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Fuzzy Modeling and Output Feedback Stabilization of a Nonlinear Magnetic Bearing with Delayed Feedback

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Abstract: This paper presents a robust fuzzy logic-base control scheme for a nonlinear magnetic bearing system that is subject to time delay in feedback loop. A new Takagi-Sugeno fuzzy model is proposed to represent the nonlinear magnetic bearing. Base on the new fuzzy model, a PDC controller is designed in terms of a proposed delay-dependent stability criterion which guarantees the asymptotic stability of the fuzzy model. The results of simulation verify the effectiveness and superiority of the proposed method.

Keywords: Magnetic Bearing, Time Delay, LMI

Introduction

Active magnetic bearings (AMBs), which support rotors without any mechanical contact and lubrication, provide the possibility for high-speed rotation of machines, and more and more it inevitably requires the introduction of modern digital controller with high control precision and advanced control strategies. However, new problems emerge unexpectedly as modern digital controller is widely used in AMB systems. One problem is the unavoidable time delay in the feedback loop of an AMB controller (e.g., network delay, computation time delay in digital controllers) [1-3]. As magnetic bearings are applied to high-speed rotating systems, and rotors vibrates in a synchronous frequency due to mass imbalance, these delays, which are neglected in most systems, may approach to period of rotor vibration and have great influences on dynamics of the system. Even though these delays are very small, they may lead to oscillatory response of larger amplitude, and even cause instability of the whole system [4, 5]. Therefore, it is important both in theory and practice to take the time delay into considerations when designing a controller for an AMB system. But due to the inherent nonlinearity of the AMBs, the stability analysis and controller design for a magnetic bearing system with delayed feedback are difficult and still a challenge.

Time delay is a main source of instability and poor performance, which usually makes the rotor supported by a magnetic bearing far away from the equilibrium point. Large vibration of the rotor may yield great model errors for controllers that are based on the assumption that the rotor oscillate in a small region and are designed by linearizing the dynamic of bearing about a nominal equilibrium point[6, 7]. Recently, the rapid development of fuzzy model-based control theory [8-17] provides the possibility to solve the control problem of highly nonlinear systems like a magnetic bearing with time delay. Through a suitable fuzzy system partitioning of the whole operation region of a complex nonlinear system, the Takagi-Sugeno fuzzy model [18], which is comprised of a family of local linear models, can provide a powerful solution for function approximation, stability analysis and controller design of nonlinear delay system in view of fruitful control theory and technique for linear delay system. The stability and control design issues about T-S fuzzy model with time delay can be classified into two categories, namely delay-independent criteria [8, 9, 14] and delay-dependent criteria[10-13, 15-17]. The study in [11, 15] extended delay-dependent approach to T-S fuzzy system with bounded time-varying delay. In [12, 13], stabilization problems for the case of input delay were investigated, and observer-based PDC controllers were successfully conducted.

In this paper, we considered the delay-dependent stabilization for a general nonlinear AMB system with delayed feedback based on a new T-S fuzzy model. The parallel distributed compensation (PDC) was adopted to synthesize a stable fuzzy controller, and the controller design problem was then converted into a linear matrix inequality (LMI) problem. Finally, the results of computer simulation verified the performance of the presented control strategy.

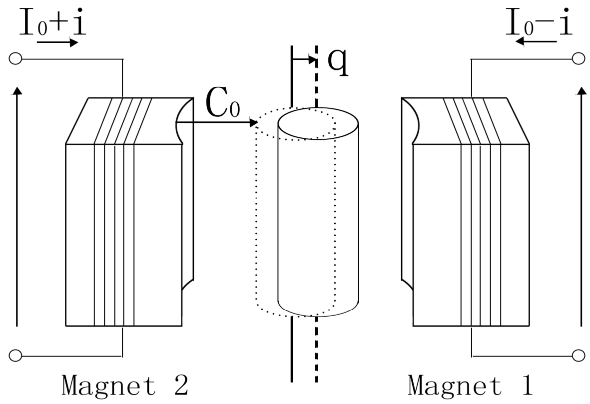

Fig 1: Current biased radial AMB

Fuzzy modeling of magnetic bearings

As shown in Figure 1, a typical 1-Dof AMB system consists of a stator and a rotor. A magnetic field is created within the stator, rotor and the air gap between the stator and the rotor when current flows in the coils that are wound around the stator. The dynamical mathematical model considering time delay in feedback loop can be written as:

