Effects and Compensations of Time Delay on the Dynamic Stiffness and Damping Coefficients of Active Magnetic Bearings

Su Wenjun[1,a], Geng Haipeng[1,b], Liu Heng[1,c], Yu Lie[1,d]

[1]Institute of Mechatronics and Information Systems, Key Laboratory of Ministry of Education for Strength and Vibration, Xi'an Jiaotong University, Xi'an, 710049, China

[a]wenjunsu@mail.xjtu.edu.cn, [b]genghaipeng@mail.xjtu.edu.cn,
[c]hengliu@mail.xjtu.edu.cn, [d]yulie@mail.xjtu.edu.cn

Abstract: Aiming at the problem of small time delay in the control system of an active magnetic bearing (AMB), the direct approach is applied to study the effects and compensation of time delay on the dynamic stiffness and damping coefficients of active magnetic bearings. Second-derivative feedback is used to compensate time delay effects, and performance of the controller has been verified. The results indicate that the time delay mainly affects the dynamic performance of AMB system in the upper frequency band, and results in the increasing of bearing dynamic stiffness coefficient and the decreasing of bearing dynamic damping coefficient with the high-speed AMB, respectively. The relationship between second-derivative and bearing performance is investigated, and it is shown that the time delay effects on the bearing properties may be effectively compensated by using the second-derivative feedback.

Keywords: Active Magnetic Bearings, Time Delay Effect, Time Delay Compensation

Introduction

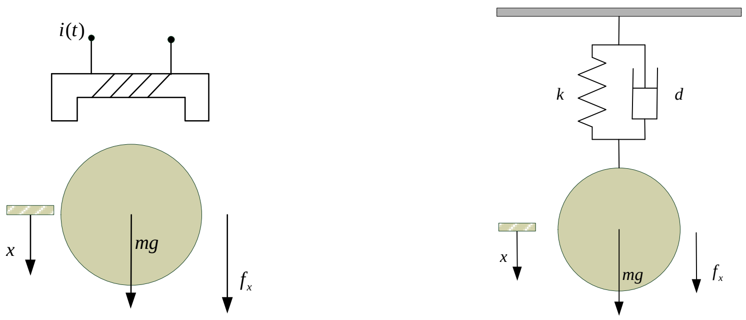
It is known that the time delay sources are force and signal transmit delay, controller delay, and power amp delay etc. Because the electromagnets are open-loop unstable, all designs of magnetic bearing require external electronic control to regulate the electromagnetic forces acting on the bearing. In addition, magnetic bearings permit active control of the bearings dynamic characteristics. Although the force and signal transmit delay are fixed values, but the controller delay can be varied. Otherwise, the time delay in active magnetic bearing system (AMBs) is impossible to avoid but to reduce. Normally, the low-speed AMBs ignores the effect of inner time delay. As high-speed and high-precision increase, the effort to reduce and compensate the time delay in AMBs can be concentrated on the critical.

Magnetic bearings offer several advantages over conventional fluid-film bearings. These advantages have necessitated the development of improved bearings capable of supporting these new high-speed rotors. Several researchers have presented results demonstrating the efforts of time-delay on the active magnetic bearings. Hisatani et al. [1,2], compared analog and digital control designs for single-axis control of a monrotating rotor and found that simple, fast digital algorithms tended to perform better for their application than slower, more complex ones. The effect of time delay in the control path on the dynamic performance of active vehicle suspension equipped with"sky-hook" damper is studied through the use of numerical simulations by Zhang Wenfeng et al.[3], It is pointed out that the time delay mainly affects the dynamic performance of suspension system in the lower frequency band and result in the instability of the first natural mode of suspension system. The time delay effect on the dynamic stiffness and damping coefficients of active magnetic bearings have not yet been investigated quantitatively. In this paper, the effects of time delay on the dynamic stiffness and dynamic damping coefficients of AMBs are further demonstrated with the implementation of digital controller.

The time delay in digital control system is impossible to avoid but to reduce. Optimal control method and variable structure control method were applied used to compensate time delay by Sun Feng[4] with single-degree-of-freedom system. It was shown that the control performance of variable structure control method is much superior to that of optimal control method. In this paper, otherwise, second-derivative feedback is used to compensate the effects of time delay on AMBs.

