

# Research on $H_\infty$ Control for Flexible Rotor System Supported by Active Magnetic Bearings and Metal Rubber Rings

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**Abstract:** To lighten burden for Active Magnetic Bearing (AMB), Metal Rubber Ring (MRR) is introduced into the AMB system. Based on mixed-sensitivity  $H_\infty$  control theory, the  $H_\infty$  controller is designed and realized based on TMS320F2812DSP for the system. The influence of Weight Functions on dynamic characteristics of the system is investigated in this paper. The theoretical analysis and experimental results show that  $H_\infty$  control can help the system operate over the first two bending critical speeds safely and stably.

**Keywords:** Active Magnetic Bearing, Flexible Rotor, Metal Rubber Ring,  $H_\infty$  Control, Weight Function

## Introduction

In order to reduce the vibration of the flexible rotor system supported by AMBs, two main control methods have been adopted in many literatures. One is the technology of inhibiting synchronous vibration, and the other is the modern control theory or robust control theory for improving the support damping.

Ulrich Schönhoff et al developed  $\mu$ -synthesis controller for a flexible AMB system with severe gyroscopic effects and substructure modes to pass the first critical speed<sup>[1]</sup>. Hiroyuki Fujiwara et al used two control modes that the translating control employing the PID plus a notch filter and the titling mode control installed the PID phase shifting filter to make the rotor run stably at the second critical speed<sup>[2]</sup>. Kai Adler et al presented an on-line closed-loop active balancing facility for a supercritical rotor on AMBs<sup>[3]</sup>. Masaki Murata presented a controller that combined PID and LQ controllers applied for the model "Extended Reduced Order Physical Model"<sup>[4]</sup>. Lei Zhao et al designed  $H_\infty$  controller to make the rotor pass through the first two flexible critical speeds<sup>[5]</sup>. Zhenyu Xie et al added Magnetic Damper in AMBsystem, and the system passed through the second bending critical speed stably<sup>[6]</sup>.

To lighten AMBs burden, MRR is introduced and set around AMB, which is called Complex Bearing (CB). Our research shows that MRR is propitious for the system to pass through the first bending critical speed<sup>[7]</sup>. In this paper,  $H_\infty$  control strategy is adopted for the system and the influence of Weight Functions on dynamic characteristics of the system is investigated.

The experimental system is shown as Fig.1, and the CB shown as Fig.2. The mechanical structure is composed of the rotor, two radial CBs, one axial AMB, and one built-in motor. The overall length of the rotor is 828mm. The external diameters of the radial CB journals are 39.7mm. The clearances of the radial and axial bearings are 0.25mm. The bias currents of the radial and axial AMBs are 2.5A. The distance between the two radial CBs is 703mm.



Fig. 1 Photo of the experimental system

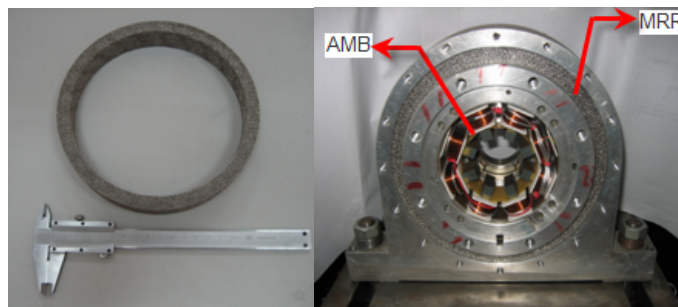


Fig.2 Photo of the Complex Bearing

### $H_\infty$ Control Strategy

Typical  $H_\infty$  mixed-sensitivity problem is shown as Fig.3.