$$m\ddot{q} = -F(q,i) = k\left[\left(\frac{I_0 - i(T-\Delta T)}{c_0 - \beta q}\right)^2 - \left(\frac{I_0 + i(T-\Delta T)}{c_0 + \beta q}\right)^2 \right] \quad (1)$$

where q denotes the displacement of the rotor, i denotes the input control current, and ΔT is time-varying feedback delay with $\Delta T \le \bar{d}$. k is the magnetic force constant; I_0 and i denote the bias and control currents, respectively; c_0 is the nominal air gap between the stator and the shaft; m is the mass of the rotor.

Taking the first order Taylor's series expansion of the magnetic force $F(q,i)$ at the operating points (q^*, i^*), and introducing new parameters:

$$x = q/c_0, u = i/I_0, t = \Omega_0 T, \bar{\Omega} = \Omega/\Omega_0, \tau(t) = \Omega_0 \Delta T \quad (2)$$

with $\tau(t) \in [\tau_0 - \sigma, \tau_0 + \sigma], \tau_0 \ge \sigma$ yields

$$\ddot{x} = -K_x(x^*, u^*)\cdot x - K_u(x^*, u^*)\cdot u(t-\tau(t)) + d(x^*, u^*) \quad (3)$$

where the displacement stiffness K_x and current stiffness K_u can be shown to be given by

$$K_x(x,u) = -\bar{k}\beta[\frac{(1-u)^2}{(1-\beta x)^3} + \frac{(1+u)^2}{(1+\beta x)^3}] = -\bar{k}f_1(x,u),$$

$$K_u(x,u) = \bar{k}[\frac{(1-u)}{(1-\beta x)^2} + \frac{(1+u)}{(1+\beta x)^2}] = \bar{k}f_2(x,u)$$

$$d(x^*,u^*) = \frac{\bar{k}}{2}[\frac{(1-u^*)^2}{(1-\beta x^*)^2} - \frac{(1+u^*)^2}{(1+\beta x^*)^2}] - K_x \cdot x^* - K_u \cdot u^* \quad \text{with} \quad \bar{k} = 2kI_0^2/(c_0^3 \cdot \Omega_0^2 \cdot m)$$

Based on linearization of nonlinear model (1) as we described above, Hong[19] propose a Takagi-Sugeno-Kang fuzzy model for a 1-DOF AMB system. A detailed introduction of the fuzzy model is given in Section IV. Unfortunately, in Hong's model, approximation accuracy of nonlinear stiffness K_x and K_u, which take an important role in system stability and dynamics, depends on number of operating points selected for fuzzy model. That is, it is difficult to express a nonlinear AMB system precisely by using only several points. However, adding more operating points leads to great computation burden for control algorithms based on LMIs, especially for delay-dependent stabilization criterion proposed in section III. As a result, we develop the follow fuzzy model:

First, we assemble the fuzzy linguistic rules for two special cases:

i) The maximum value of K_u occurs at $u=0$ and $x = x_{\max}$,

ii) The minimum value of K_u occurs at $u=1$ and $x = x_{\max}$,

where $x_{\max}$ is the maximum vibration amplitude of the rotor that is limited by a touchdown bearing.

Remark 1. For the sake of brevity, the above analysis are focused only on the region $x \in [0, x_{\max}]$, and control input u is restricted on $[0,1]$ to keep system stable. The result is also hold for $x \in [-x_{\max}, 0], u \in [-1,0]$ since the dynamics of an AMB system is symmetric about zero displacement.

