0.5b}^{z_2+0.5b} (R_1 = R, R_2 = R-a) \tag{20}$$

Dynamic Analysis of Permanent Magnetic Bearing - Rotor System

The paper organizes the dynamic calculation program by using program Matlab and analyzes the dynamic influences based on different stiffness coefficients and bearing arrangement.

To validate the feasibility of the design of permanent magnetic bearing, and explore its actual prospects for industrial applications, we designed simulated test bed of the permanent magnetic bearing - rotor system as shown in Fig. 4.

Due to the structural arrangement of same-sex attraction and the mutual attraction of the rare earth permanent magnet rings is strong when working in a small air gap, the factors as magnetic isolation and ease of installation must be considered into design. It is to be noted that the axial magnetic force between the magnetic ring changes greatly along with air gap, so how to ensure that the system works in optimal conditions by adjusting the magnetic gap in the installation is very important.

Based on the above considerations, it is expected that permanent magnet bearing will become a standard part for users. And the paper tries to reflect this design philosophy in structural design in Fig.5. Structural parameters of rotor shaft No. in different section are revealed in Table 1.

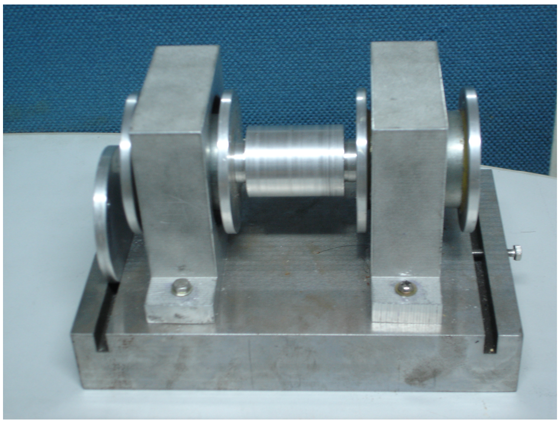

Fig. 4 Simulated Test Bed of Permanent Magnetic Bearing - Rotor System

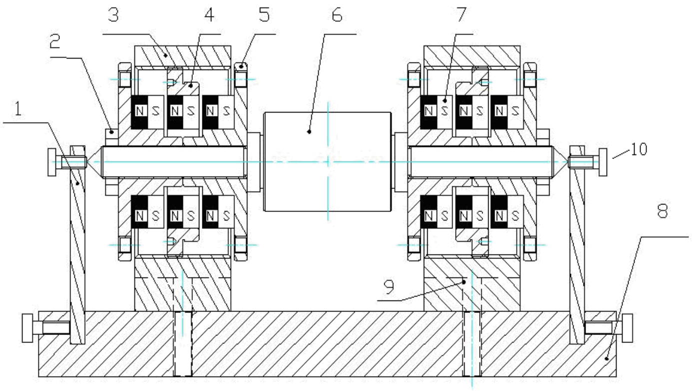

Fig.5 Test Bed of Permanent Magnetic Bearing -

Table.1 The Ladder Structure of the Rotor Shaft of Test Bed

Shaft No.	1	2	3	4	5	6	7	8	9
Diameter d(mm)	10	60	20	15	15	20	60	18	30
Length l(mm)	10	4	10	6	6	10	4	5	40
Shaft No.	10	11	12	13	14	15	16	17	
Diameter d(mm)	18	60	20	15	15	20	60	10	
Length l(mm)	5	4	10	6	6	10	4	10	

The change of bearing point affects tilt stiffness and coupling stiffness. So if choosing rotor mass center as bearing point with permanent magnets bearing point coordinates (x , y , z , λ , φ , ψ) and centroid coordinates (x',y',z',λ', φ', ψ'), The stiffness matrix of supporting point after translation is expressed as:

$$\begin{pmatrix} K_r & 0 & 0 & 0 & -L\cdot K_r+K_{\varphi r} & 0 \\ 0 & K_r & 0 & L\cdot K_r+K_{\varphi r} & 0 & 0 \\ 0 & 0 & K_z & 0 & 0 & 0 \\ 0 & L\cdot K_r+K_{\varphi r} & 0 & L^2K_r-L\cdot(2K_{\varphi r}+F_z)+K_\varphi & 0 & 0 \\ -L\cdot K_r+K_{\varphi r} & 0 & 0 & 0 & L^2K_r-L\cdot(2K_{\varphi r}+F_z)+K_\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (21)$$

