# Gimballing Control and Its Implementation for a Magnetically Suspended Flywheel

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**Abstract**: The idea of utilizing the gyroscopic effects in rotor dynamics level was proposed to improve the precision of gimbaling control of a magnetically suspended flywheel(MSFW). The dynamics couplings of a rotor are the main problems of tracking control. A feed forward matrix was proposed. Implementations of gimballing control were discussed. Angular displacement stiffness will degrade the precision of gimballing control, and additional currents were added to coils to compensate the disturbance. The experimental results show that adopting the proposed methods, the tested MSFW can generate a torque of 1.92Nm.

Keywords: Magnetically Suspended Flywheel, Gimballing Control, Gyroscopic Effects

## [ .Introduction

A flywheel is an inertial actuator in the spacecraft attitude control system. According to the attitude control system(ACS) command, flywheels generate suitable attitude control torques to correct spacecraft attitude deviation, or to adjust to an assigned attitude. Conventional flywheels are supported by ball bearings. The attitude control precision and wheels' lives are limited by the mechanical friction between rotor and stator. Magnetically suspended flywheel(MSFW) is a type of novel attitude actuator supported by magnetic bearings. Compared with conventional flywheels, MSFWs have the advantages of no friction, no lubrication, and long lives<sup>[1][2]</sup>. Furthermore, being actively controlled, MSFWs can achieve high attitude control precision when adopting specific algorithms.

If the AMBs of a MSFW have two tilt active control degrees, MSFW is able to tilt(gimballing) its spin axis of the rotor within a clearance between the rotor and the stator to generate large torque along the radial plane, without any additional device. A MSFW with gimballing capability is promise to fulfil the requirements of both precision and maneuvers, which can express the superiority over other attitude actuators better<sup>[3]</sup>.

A MSFW with gimballing capability is superior to other attitude actuators in maneuvering, integrated level or mass. The structure, control and applications are main technologies of gimballing MSFW. The structures and configurations of gimballing MSFWs which had no additional devices have been studied<sup>[3]</sup>. But the gimballing control problems of a MSFW have not been discussed yet, which are studied in this paper.

### **II**. Description Of Gimballing Control

A. The AMB-rotor system arrangement of a MSFW

Tilt bearings are always combined with radial or axial bearings<sup>[3]</sup>. Figure 2 shows the arrangement of a typical radial/tilt 4 DOF AMB-rotor system(2 radial DOF, 2 tilt DOF). The arrangement contains four bearing pairs: two identical radial bearing pairs, parallel to X axis in upper and lower location of XGZ plane, which are shown in Fig. 1, and two other radial bearing pairs, parallel to Y axis placed in YGZ plane symmetrically, which are not shown in Fig. 1. Define the rotations about X, Y, and Z axis are  $\alpha$ ,  $\beta$ , and  $\theta$ , respectively. The rotation  $\theta$ 

is controlled by motor, and  $\alpha$ ,  $\beta$  is controlled by magnetic bearings. The translation about axial axis is independent of the translation and rotation about radial axes. Assuming that the rotor is rigid and symmetrical about axial axis, magnets, sensors, controllers and power amplifiers are identical, then translation along and rotation about radial axes are decoupling(seeing Appendix).



Fig. 1 Arrangement of a four DOF AMB-Rotor system

B.A summary of gimballing control of a MSFW

The physical process of gimballing control of a MSFW is tilting the spin axis of the rotor to the command angle at the command rate, controlled by AMBs. It is about tracking control in control theory. The tracking objectives are to achieve the desired(commanded) angle and rate. Usable gimballing angle determines transverse momentum and tolerant gimballing rate determines maximum gimbal torque.

Yohji Okada et al put up a inclination control method to control the rotation as well as the translation of a rotor<sup>[4]</sup>. The tested rotor is slender and gyroscopic effects were not considered in this method. But a flywheel rotor's ratio of the inertia moment is not small  $(J_z/J_e>1)$ , and considering the high rotor speed, gyroscopic effects will degrade the tracking precision.