Choosing the (x,u) as the antecedent variables of the T-S fuzzy model yields:

PLANT RULE 1: IF $K_u(x,u) = PB$

THEN $\dot{X}(t) = A_1 X(t) + B_1 \bar{u}(t-\tau(t))$

PLANT RULE 2: IF $K_u(x,u) = PS$

THEN $\dot{X}(t) = A_2 X(t) + B_2 \bar{u}(t-\tau(t))$

where 'PB' denotes 'Positive Big', 'PS' denotes 'Positive small', $X = [x \quad \dot{x}]^T$, $\bar{u}(t) = u(t) + d_i$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -K_x(x_{\max},1) & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -K_u(x_{\max},1) \end{bmatrix}, \quad d_1 = d(x_m,1)/K_u(x_m,1)$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -K_x(x_{\max},0) & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -K_u(x_{\max},0) \end{bmatrix}, \quad d_2 = d(x_m,0)/K_u(x_m,0)$$

Then the membership function is designed as:

$$h_1(\theta(t)) = \frac{K_u(x_{\max},0) - K_u(x,u)}{K_u(x_{\max},0) - K_u(x_{\max},1)}, \qquad h_2(\theta(t)) = 1 - h_1(\theta(t)), \qquad \theta = [x,u]^T$$

Similar to $h_i(\theta(t))(i=1,2)$, we define $H_i(\theta(t))$ as

$$H_1(\theta(t)) = \frac{K_x(x_{\max},0) - K_x(x,u)}{K_x(x_{\max},0) - K_x(x_{\max},1)}, \qquad H_2(\theta(t)) = 1 - H_1(\theta(t))$$

and (3) can be written as:

$$\dot{X} = \sum_{i=1}^{2}[H_i(\theta(t))A_i X + h_i(\theta(t))B_i \bar{u}(t-\tau(t))] \tag{4}$$

Then let us introduce the function $\delta_i(\theta(t))$ as:

$$\delta_i = \begin{cases} \dfrac{H_i(\theta(t)) - h_i(\theta(t))}{h_i(\theta(t))} & h_i \neq 0 \\ 0 & h_i = 0 \end{cases} \quad \text{(as shown in Fig.2)} \tag{5}$$

Remark 2. It is not hard to verify that $\delta_i(x,u)$ is bounded on $x \in [0, x_{\max}]$, $u \in [0,1]$. That is, there exist constants a_i, b_i ($a_i \le b_i$) such that $\delta_i(t) \in (a_i, b_i)$ under $x \in [0, x_{\max}]$, $u \in [0,1]$. Denote $\delta_i(t) = \rho_i + e_i(t)$ where $\rho_i = (a_i + b_i)/2$. It is obvious that

$$|e_i(t)|^2 \le \rho_i^2 \qquad i = 1,2 \tag{6}$$

Combine (4) and (5), we obtain the final output of the fuzzy system which is structured by a set of linear parameter-varying systems as following:

$$\dot{X} = \sum_{i=1}^{2} h_i(\theta(t))[\bar{A}_i X + B_i \bar{u}(t - \tau(t))] \tag{7}$$

where $\bar{A}_i = (1 + \delta_i(t))A_i$

Remark 3. Besides AMB systems, the fuzzy model design presented above fits other nonlinear systems that satisfy two conditions as follow:

i) One of stiffness functions (e.g K_x, K_u) is bounded on $X \in \Re^n$, $u \in \Re^m$

ii) $|\rho_i| (i = 1,2)$ is small enough to obtain proper control gains by solving LMIs presented in next section.

