The Twelfth International Symposium on Magnetic Bearings (ISMB 12)
Wuhan, China, August 22-25, 2010

Analysis of Cross Feedback Control for the Magnetically Suspended Flywheel Rotor

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Abstract: For the magnetically suspended flywheel rotor of big moment of inertia, gyro effect is significant when it is rotating at high speed. Traditional decentralized PD control cannot insure the stability of the system. This paper mainly studies the cross feedback control, analyzes the velocity and displacement cross feedback, explains the control principle of cross feedback. The poles of precession and nutation, and the damping curves are given, showing the trends. And the results of the simulation experiments show that the cross feedback control is effective to restrain the gyro effect.

Flywheel systems are widely used in the energy storage, uninterruptible power supply, satellite attitude control, adopting magnetically suspending is an important developing way for the flywheel systems. Magnetic bearing is a new kind of electromagnetic bearing using magnetic force to balance the rotor between the stator. Magnetically suspended machines have the unique advantages of contact-free, without lubrication, adjustable stiffness and damping that traditional bearing cannot substitute.

Magnetically suspended flywheel systems are stable in theory adopting decentralized PD control. However, for the flywheel rotor of big moment of inertia, gyro effect is significant when rotating at high speed, constrains to raise the rotating speed, the stability is deteriorated, new control strategy is needed to restrain the gyro effect.

This paper mainly studies the cross feedback control aimed at the gyro effect. The control principle of velocity and displacement cross feedback are introduced, analyzed the poles, frequencies and damping of precession and nutation, and the results of simulation experiments are given.

Keywords: Magnetic Bearing, Cross Feedback, Gyro Effect, Precession and Nutation

1. Model of the flywheel rotor:

Magnetically suspended rotor is shown in Fig.1. To simply the problem, following assumptions are made. The rotor is symmetrical, a rigid body model is used, and the radial PMBs are isotropic. This paper only consider the radial motion, the axial motion is neglected. And the bearing force is regarded as a linear model.

The equations of motion are given as follows:

$$\begin{cases} \ddot{x}_c = (F_{xa} + F_{xb}) / m \\ \ddot{y}_c = (F_{ya} + F_{yb}) / m \\ J_d \ddot{\theta} x = -J_z \omega \dot{\theta}_y - (F_{ya} - F_{yb}) L / 2 \\ J_d \ddot{\theta} y = J_z \omega \dot{\theta}_x + (F_{xa} - F_{xb}) L / 2 \end{cases} . \qquad (1)$$

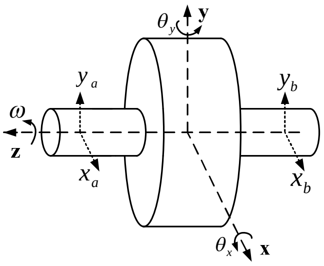

Fig.1 Flywheel rotor

The gyro torque T increases with the rotating speed. It influences the translational mode little, however, it influences the angular mode heavily. This paper mainly considers the angular mode.

2.1 Decentralized PD control:

Two DOF magnetic bearings are mounted at a and b, PD control is used to control the displacement error. In order to focus on the control performance, the time delay of the controller and power amplifier is neglected; the system is considered as an ideal elastic damping system.

As the rotating speed goes up, the gyro torque increases. It affects the precession and nutation significantly. The precession and nutation are the key factors that affect the system stability. Here the poles of the precession and nutation are plotted; the changing trends and stability are analyzed. The characteristic equation of the angular mode is as follows:

$$(J_d s^2 + DL^2 s/2 + PL^2/2)^2 + (J_z \omega s)^2 = 0 . \qquad (2)$$

The poles of the precession and nutation, and the frequency changing trends are plotted as follows:

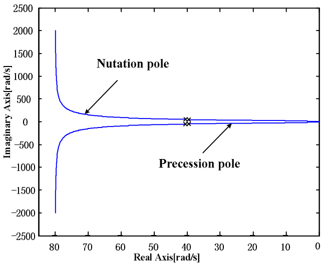

Fig.2 Poles of precession and nutation(PD)