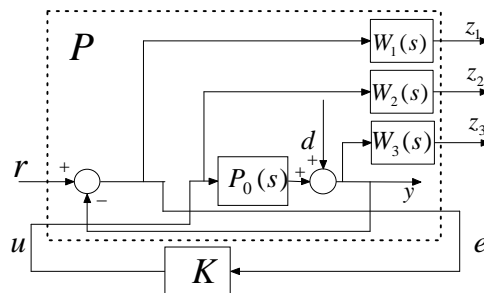


Fig.3 Typical  $H_\infty$  mixed-sensitivity problem

where,  $P_0$  is the transfer function of the mechanical structure and  $K$  is the control system.  $W_1$ ,  $W_2$ ,  $W_3$  are Weight Functions.  $P$  is the generalized controlled subject. The solution of  $H_\infty$  problem is deduced as

$$\left\| \begin{array}{c} W_1 S \\ W_2 K S \\ W_3 T \end{array} \right\| \leq \gamma \quad (\text{generally } \gamma = 1). \quad (1)$$

where, Sensitivity function  $S$  is the transfer function from disturbance  $d$  to output  $y$ . Complementary sensitivity function  $T$  is the transfer function from referenced input  $r$  to

output  $y$ .

$W_1$  is the measurement of the ability of interfering suppression.  $W_2$  controls the output size, and commonly it is a constant.  $W_3$  denotes perturbation norm bound. Usually  $W_1$  and  $W_3$  are represented as follows,

$$W_1 = \frac{a}{s+b}, \quad (2)$$

$$W_3 = \frac{cs}{ds+2000}. \quad (3)$$

Supposing  $W_2=2.5e-7$ ,  $W_3 = \frac{s}{2s+2000}$ , the curves of the step response and bode diagram of the system with various value of 'a' and 'b' are shown in Fig.4, Fig.5, Fig.6, Fig.7 respectively.

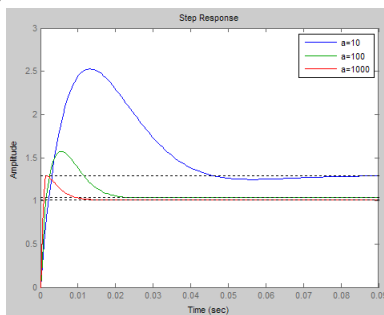


Fig.4 Step response ( $a=10,100,1000$ )

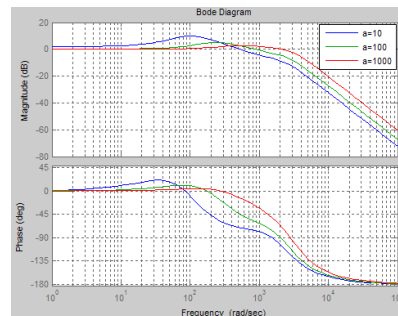


Fig.5 Bode diagram ( $a=10,100,1000$ )

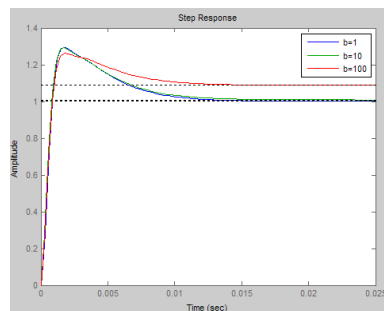


Fig.6 Step response ( $b=1,10,100$ )

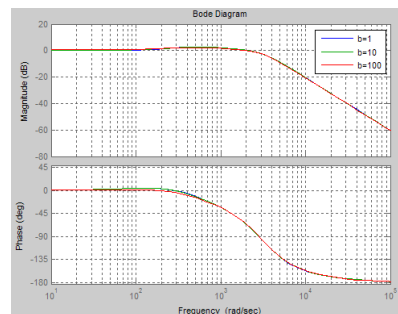


Fig.7 Bode diagram ( $b=1,10,100$ )

Note that 'a' has great influence on overshoot, response time, steady state error, magnitude and phase of the close-loop system while 'b' only has little influence on dynamic characteristics of the system.

Supposing  $W_1 = \frac{10}{s+1000}$ ,  $W_3 = \frac{s}{2s+2000}$ , the curves of the step response and bode diagram of the system with various value of ' $W_2$ ' are shown in Fig.8 and Fig.9 respectively.