Magnetic bearing characteristics

Without loss of generality, single degree-of-freedom(DOF) thrust bearing is used in proof to study that the effect of time delay on AMB characteristics, which without the cross-term of dynamic stiffness and damping coefficient. , Then, the performance of thrust bearing with time delay can directly reflects the effect of time delay on AMBs.



(a) (b)

Fig.1 (a) Single DOF bearing model; (b) Single DOF spring-mass model.

Fig.1(a) shows the magnetic bearing system differential equations of motion are:

$$m\ddot{x}(t)+k_s x(t)+k_i i(t)=f_x(t) \tag{1}$$

where f_x is the external force, x is the displacement, i is the control current exerted by the power amplifiers, and m representing the mass of the rotor. The coefficient k_s is called the open loop gain of the actuator and is negative; the coefficient k_i is called the current gain and is positive. These coefficients are assumed to be constant, and are determined experimentally.

Assuming that the bearing can be modeled as a first-order device, the single degree-of-freedom rotor model is shown in Fig.1(b), where the corresponding equation of motion is

$$m\ddot{x}+kx+d\dot{x}=f_x \qquad (k>0, d>0) \tag{2}$$

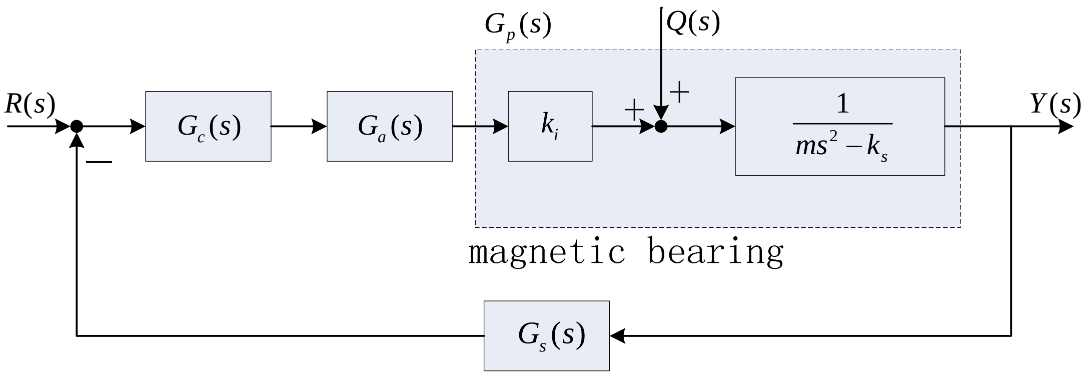
with k and d representing the stiffness and damping coefficients of the bearing. This equation can be rewritten in the form of a second-order system:

$$\frac{X(s)}{F_x}=\frac{1}{s^2+2\xi\omega_n s+\omega_n^{\ 2}} \tag{3}$$

where

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$\xi = \frac{d}{2\sqrt{mk}}$$
.

Magnetic bearing systems are open-loop unstable; therefore, active feedback control is required to stabilize the system. Using a controller $G_c(s)$, which relates rotor position x to actuator current i, yields the block diagram of closed-loop magnetic bearing system is shown in Fig.2.



$Q(s)$:perturbation; $G_c(s)$:digital controller; $G_a(s)$:Power amp; $G_p(s)$:magnetic bearing; $G_s(s)$: displacement sensor

Fig.2. Block diagram of closed-loop magnetic bearing system

The two most important mechanical properties of a typical conventional bearing are its stiffness and damping. Stiffness is the component of the baring force applied in proportion to the displacement of the rotor, and damping is the component of the bearing force applied in proportion to the radial velocity of the rotor. It will be helpful to relate the feedback controller transfer characteristic $G_c(s)$ to the dynamic stiffness and dynamic damping coefficients of the magnetic bearing system. Because the magnetic forces are function of a frequency-dependent transfer function the dynamic stiffness and dynamic damping coefficients of active magnetic bearings, the dynamic stiffness and dynamic damping properties are called $k(\omega)$ and $d(\omega)$, respectively. To find the relation of the feedback controller $G_c(s)$ to the dynamic stiffness and damping of the controlled magnetic bearing system, the equations for the magnetic bearing stiffness and damping are:

$$\begin{cases} k(\omega) = k_s + k_i \operatorname{Re}[G_c(j\omega)] \\ d(\omega) = k_i \dfrac{\operatorname{Im}[G_c(j\omega)]}{\omega} \end{cases} \tag{4}$$