It can be seen from equation (21) that radial force of permanent magnetic bearings has relation with radial stiffness coefficient and coupling stiffness coefficient and torque has relation with tilt stiffness coefficient, coupling stiffness coefficient, radial stiffness coefficient and the axial force. Consequently, the influence of each stiffness coefficient on dynamics of rotor system is analyzed correspondingly.

By using a pair of magnetic rings bearing and considering the influence of coupling stiffness and axial force, critical speed value is found as(work space is 1mm) in Fig. 6.

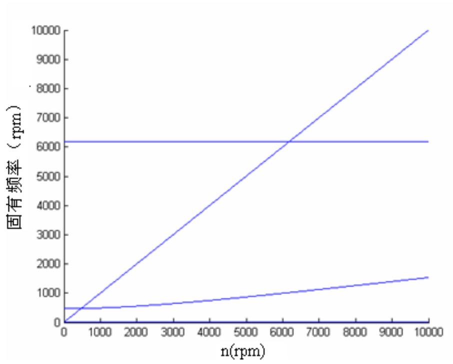

Fig. 6 Critical Speed

Table 2. Critical Speed of Single Permanent Magnetic Bearing Supporting (Work space is 1mm)

Critical Speed	First-order	Second-order	Third-order
1	6	478	6180(Re-root)
2	6	498	6180
3	0.5	498	6180

The critical speed is calculated as in Table 2 when axial force and coupling stiffness coefficient is zero. Where 1 represents taking coupling stiffness coefficient and axial force into consideration, 2 regardless of the influence of coupling stiffness coefficient and 3 without axial force into consideration. The result shows: low mode of magnetic bearings -rotor system is in tilt vibration and higher modes are radial vibration. Axial force and coupling stiffness affect tilt stiffness. Among the effect of coupling stiffness can be ignored. Axial force largely affects low critical speed. But the increase of the axial force can enhance the tilt stiffness of the system.

The critical speed is shown in Fig. 7 and Table 3 when changing work space to 0.5mm.

Table 3 Critical Speed of Single Permanent Magnetic bearing supporting (Work space is 0.5mm)

	First-order	Second-order	Third-order
Critical Speed	5	551	7228

Table 3 shows that the reduction of air gap causes the increase of radial stiffness of magnetic bearing and higher-order critical speed, but has no effect on low-speed. Tilt stiffness of magnetic bearings - rotor system is relatively small, thus the optimization of structural size has little effect on the tilt stiffness. So in design, the support span should be maximized.

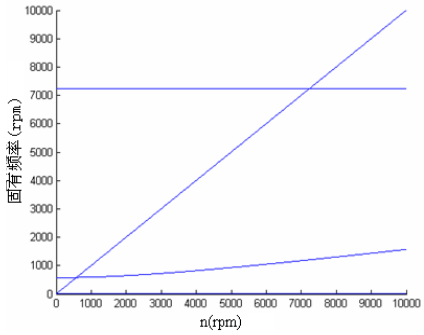

Fig.7 Critical Speed

Coupling stiffness coefficient of stiffness matrix will disappear when the permanent magnet bearings are in even number, that is, $K_{\varphi r}$=0. In Table 4, 1 and 2 are the critical speed calculated when respectively using two pairs and four pairs of permanent magnet bearings supporting form.

Table .4 Critical speed of multiple pairs of the permanent magnet bearing supporting

Critical Speed	First-order	Second-order	Third-order
1	6	829	10222
2	5	1154	14456

Table 4 shows that the application of multiple pairs of permanent magnet rings can increase the value of high-order critical speed, but basically has no effect on low critical speed. By reducing work space and increasing the number of magnetic rings can improve the bearing stiffness and the critical speed.

As can be seen from Fig.8, the movement mode of rigid rotor is simple. It consists of two types: translational type and the rotation in the plane *XZ* and plane *YZ* and the combination of these two movement types. Mode 1 and model 3 are cone whirl while model 2 and model 4 reverse eddy.