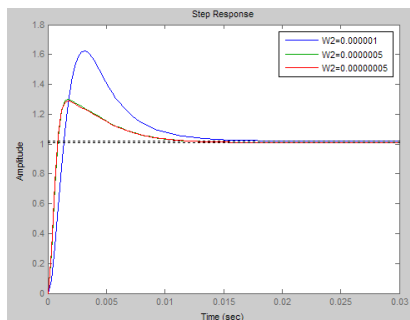


Fig.8 Step response ( $W_2=1e-5,5e-6,5e-7$ )

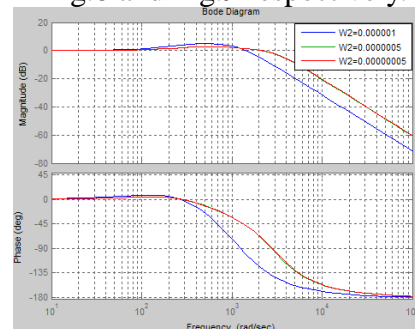


Fig.9 Bode diagram ( $W_2=1e-5,5e-6,5e-7$ )

Note that smaller  $W_2$  can reduce the overshoot and rise time of the system.

Supposing  $W_1 = \frac{10}{s+1000}$ ,  $W_2=2.5e-7$ , the curves of the step response and bode diagram of the system with various value of 'c' and 'd' are shown in Fig.10, Fig.11, Fig.12, Fig.13 respectively.

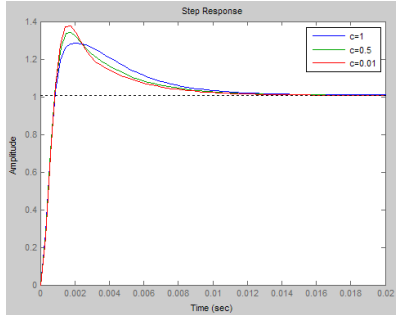


Fig.10 Step response ( $c=1,0.5,0.01$ )

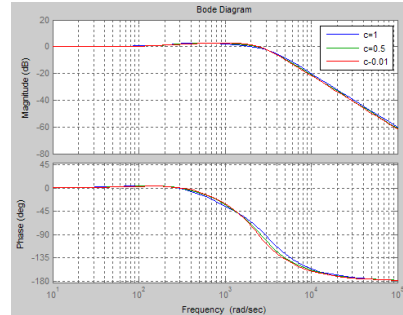


Fig.11 Bode diagram ( $c=1,0.5,0.01$ )

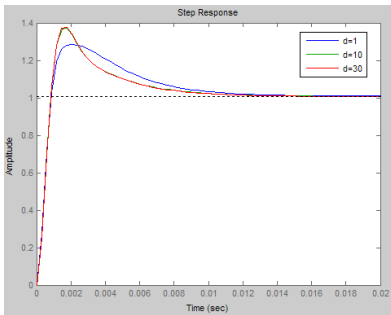


Fig.12 Step response ( $d=1,10,30$ )

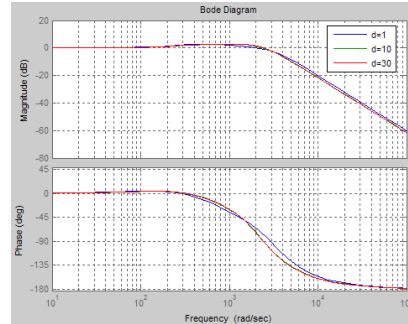


Fig.13 Bode diagram ( $d=1,10,30$ )

Note that c and d both have influence on overshoot of the system.

In order to match the system, the Weight Functions should be improved time after time.

According to the  $H_{\infty}$  Control strategy above, the program based on TMS320F2812DSP is as Fig.14.

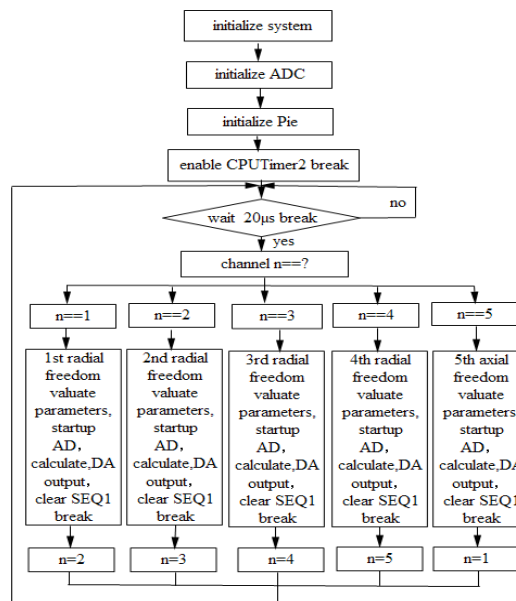


Fig.14 Program based on TMS320F2812DSP

### Experimental Modal Analysis

The rotor is dealt with twelve positions, on which the pulse excitations are imposed separately. The vibrations of the 6th and 10th positions are measured by the two acceleration transducers synchronously.