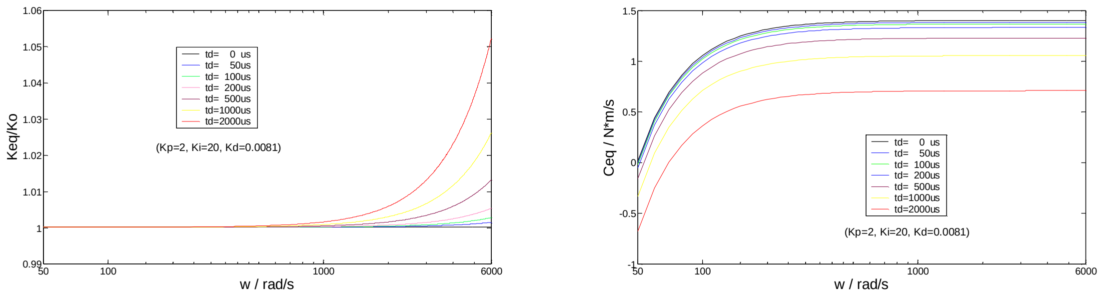
Effect of Delay

Traditional PID control algorithms have been implemented on the prototype magnetic bearing rig presented in this paper[5]. Time delay is intrinsic to all existing devices and control feedback paths. The delay problem occurs when the time delay can not be ignored but smaller than the sampling interval in the bearing system. Due to the presence of the exponential term $e^{-\tau s}$ of the time delay, the number of zeros of homogeneous algebraic equation is infinite and this makes such a bearing characteristics analysis extremely difficult. When the stable controller is actually implemented on a digital controller, we can get $\tau << T_s$, with T_s is the sampling interval of the digital control system, and small enough to recover the continuous-time signals. Thus, the equations for the dynamic stiffness and damping of the magnetic bearing with time delay are:

$$\begin{cases} k_{eq}(\omega) = k_s + k_i \operatorname{Re}[G_c(j\omega)e^{-j\tau\omega}] \\ d_{eq}(\omega) = k_i \dfrac{\operatorname{Im}[G_c(j\omega)e^{-j\tau\omega}]}{\omega} \end{cases} \tag{5}$$

Since $\tau << T_s$, so $e^{-\tau s}$ can be expanded as Taylor series with first-order approximation. Substituting the approximation $e^{-\tau s} \approx 1 - \tau s$. Thus, using this approximation for the Laplace variable $e^{-\tau s}$ yields the following Fig.3.

Fig.3 shows the effects of varying time delay on dynamic stiffness and damping of bearing for the digital PID controller. The effects of various of time delay on the shapes of the calculated stiffness and damping cures are illustrated in Fig.4. Inspection of Fig.3(a) reveals that time delay is dramatic changes the high-frequency ports(around 1000 rad/s) of the dynamic stiffness curves. The tends in these stiffness curves suggest that the rotor response will exhibit more stiffness at higher running speeds with almost no effect at the lower speeds as the amount of time delay is increased.



K_{eq} and K_o represent the dynamic stiffness of bearings with and without time-delay, respectively.

(a) (b)

Fig.3 (a) Stiffness effect of varying time-delay in a PID controller; (b)Damping effect of varyingtime-delay in a PID controller

Time delay of the system significantly affects the dynamic damping coefficient. Can be seen from Fig.3(b), in the whole frequency band, time delay makes the dynamic damping coefficient significantly reduced, while the increase of time delay, significantly reduced the damping coefficient. In the low frequency region, and even the damping coefficient is negative, due to the dynamic stiffness and damping bearing is used to predict bearing on the response to external excitation, and not be used to predict the stability of the system, so can not say at this time the system instability.