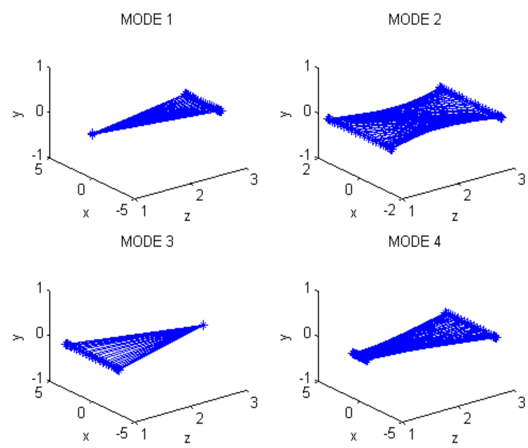

Fig.8 Vibration Mode at 5000rpm

The following is the analysis of unbalance response of system. The unbalance response of system at 5000rpm can be obtained when focus shift of rotor is not more than 0.3μm according to drawing, supposing e_x=e_y=0.212μm and eccentric quality △m=0.3g. Fig.9 is

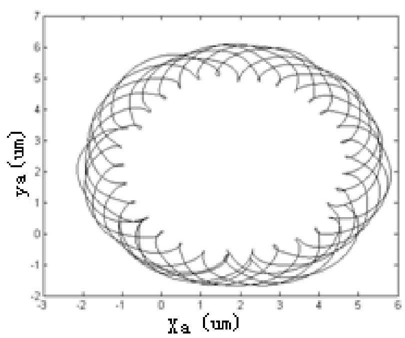

Fig.9 Axis Orbit Diagram of Left Bearing

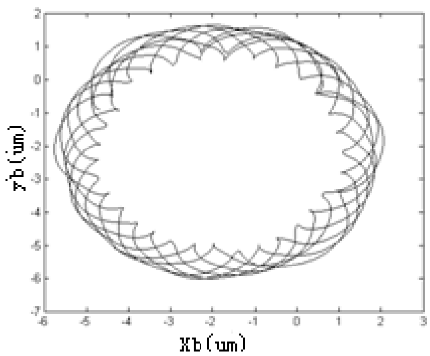

Fig.10 Axis Orbit Diagram of Right Bearing

the axis orbit diagram of left bearing and the amplitudes are between -2~+5μm；Fig.10 is the axis orbit diagram of right bearing and the amplitudes are between -6 ~ +2μm；Fig.11 is the unbalance response of the bearing in radial vibration with the change of rotation speed.

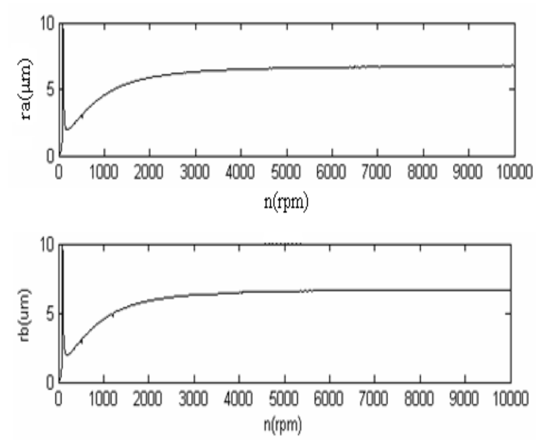

Fig. 11 The value of unbalance response with change of rotation speed

Conclusion

In this paper, the dynamic model of the permanent magnetic bearing-rigid rotor system is established. Calculation of the stiffness matrix of permanent magnetic bearing system by the method of equivalent current and the analysis about the dynamics of magnetic bearing - rotor system have been done. The complex modal map and the natural frequency of the rigid rotor system are calculated by Matlab program. The analysis shows that axial force makes a great influence on low critical speed of system while the coupling stiffness coefficient can be ignored in analysis. The critical speed would be raised while the stiffness of bearing enhanced by reducing the number of air gap and increasing the magnetic ring. The mode is very simple which include cone and reverse eddy, the unbalanced response amplitude is relatively small.

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