The rotor is suspended by the two radial CBs and one axial AMB. By the exciting tests, the first four mode shapes can be obtained, as shown in Fig.14.

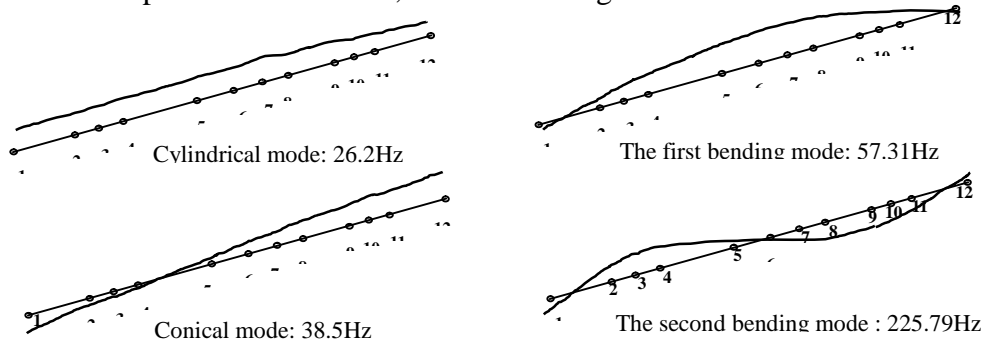


Fig.14 First four mode shapes

### Operation Test of the System

The whole system is shown in Fig.1. The rotor is also suspended by two radial CBs and one axial AMB with  $H_{\text{reg}}$  control above, and driven by the built-in motor from 0 to 230Hz.

The vibrations of the rotor on the left radial CB are measured by the displacement sensor, and the same frequency vibrations as the rotation speed can be obtained by the Dynamic Signal Analyzer in real-time, as shown in Fig.15.

Fig.15 shows that, the system can pass through the first two bending critical speed safely and stably.

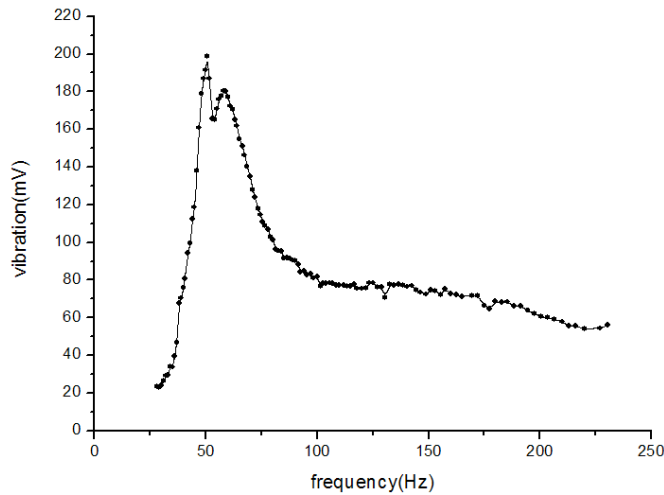


Fig.15 Same frequency vibration

### Conclusion

In this paper, dynamic characteristics of the flexible rotor supported by Active Magnetic Bearings and Metal Rubber Rings are investigated by theoretical calculation, Experimental Modal Analysis, and actual operation of the system. Main results are obtained as follows,

- 1) Greater Weight Function  $W_1$  can improve the dynamic characteristics of system while  $W_2$  and  $W_3$  should better be small.

- 2) According to the Experimental Modal Analysis, the first two bending critical speeds of the AMB system are around 3450r/min (57.3Hz) and 13550r/min (225.8Hz).
- 3) With the help of MRRs and AMBs with  $H_{\infty}$  control, the flexible rotor system can pass through the first two bending critical speed safely and stably.

### Acknowledgement

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