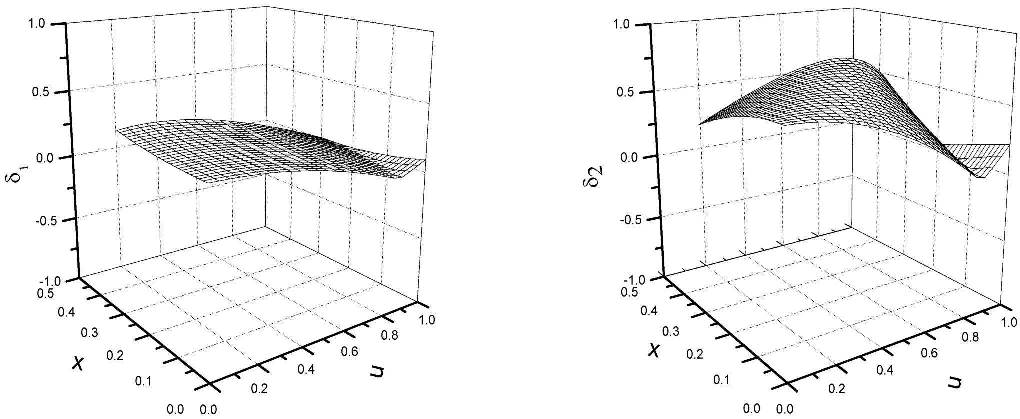

Fig 2: Characteristics of δ_i

Fuzzy Delay-Dependent stabilization

Observer Design. For an AMB system, only the displacement of the rotor is available to measure by using position sensors, while the velocity is not obtainable. Therefore, it is necessary to introduce an observer for unmeasured state $\dot{x}$ and design an output feedback control law. First, submitting (2) to (1) with $\tau(t) = 0$ yields the dimensionless form of an AMB model:

$$\ddot{x} = -\tilde{F}(x,u) = \bar{k}\left[(\frac{1-u}{1-\beta x})^2 - (\frac{1+u}{1+\beta x})^2 \right] \tag{8}$$

Then, a nonlinear observer is given as following:

$$\hat{\dot{x}} = \theta + l \cdot x \tag{9}$$

where l is constant observer gain to be determined and θ satisfies:

$$\dot{\theta} = -l\theta - \tilde{F} - l^2 x \tag{10}$$

Let $e = \dot{x} - \hat{x}$ denote the observation error. Therefore, we have:

$$\begin{aligned} \dot{e} &= (-\tilde{F}) - (\dot{\theta} + l\dot{x}) \\ &= l(\theta + lx) - l\dot{x} \\ &= -le \end{aligned} \tag{11}$$

which implies that the error e converges to zero with a proper gain $l > 0$.

Controller Design. For system (7), based on the parallel distributed compensation (PDC), the following fuzzy control law is employed to deal with the problem of delay-dependent stabilization. The PDC rules share the same fuzzy sets with the antecedent parts of model (7) and can be represented as:

$$\bar{u}(t) = \sum_{j=1}^{2} h_j(\mu(t)) K_j \hat{X}(t) \tag{12}$$

with $\hat{X} = [x \quad \hat{x}]^T$

For convenience, let $h_i = h_i(\theta(t))$, $h_j(\tau) = h_j(\theta(t-\tau))$ and define $X_s = [X^T \quad e]^T$ and

$$A_{is} = \begin{bmatrix} \bar{A}_i & 0 \\ 0 & -l \end{bmatrix}, \quad B_{is} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad K_{js} = [K_j \quad -K_j C], \quad C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then, associated with the control law (12), AMB system can be expressed in the descriptor form [20] as follows:

$$E\dot{\bar{X}}(t) = \begin{bmatrix} Y(t) \\ -Y(t) + \sum_{i,j=1}^{2} h_i h_j(\tau)[(\bar{A}_{is} + B_{is}K_{js})X_s(t) - B_i K_j \int_{t-\tau_0}^{t} Y(s)ds - B_i K_j \int_{t-\tau_0-\varepsilon(t)}^{t-\tau_0} Y(s)ds] \end{bmatrix} \tag{13}$$

where $Y(t) = \dot{X}_s(t), \bar{X}(t) = \begin{bmatrix} X_s(t) \\ Y(t) \end{bmatrix}, E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\varepsilon(t) = \tau(t) - \tau_0 \in (-\sigma, \sigma)$

By combining the descriptor form (13) and the corresponding LKF:

$$\begin{aligned} V(t) &= \bar{X}^T(t)EP\bar{X}(t) + \int_{t-\tau_0}^{t} X_s^T(s)\tilde{Q}X_s(s)ds + \int_{-\tau_0}^{0} ds \int_{t+s}^{t} Y^T(\theta)\tilde{R}_1 Y(\theta)d\theta \\ &+ \int_{-\tau_0-\sigma}^{-\tau_0+\sigma} ds \int_{t+s}^{t} Y^T(\theta)\tilde{R}_2 Y(\theta)d\theta \end{aligned} \tag{14}$$

we obtain sufficient delay-dependent conditions in the form of LMIs as follow:

lemma 1. Suppose that τ_0, σ are two given positive scalars, where $\tau_0 > \sigma$. System (13) with is asymptotic stable for any $\tau(t) \in [\tau_0 - \sigma, \tau_0 + \sigma]$, if there exist a scalar $\mu > 0$, and common matrices $\tilde{Q} > 0$, $\tilde{R}_1 > 0$, $\tilde{R}_2 > 0$, $\tilde{N} = [\tilde{N}_1 \quad \tilde{N}_2]$, $P = \begin{bmatrix} P_1 & 0 \\ P_2 & \mu P_2 \end{bmatrix}, P_1 > 0$ such that the following LMI (15) and (16) hold:

$$\begin{bmatrix} P_2^T A_{is} + A_{is}^T P_2 + \tilde{Q} + \tilde{N}_1 + \tilde{N}_1^T + \tau_0 \tilde{Z}_1 & P_1 - P_2^T + A_{is}^T P_3 + \tilde{N}_2 + \tau_0 \tilde{Z}_2 & P_2^T B_{is} K_{js} - \tilde{N}_1^T & \sigma P_2^T B_{is} K_{js} \\ * & -P_3 - P_3^T + \tau_0 \tilde{Z}_3 + \tau_0 \tilde{R}_1 + 2\sigma \tilde{R}_2 & \mu P_2^T B_{is} K_{js} - \tilde{N}_2^T & \mu\sigma P_2^T B_{is} K_{js} \\ * & * & -\tilde{Q} & 0 \\ * & * & * & -\sigma \tilde{R}_2 \end{bmatrix} < 0 \tag{15}$$

$$i, j = 1, 2, 3$$

$$\begin{bmatrix} \tilde{R}_1 & \tilde{N} \\ \tilde{N}^T & \tilde{Z} \end{bmatrix} \ge 0 \tag{16}$$

Proof: Proof is similar to [21,theorem 1],

Since A_{is}, K_{js} actually contains the designed variables K_j and l, we next transform (15) (16) to decoupled LMIs which gives the computation of K_j and l.

Proposition 1: Suppose that $\rho_i\,(i=1,2)$, τ_0 and σ are given positive scalars and $\tau_0>\sigma$. If there exist scalars $\mu>0$, $d_{1i}>0$, $d_{2i}>0$, $g\neq 0$, $q>0$, $p_1>0$, $r_1>0$, $r_2>0$, z_1,z_2,z_3,n_1,n_2 and common matrices $Q>0$, $R_1>0$, $R_2>0$, $M>0$, V, L_j (j=1,2), $N=[N_1 \quad N_2]$, $Z=\begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}$, such that the following LMI (17), (18)and (19) hold:

$$\begin{bmatrix} \multicolumn{2}{c}{\Psi_{ij}} & \begin{matrix} B_iL_j-N_1^T \\ \mu B_iL_j-N_2^T \end{matrix} & \begin{matrix} \sigma B_iL_j \\ \mu\cdot\sigma B_iL_j \end{matrix} & \begin{matrix} V^T \\ 0 \end{matrix} & \begin{matrix} \mu V^T \\ 0 \end{matrix} \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & -\sigma R_2 & 0 & 0 \\ * & * & * & * & -d_{1i}I & 0 \\ * & * & * & * & 0 & -d_{2i}I \end{bmatrix}<0 \tag{17}$$

$$i,j=1,2$$

$$\begin{bmatrix} -2\varphi+q+2n_1+\tau_0 z_1 & p_1-g+\mu\varphi+n_2+\tau_0 z_2 & 0 & 0 \\ * & -2\mu g+\tau_0 z_3+\tau_0 z_1+2\sigma r_2 & 0 & 0 \\ * & * & -q & 0 \\ * & * & * & -\sigma r_2 \end{bmatrix}\leq 0 \tag{18}$$

$$\begin{bmatrix} R_1 & N \\ * & Z \end{bmatrix}\geq 0 \tag{19}$$

$$\begin{bmatrix} r_1 & n_1 & n_2 \\ * & z_1 & z_2 \\ * & * & z_3 \end{bmatrix}\geq 0 \tag{20}$$

where

$$\Psi_{ij}=\begin{bmatrix} (1+\rho_i)A_iV+V^T(1+\rho_i)A_i^T+Q+N_1+N_1^T+\tau_0 Z_1+d_{1i}\rho_i^2A_i^TA_i & M-V+\mu V^T(1+\rho_i)A_i^T+N_2+\tau_0 Z_2 \\ * & -\mu V-\mu V^T+\tau_0 Z_3+\tau_0 R_1+2\sigma R_2+d_{2i}\rho_i^2A_i^TA_i \end{bmatrix}$$

then under the control law (12) with feedback gains K_j and observer gain l given by

$$K_j=L_jV^{-1}, \qquad l=\varphi/g \tag{21}$$

the fuzzy system (7) is asymptotically stable for any time delay $\tau(t)\in[\tau_0-\sigma,\tau_0+\sigma]$.