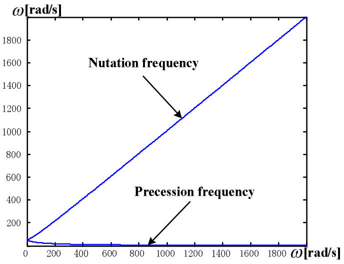

Fig.3 Frequency changing trends

The precession mode is called backward precession, the rotating direction is inverse of the rotor's; the nutation mode is called forward precession, the rotating direction is the same as the rotor's. As is shown in Fig.2 and 3, as the rotating speed increases, the precession pole goes to the imaginary axis, the frequency decreases to 0（$lim_{\omega \to \infty} \omega_P = 0$）. The nutation pole goes off the imaginary axis, the frequency increases, and is approximately proportional with the product of the speed and ratio of moments of inertia.（$lim_{\omega \to \infty} \omega_N = J_z \omega / J_d$）. Theory and simulation have proved that parameter P determines the precession and nutation frequency, the frequencies increase with the P; parameter D determines the precession and nutation damping. The changing trends of the poles can be explained on the rotational phase plane as follows:

In the precession mode, the proportional feedback torque produced by elastic support followed the tangent direction of the circle, velocity feedback torque produced by damping support points to the centre of the circle. The gyro torque T increases as the rotation speed. The direction of gyro torque is opposite with the proportional feedback torque, so the precession frequency decreases with the speed goes up, and is nearly to zero when rotating at extremely high speed. In the nutation mode, the direction of gyro torque is coincident with the proportional feedback torque, so the nutation frequency increases rapidly with the rotating speed.

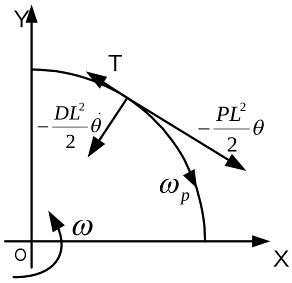

Figure 4.1 Precession torque

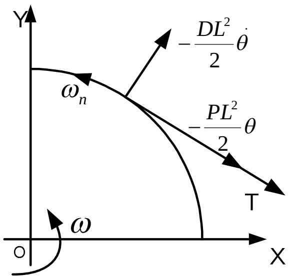

Figure 4.2 Nutation torque

The damping of the precession and nutation are approximately the same, they both decrease fast as the speed up. When the rotor goes to a high speed, the nutation frequency exceeds the working frequency range of the controller; phase lag is severe, destabilizing the system obviously. The precession frequency goes to the working region of the integrator, damping deteriorates, and the rotor is destabilized.

2.2 Velocity cross feedback control:

In order to compensate the gyro torque that does harm to the system, velocity cross feedback is appended in the decentralized PD as is shown in Fig.5. This method only adds a forward channel, the expansion of the controller is small, and the computing time is short, so it is easy to be realized.

The translational mode is not affected when cross feedback is introduced. The characteristic equation of angular mode is:

$$(J_d s^2+\frac{DL^2}{2}s+\frac{PL^2}{2})^2+(J_z\omega-D_cL^2)^2s^2=0. \qquad (3)$$

If $J_z\omega-D_c=0$ ($D_c=J_z\omega/L^2$), the gyro effect is completely compensated. Complete compensation is possible in theory, but may lead to instability in practice mainly due to time delay in controller and power amplifier. An attenuation factor C is used to improve the robustness. It can be written as $D_c=CJ_z\omega/L^2$ (0<C<1).