Gyroscopic effects will degrade the precision of gimballing control and must be considered. Instability caused by gyroscopic effects is one of the key problems of AMB research. Researchers put forward the following methods: differential feedback compensation<sup>[5]</sup>, and so on. But due to time delay, a completed compensation of gyroscopic effects in practice will lead to instability. Literature [5] put up an attenuation factor  $C_{\text{att}}$  to indicate the ratio of the implemented compensation compared to a complete compensation. C. AMB-Rotor System Dynamic Model

The equation of motion of AMB-Rotor system with reduced compensation can be described as follows according to appendix:

$$M\ddot{\overline{q}} + (D + \omega'G)\dot{\overline{q}} + K\overline{q} = F'$$
<sup>(1)</sup>

(2)

In equation(1), M is the symmetric mass matrix, D the symmetric damping matrix, G the skew-symmetric gyroscopic matrix, K the symmetric matrix. For the rigid body model the matrices are:

$$M = \begin{bmatrix} J_x & 0 \\ 0 & J_x \end{bmatrix}, \ \overline{q} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \ \omega' = (1 - C_{att})\omega, \ G = \begin{bmatrix} 0 & -J_z \\ J_z & 0^z \end{bmatrix}, \ D = K = 0$$

where  $C_{\text{att}}$  is the attenuation factor of [5].

The bearing force *F*' can be written as:  $F' = B_{r} f$ 

where

$$B_{z} = \begin{bmatrix} 0 & 0 & -l_{a} & -l_{b} \\ l_{a} & l_{b} & 0 & 0 \end{bmatrix}, \quad f = \begin{bmatrix} F_{xa} & F_{xb} & F_{ya} & F_{yb} \end{bmatrix}^{T}.$$

### **III.** Algorithm of gimballing control

A new idea of utilizing gyroscopic effects is proposed to eliminate the degradation of gimballing control caused by gyroscopic effects, which is applying extra torque actively to make a rotor precessing to improve the precision of gimballing control.



Fig. 2 A block diagram of gimballing control

Figure 2 shows a block diagram of gimballing control with a feed forward matrix to improve the tracking accuracy.

 $G_{\rm p}$  is the dynamic matrix of rotor rolling motion with uncompleted compensation,  $G_{\rm c}$  the decentralized control matrix,  $G_{\rm r}$  the feed forward matrix.  $\overline{q}_r = [\alpha_r \ \beta_r]^T$  indicates the attitude commands of output torque.

The feed forward matrix is as follows:

$$G_r = \begin{bmatrix} J_e s^2 & H's \\ -H's & J_e s^2 \end{bmatrix}$$
(3)

The feed forward compensation torque generated by the matrix is as follows:

$$M_r = G_r \overline{q}_r$$

(4)

The physical meaning of the matrix is to improve the tracking precision by added torques according to angular velocity and acceleration signals.

Define the tracking error  $\overline{e}$  as follows:

$$\overline{e} = \overline{q} - \overline{q}_r = \begin{bmatrix} \alpha_e \\ \beta_e \end{bmatrix}$$
(5)

### **IV. Implementation of gimballing control**

The motion equations of the rotor of reluctance AMBs can be described as follows:

$$\overline{q}(s) = G_p(s) \cdot (M_c + M_r)$$

$$M + M = B f$$
(6)
(7)

$$f = K \cdot q + K \cdot i \tag{8}$$

Where  $K_s$ ,  $K_i$ , q, and i has the same expression as that in [5], but the meaning of q and i is the displacement and current only caused by tilting the rotor about X and Y axis, which is different from that in [5].

 $M_{\rm c}$  is the torque generated by feedback control according to Appendix:

$$M_c = B_z \cdot (K_s q + K_i i_c)$$

$$i_c = -K_v e - K_v \dot{e}$$
(9)
(10)

### Where

 $q = T\overline{q}$ , is the rotation about X and Y axis of bearing coordinate,  $e = T\overline{e}$ , is the tracking error of rotation about X and Y axis of bearing coordinate,  $T = \begin{bmatrix} 0 & 0 & -l_a & -l_b \\ l_a & l_b & 0 & 0 \end{bmatrix}^T$ , is the

transformation matrix from generalized coordinate to bearing coordinate.  $i_c$  is the feedback control currents,  $K_x$  and  $K_r$  is is given in Appendix.