Proof. Assume a scalar $\gamma>0$,letting

$P_2=\begin{bmatrix} G & 0 \\ 0 & \gamma g \end{bmatrix}$, $\tilde{Q}=\begin{bmatrix} \tilde{Q}_1 & \tilde{Q}_2 \\ * & \gamma q \end{bmatrix}$, $P_1=\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ * & \gamma p_1 \end{bmatrix}$, $\tilde{R}_1=\begin{bmatrix} \tilde{R}_{11} & 0 \\ 0 & \gamma r_1 \end{bmatrix}$ $\tilde{R}_{21}>0$ $\tilde{R}_2=\begin{bmatrix} \tilde{R}_{21} & \tilde{R}_{22} \\ * & \gamma r_2 \end{bmatrix}$, $\tilde{N}_1=\begin{bmatrix} \tilde{N}_{11} & 0 \\ 0 & \gamma n_1 \end{bmatrix}$,

$\tilde{N}_2=\begin{bmatrix} \tilde{N}_{21} & 0 \\ 0 & \gamma n_2 \end{bmatrix}$, $\tilde{Z}_1=\begin{bmatrix} \tilde{Z}_{11} & 0 \\ 0 & \gamma z_1 \end{bmatrix}$, $\tilde{Z}_2=\begin{bmatrix} \tilde{Z}_{21} & 0 \\ 0 & \gamma z_2 \end{bmatrix}$, $\tilde{Z}_3=\begin{bmatrix} \tilde{Z}_{31} & 0 \\ 0 & \gamma z_3 \end{bmatrix}$

where G is nonsingular and $g\neq 0$.

and submitting into (15) and after exchanges of rows and columns, yields the following matrix:

$$\begin{bmatrix} \Delta_{ij} & \Gamma_{ij} \\ * & \gamma\Lambda \end{bmatrix}<0 \tag{22}$$

where

$$\Delta_{ij}=\begin{bmatrix} G^T\bar{A}_i+G_i^TT+\tilde{Q}_1+\tilde{N}_{11}+\tilde{N}_{11}^T+\tau_0\tilde{Z}_{11} & \tilde{P}_{11}-G^T+\mu\bar{A}_i^TG+\tilde{N}_{21}+\tau_0\tilde{Z}_{21} & G^TB_iK_j-\tilde{N}_{11}^T & \sigma G^TB_iK_j \\ * & -\mu G-\mu G^T+\tau_0\tilde{Z}_{31}+\tau_0\tilde{R}_{11}+2\sigma\tilde{R}_{21} & \mu G^TB_iK_j-\tilde{N}_{21}^T & \mu\sigma G^TB_iK_j \\ * & * & -\tilde{Q}_1 & 0 \\ * & * & * & -\sigma\tilde{R}_{21} \end{bmatrix}$$

,

$$\Lambda=\begin{bmatrix} -2gl+q+2n_1+\tau_0z_1 & p_1-g+\mu gl+n_2+\tau_0z_{21} & 0 & 0 \\ * & -2\mu g+\tau_0z_3+\tau_0z_1+2\sigma r_2 & 0 & 0 \\ * & * & -q & 0 \\ * & * & * & -\sigma r_2 \end{bmatrix}$$

$$\Gamma_{ij}=\begin{bmatrix} \tilde{Q}_2 & P_{12} & G^TB_iK_jC & \sigma G^TB_iK_jC \\ * & 2\sigma\tilde{R}_{22} & \mu G^TB_iK_jC & \mu\sigma G^TB_iK_jC \\ * & * & -\tilde{Q}_2 & 0 \\ * & * & * & -\sigma\tilde{R}_{22} \end{bmatrix}$$