Delay Compensation

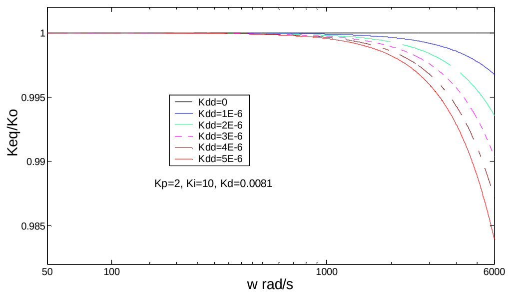
Digital control algorithms was investigated by Ronald D.W et al.[6], it was shown that digital PD, PDD, and PIDD controllers have exhibited bearing damping characteristics that drop off at higher frequencies, and this roll off is due to the phase lag introduced into the system by the various low-pass filters and delay elements present. Therefore, all these results present the time-delay factors should be compensated for building a better digital controller of active magnetic bearing system. Because of the distinctive characteristics, time delay is even now a difficult and unresolved completely problem in control engineering. Silva[7]has presented results demonstraton the efficient computation of the entire set of PID controllers achieving stability and various performance specifications for linear time-invariant plants. A PID controller was implemented using Taylor approximation presented here.

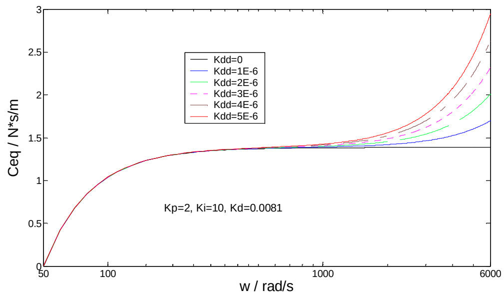
Incorporating second derivative feedback into the basic PID controller helps to overcome the high-frequency phase lag effects by adding phase lead to the controller signal over a range of higher frequencies was implemented and tested, and this algorithms is shown to provide the bearing designer with powerful tools for compensation effects of time delay on the bearings. In this paper, the addition of second derivative to the PID controller has been used to compensate the inverse effect of increasing the time delay.

Second derivative feedback is easily added to a digital PID controller. The continuous frequency domain representation of a proportional-integral-derivative–derivative (PIDD) control would be represented by:

$$G_c(s) = K_p + K_d s + K_{dd} s^2 + K_i \frac{1}{s} \tag{6}$$

The effects of varying amount of the second derivative on the shapes of the calculated stiffness and damping cures are illustrated in Fig.5.





K_{eq} and K_o represent the dynamic stiffness of bearings with and without time-delay, respectively.

(a) (b)

Fig.4. (a) Stiffness effects of varying amount of the second derivative feedback in a PIDD controller; (b) Damping effect of varying amount of the second derivative feedback in a PIDD controller

Fig.4 shows only very small changes in the low-frequency portion of the stiffness and damping of bearings as the amount of second derivative is increased. However, note the noticeable changes in both the high-frequency portion(around 1000 rad/s) of the stiffness and damping performance of bearings: as the amount of second derivative is increased, the dynamic stiffness decreases noticeably, and the dynamic stiffness increases noticeably.

Comparison of the effects of second derivative and the effects of time delay afford an approach to compensate the inverse effects of time delay on high-frequency portion for magnetic bearing characteristics.

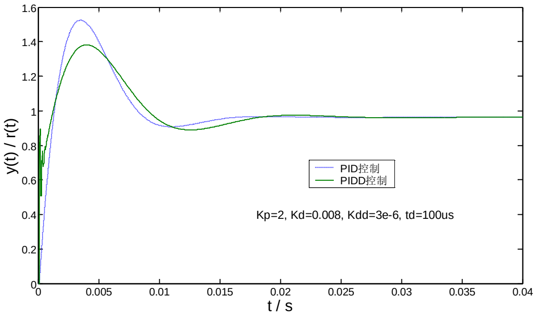


Fig.5 The rotor step response of varying second derivative feedback in a PIDD controller

The rotor step responses shows in Fig.5 confirm this prediction: As the amount of the second derivative is increased, the amplitude response of the rotor decreases noticeably. Form Eq.3, the decreasing of rotor response even confirms the increasing of the damping of bearings. All these shown that the time delay effects on the bearing properties may be effectively compensated by using the second-derivative feedback.

Conclusion

The time delay effect on the dynamic stiffness and damping coefficients of active magnetic bearings have been investigated in this paper. The results presented in this paper show that time delay mainly affects the bearing characteristics of magnetic bearings in the high-frequency band, and result in the noticeable changes of the dynamic stiffness and damping curves. The addition of a second derivative to the PID controller has be employed to compensate the neglect effect of time delay on the high-frequency portion of the dynamic stiffness and damping of magnetic bearings. The performance of this new approach has been confirmed successfully by the rotor step response.

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