 $M_{\rm r}$  is the torque generated by the feed forward matrix in Equ.(4). The feed forward current of coils compensated the tracking error caused by gyroscopic effects is as follows considering Equ.(4), (7) and (8):

$$i_r = T(B_z K_i T)^{-1} G_r \overline{q}_r \tag{11}$$

AMB-Rotor system dynamics with uncompleted compensation in [5] can be described by

$$\begin{aligned} M\ddot{q} + (D + \omega'G)\dot{\bar{q}} + K\bar{q} \\ = B_{z}[K_{s}T\bar{q} + K_{i}(-K_{x}T\bar{e} - K_{y}T\dot{\bar{e}})] \end{aligned} \tag{12}$$

The feed forward matrix improves the tracking characters of the first order differentiation ( $\omega' G \dot{q}$ ) and the second differentiation ( $M \ddot{q}$ ). These two terms represent the rotor dynamical characters.

 $B_{r}K_{s}T\bar{q}$  in Equ.(12) is the torque caused by equivalent angular displacement stiffness. It reflects the force-displacement characters of AMBs. The principle of compensation is to decrease the angular displacement torques by force-current torques. The current compensating angular displacement torques is described by

$$i' = K_i^{-1}K_s T \overline{q}$$
 (13)  
The currents of reluctance AMB magnet is the sum of three terms:  
 $i = i_c + i_r + i'$  (14)

V. EXPERIMENTS

The validation was undertaken on a MSFW system FW50D. The device, shown in Fig. 3, has five DOF AMBs. The configuration of the radial and tilt bearings is the same as that in Fig. 1. The key technical parameters of the MSFW were shown in Table 1.

Table 1 Technical characte		
Item	Value	
Angular momentum H	50[Nms]	
Normal speed $\omega$	±5000[r/min]	
Equator moment of inertia Je	$0.0575[kgm^{2}]$	
Pole moment of inertia Jz	0.0956[kgm <sup>2</sup> ]	
Displacement stiffness Kh	6.68×10 <sup>5</sup> [N/m]	
Current stiffness Ki	300[N/A]	
	Rotor	Sensor(AX)
		Sensor(BX)

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Fig. 4 Sensing of gimballing angles

(14)

The displacements and gimballing angles are measured by eddy current sensors. The gimballing angles are calculated by the two sensing signals' difference, as shown in Fig. 4. Because the gimballing angle is small, the description of the angles can be approximated as follows:

$$\begin{cases} \alpha = \arctan\left((S_{AY} - S_{BY})/l_s\right) \approx (S_{AY} - S_{BY})/l_s\\ \beta = \arctan\left((S_{AX} - S_{BX})/l_s\right) \approx (S_{AX} - S_{BX})/l_s \end{cases}$$
(15)

Real time control is implemented on a TMS320C6713B+FPGA digital control board<sup>[6]</sup>. The gimballing rate of the given experimental results was 2.2°/s, with which the wheel could generate the maximum gimballing torque of  $\pm 1.92$ Nm. The usable gimballing angle of FW50D is 0.055°.

Fig. 5 to Fig. 8 shows the gimballing control effects with and without the proposed method. The methods could tilt the wheel about any axis on the plane normal to the spin axis. In order to demonstrate the compensation of rotor dynamic coupling better, we choose to tilt about the X axis in the following experiments, in which the angle  $\alpha$  should be stable when the angle  $\beta$  changes. Note that the wheel rotor has not been dynamic balanced. The results contained synchronous components obviously.

The feed forward matrix compensates rotor dynamic coupling, which is mainly expressed during the tilting moment. Fig. 5 and Fig. 6 show a transient of gimballing angles by receiving a gimballing command, without and with the proposed method, at the rotational speed of 5000rpm.



Fig. 5 Gimballing angle without the proposed method(transient)



Fig. 6 Gimballing angle with the proposed method(transient)

The angular stiffness compensation compensates the angular stiffness torques, which sustain if the rotor deviates the equilibrium position during gimballing control. Fig. 7 and Fig. 8 show a period of time from receiving the gimballing command at the rotational speed of 5000rpm.