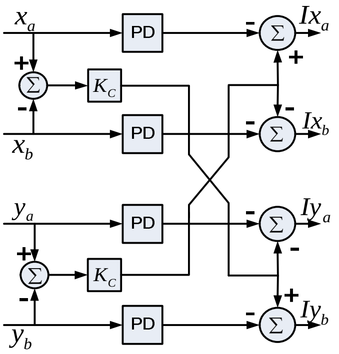

Fig.5 Control block of cross feedback

Compared with equation (2), when velocity cross feedback is introduced, the rotor seems to rotate at a low speed, the stability is enhanced. From the below figures, the location of the poles of precession and nutation is improved. The precession frequency increases while the nutation frequency decreases, damping is both strengthened.

On the rotational phase plane, the gyro torque is counteracted by the compensation torque, so the frequencies of precession and nutation are kept in the intermediate frequency range that is easy to be controlled. The operating performance is improved.

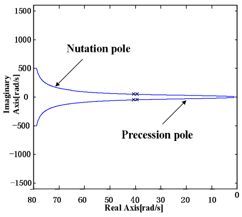

Fig.6 Poles of precession and nutation

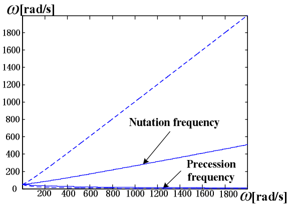

Fig.7 Frequency changing trends

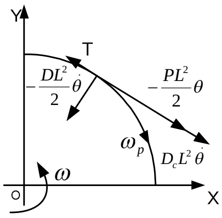

Fig.8.1 Precession torque

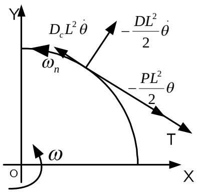

Fig.8.2 Nutation torque

2.3 displacement cross feedback control:

Velocity cross feedback can improve the system's performance significantly, raise the rotating speed. However, when the rotor runs to a higher speed, only with velocity cross feedback cannot insure the system stability. Mainly it is for that the frequency of the precession mode is decreased into the working range of the integrator, the damping is deteriorated. So it is needed to improve the precession damping. Hence, the displacement cross feedback is added, the implementing principle and block is the same.

Under velocity and displacement cross feedback, the equation of angular mode is as follows:

$$(J_d s^2+\frac{DL^2}{2}s+\frac{PL^2}{2})^2+((J_z\omega-D_cL^2)s+P_cL^2)^2=0. \quad (4)$$

The displacement cross feedback parameter P_c is proportional with the speed. Adding displacement cross feedback influences the frequencies of precession and nutation little, however, the precession damping increases notable, while the nutation damping decreases a little. The damping curves of the nutation and precession are shown in Fig.9 and compared with the control only with velocity cross feedback.

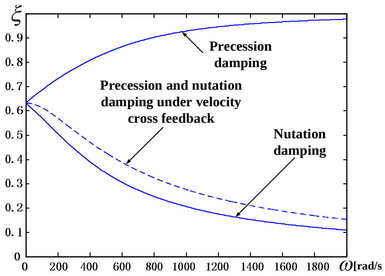

Fig.9 Damping of precession and nutation

Analyze the torque on the rotational phase plane. In the precession mode, the displacement cross feedback torque is in the same direction with the damping torque, damping is strengthened. The precession frequency is low, the damping increases obviously. In the nutation mode, new adding torque is in the opposite direction of damping torque, damping is weakened. The nutation frequency is high, so the damping decreases only a little.

In order to eliminate the bad influence of displacement cross feedback on the nutation damping, displacement cross feedback can be improved. Using low-pass and high-pass filters to obtain precession and nutation signals, then adopt cross feedback separately in opposite direction.