It is easy to verify that (22) holds for sufficiently small $\gamma>0$ when $\Delta_{ij}<0$ and $\Lambda\le 0$.First, assume that Δ_{ij} is negative definite:

$$\begin{bmatrix} G^T\bar{A}_i+G_i^TT+\tilde{Q}_1+\tilde{N}_{11}+\tilde{N}_{11}^T+\tau_0\tilde{Z}_{11} & \tilde{P}_{11}-G^T+\mu\bar{A}_i^TG+\tilde{N}_{21}+\tau_0\tilde{Z}_{21} & G^TB_iK_j-\tilde{N}_{11}^T & \sigma G^TB_iK_j \\ * & -\mu G-\mu G^T+\tau_0\tilde{Z}_{31}+\tau_0\tilde{R}_{11}+2\sigma\tilde{R}_{21} & \mu G^TB_iK_j-\tilde{N}_{21}^T & \mu\sigma G^TB_iK_j \\ * & * & -\tilde{Q}_1 & 0 \\ * & * & * & -\sigma\tilde{R}_{21} \end{bmatrix}<0 \quad (23)$$

Let $V=G^{-1}$, pre- and postmultiply $diag\{V^T \quad V^T \quad V^T \quad V^T\}$ and its transpose to (23), and apply the change of variables such that

$K_jV=L_j$, $V^T\tilde{P}_{11}V=M$, $V^T\tilde{Q}_1V=Q$, $V^T\tilde{R}_{11}V=R_1$, $V^T\tilde{N}_{11}V=N_1$, $V^T\tilde{N}_{21}V=N_2$, $V^T\tilde{Z}_{11}V=Z_1$, $V^T\tilde{Z}_{21}V=Z_2$, $V^T\tilde{Z}_{31}V=Z_3$ to get the following inequality:

$$\begin{bmatrix} \bar{A}_iV+V^T\bar{A}_i^T+Q+N_1+N_1^T+\tau_0Z_1 & M-V+\mu V^T\bar{A}_i^T+N_2+\tau_0Z_2 & B_iL_j-N_1^T & \sigma B_iL_j \\ * & -\mu V-\mu V^T+\tau_0Z_3+\tau_0R_1+2\sigma R_2 & \mu B_iY_j-N_2^T & \mu\cdot\sigma B_iL_j \\ * & * & -Q & 0 \\ * & * & * & -\sigma R_2 \end{bmatrix}<0 \quad (24)$$

Replace $\bar{A}_i$ with $\bar{A}_i=[1+\delta_i(t)]A_i=[1+\rho_i+e_i(t)]A_i$ and multiply both sides of (24) by vectors x_i ($i=1,2,3,4$).If we define $q_1=e_i(t)A_i^Tx_1$, $q_2=e_i(t)A_i^Tx_2$, then we have the following inequality:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ q_1 \\ q_2 \end{bmatrix}^T \begin{bmatrix} \multicolumn{2}{c}{\Xi_{ij}} & B_iL_j-N_1^T & \sigma B_iL_j & V^T & \mu V^T \\ & & \mu B_iY_j-N_2^T & \mu\cdot\sigma B_iL_j & 0 & 0 \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & -\sigma R_2 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ q_1 \\ q_2 \end{bmatrix}<0 \quad (25)$$

where $\Xi_{ij}=\begin{bmatrix} (1+\rho_i)A_iV+V^T(1+p_i)A_i^T+Q+N_1+N_1^T+\tau_0Z_1 & M-V+\mu V^T(1+\rho_i)A_i^T+N_2+\tau_0Z_2 \\ * & -\mu V-\mu V^T+\tau_0Z_3+\tau_0R_1+2\sigma R_2 \end{bmatrix}$

Since the conditions (6) can be replaced with the existence condition $d_{1i}>0$ and $d_{2i}>0$ such that

$$d_{1i} q_1^T q_1 \le d_{1i} \cdot \rho_i^2 x_1 A_i^T A_i x_1 , \qquad d_{2i} q_2^T q_2 \le d_{2i} \cdot \rho_i^2 x_2 A_i^T A_i x_2 \tag{26}$$

Applying the Schur complement to (25) results the first LMI (17)in *Proposition 1*.