Fig. 7 Gimballing angle without the proposed method(long term)



Fig. 8 Gimballing angle with the proposed method(long term)

### **VI.Conclusion**

Gimballing control of a MSFW could meet the maneuverability requirements of space applications, for example, earth observation of satellite platforms. Gimballing control of a MSFW could reduce the mass, volume and cost of spacecraft attitude control system, having significant application value for nimble maneuvering spacecrafts and small satellites.

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### Appendix

The equations of motion for a rotor can be described according to [5]:

$$M\ddot{\overline{q}} + (D + \omega G)\dot{\overline{q}} + K\overline{q} = F$$
(16)

The matrixes in Equ.(16) and the following equations are the same as these in [5]. Equ. (16) can be expanded as follows:

$$\begin{cases} J_x \ddot{\beta} - \omega J_z \dot{\alpha} = l_a F_{xa} + l_b F_{xb} \\ m\ddot{x} = F_{xa} + F_{xb} \\ -J_x \ddot{\alpha} - \omega J_z \dot{\beta} = l_a F_{ya} + l_b F_{yb} \\ m\ddot{y} = F_{ya} + F_{yb} \end{cases}$$
(17)

(18)

(21)

The vector of generalized bearing force *F* can be described by

 $F = B_z f$ 

Where f can be written as:

$$=K_{s}q+K_{i}i$$
(19)

Differential cross feedback control is realized by coil currents. The current can be written as follows according to Figure 11 of [5]:

$$=-K_{x}q-K_{r}\dot{q} \tag{20}$$

Bearing coordinates q can be written as:  $q = T\overline{q}$ 

Let substitute (19)(20)(21) into (18):

$$F = B_z (K_s T \overline{q} - K_x \overline{q} - K_r \dot{\overline{q}})$$
(22)

Let substitute generalized bearing force(22) into (17) and  $k_c ext{ is } \frac{C_{att} \omega J_z}{k_i (l_a - l_b)^2}$ . Assume the rotor

is symmetrical and the characters of magnets and coils are identical, then  $l_a = -l_b$ ,  $k_{s,a} = k_{s,b}$ ,  $k_{i,a} = k_{i,b}$ .

Finally the equations of motion for a rotor can be described by

 $\begin{cases} J_{x}\ddot{\beta} - (1 - C_{att})\omega J_{z}\dot{\alpha} = 4k_{s,a}l_{a}^{2}\beta - 4k_{i,a}(k_{x}l_{a}^{2}\beta + k_{v}l_{a}^{2}\dot{\beta}) \\ m\ddot{x} = 4k_{s,a}x - 4k_{i,a}(k_{x}x + k_{v}\dot{x}) \\ -J_{x}\ddot{\alpha} - (1 - C_{att})\omega J_{z}\dot{\beta} = -4k_{s,a}l_{a}^{2}\alpha + 4k_{i,a}(k_{x}l_{a}^{2}\alpha + k_{v}l_{a}^{2}\dot{\alpha}) \\ m\ddot{y} = 4k_{s,a}y - 4k_{i,a}(k_{x}y + k_{v}\dot{y}) \end{cases}$ (23)

Equation (23) shows:

(1) The translation and the rotation(about X and Y axes) for a rotor has no coupling, and two translations about X and Y axis has no coupling. These are the foundation of analyzing the rotations alone for a four DOF(2 radial, 2 tilting) AMB-Rotor system.

(2) The motion equations for a rotor of an uncompleted differential cross feedback compensated AMB-Rotor system can be written as follows:

$$M\ddot{\overline{q}} + [D + (1 - C_{att})\omega G]\dot{\overline{q}} + K\overline{q} = F'$$
(24)

$$F' = B_z \left( K_s T \overline{q} - K_x \overline{q} - K_v \dot{\overline{q}} \right)$$
(25)

Where *F*' is the vector of the generalized bearing force with the decentralized control. With the differential cross feedback compensation, the dynamic coupling becomes  $1-C_{att}$  as these with decentralized control. Equation (24) is the motion equation of a rotor with uncompleted differential compensation, and the feed forward matrix in this paper is based on (24).

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