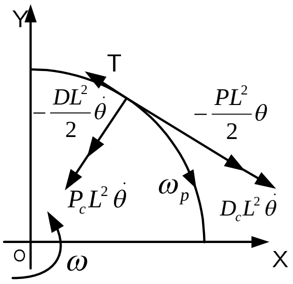

Fig.10.1 Precession torque

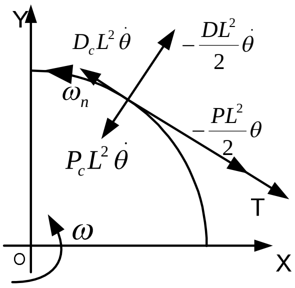

Fig.10.2 Nutation torque

The bearing force can be written as follows:

$$\begin{cases} F_{xa} = -Px_a - D\dot{x}_a - P_{cl}(y_{al} - y_{bl}) + P_{ch}(y_{ah} - y_{bh}) + D_c(\dot{y}_a - \dot{y}_b) \\ F_{xb} = -Px_b - D\dot{x}_b + P_{cl}(y_{al} - y_{bl}) - P_{ch}(y_{ah} - y_{bh}) - D_c(\dot{y}_a - \dot{y}_b) \\ F_{ya} = -Py_a - D\dot{y}_a + P_{cl}(x_{al} - x_{bl}) - P_{ch}(x_{ah} - x_{bh}) - D_c(\dot{x}_a - \dot{x}_b) \\ F_{yb} = -Py_b - D\dot{y}_b - P_{cl}(x_{al} - x_{bl}) + P_{ch}(x_{ah} - x_{bh}) + D_c(\dot{x}_a - \dot{x}_b) \end{cases} . \qquad (5)$$

(h means high frequency, indicates the nutation signal; l means low frequency, indicates the precession signal.)

As is shown in figure 11, adopting the improved cross feedback, precession and nutation damping can be enhanced simultaneously. In practice the nutation damping is difficult to be increased. In order to lighten the controller's burden, we can neglect the nutation, only use displacement cross feedback on precession, strengthening the precession damping.

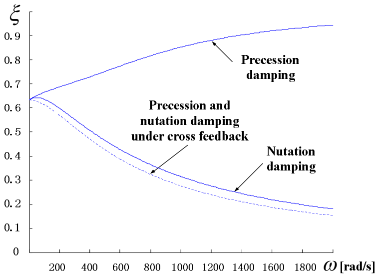

Fig.11 Damping of precession and nutation

3. Simulation experiment:

The magnetically suspended flywheel system model is built in the matlab/simulink environment, and simulated at 30000rpm speed. The rotor behaviors are shown separately under decentralized PD control, velocity cross feedback control, and both the velocity and displacement cross feedback control. As is shown in Fig.12 under the traditional decentralized PD control, the precession rotation is in clockwise, inverse of the rotor's rotating direction. The precession displacement is large, the nutation component is abundant. The system tends to loose stability, so velocity cross feedback is appended. Then the precession displacement is restrained significantly, nutation component is reduced greatly. The system's

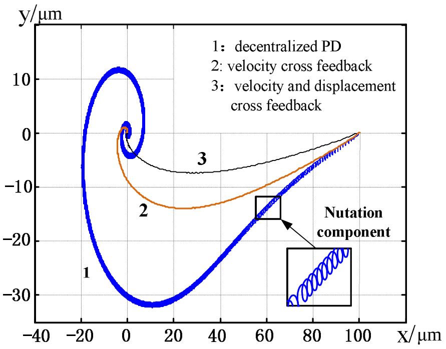

Fig.12 Rotor locuses

stability is strengthened. Displacement cross feedback is added on the base of velocity cross feedback. The displacement is reduced more, precession damping is strengthened, and the system performance is improved. Simulation experiment proved that cross feedback is a simple and effective way to constrain the gyro effect of flywheel rotor.

4．Conclusion:

In this paper, the flywheel rotor’s rotation model is built on the base of rotor dynamics, then decentralized PD, velocity cross feedback, displacement cross feedback are studied. The system stability and the poles changing trends are analyzed. The control effect is explained on the rotational phase plane, and the system is simulated. It can be concluded that the control of cross feedback is effective for the flywheel rotor of magnetically suspended with big moment of inertia. The stability of the system is improved, and the rotating speed can be increased to a higher level.

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