The inequality (18) in *Proposition 1* can be established by assuming Λ is negative semidefinite:

$$\begin{bmatrix} -2gl + q + 2n_1 + \tau_0 z_1 & p_1 - g + \mu gl + n_2 + \tau_0 z_{21} & 0 & 0 \\ * & -2\mu g + \tau_0 z_3 + \tau_0 z_1 + 2\sigma r_2 & 0 & 0 \\ * & * & -q & 0 \\ * & * & * & -\sigma r_2 \end{bmatrix} \le 0 \tag{27}$$

and letting $gl = \varphi$.

Finally rewriting LMI (19) as:

$$\begin{bmatrix} \tilde{R}_{11} & 0 & \tilde{N}_{11} & 0 & \tilde{N}_{21} & 0 \\ * & \gamma r_1 & 0 & \gamma n_1 & 0 & \gamma n_2 \\ * & * & \tilde{Z}_{11} & 0 & \tilde{Z}_{21} & 0 \\ * & * & * & \gamma z_1 & 0 & \gamma z_2 \\ * & * & * & * & \tilde{Z}_{31} & 0 \\ * & * & * & * & * & \gamma z_3 \end{bmatrix} > 0 \tag{28}$$

and pro- and post both sides of (28) by $diag[V^T \quad 1/\sqrt{\gamma} \quad V^T \quad 1/\sqrt{\gamma} \quad V^T \quad 1/\sqrt{\gamma}]$, and after exchanges of rows and columns, yields the LMI (19) and (20).This completes the proof.

By using the above results, a stabilizing PDC-type controller can be obtained directly by solving those LMIs numerically using the interior-point algorithm[22].

Simulation

To verify the effectiveness of the proposed controller, numerical simulation is carried out. Since the AMB-rotor system (1) usually operates at a high rotation speed, the harmonic disturbance created by imbalance of the rotor is considered in this simulation with the form:

$$w(T) = mc_e \Omega^2 \cos(\Omega T) \tag{29}$$

where c_e and Ω are the eccentricity and rotation speed of the rotor, respectively.

The model parameters are selected as

$$k = 0.00186 lb \cdot in^2 / amp^2 , \quad \beta = 0.974 , \quad I_0 = 0.3 amp , \quad c_0 = 0.02 in , m = 0.0126 lb \cdot \sec^2 / in .$$

Note that if $\Delta T = 0$, then the system (1) is the same as one given in [19]. According to [19], the fuzzy model for an AMB system is also constructed by linearizing (1) at several operation points as described in Section II:

PLANT RULE i: IF $x = C_1^i$ and $u = C_2^i$

THEN $\dot{x}(t) = \tilde{A}_i x + \tilde{B}_i \tilde{u} \quad i = 1,2,3,4,5$ (30)

where $C_1^1 = ZE$, $C_1^2 = C_1^4 = PM$, $C_1^3 = C_1^5 = PB$, $C_2^2 = C_2^4 = ZE$, $C_2^3 = C_2^5 = PB$, $\tilde{u}(t) = u(t) + K_{0i}$, and $B_i K_{0i}$ is a bias term resulted from model linearization.

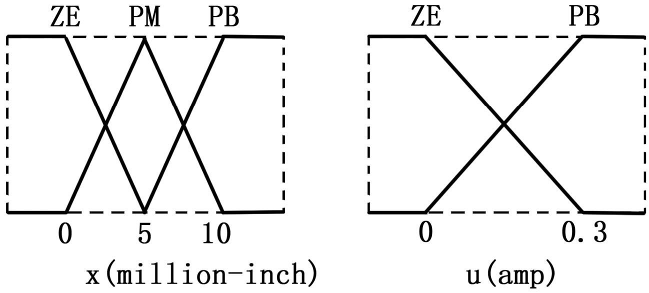

Fig 2: Membership Function

Differently from (7), the membership function in [19] is designed as follow:

$$g_i(\mu(t)) = \frac{\Pi(\mu_{ix}(x)\cdot\mu_{iu}(u))}{\sum_{i=1}^{r}\Pi(\mu_{ix}(x)\cdot\mu_{iu}(u))} \tag{31}$$

where $\mu_i\cdot(\cdot)$ is the grade of membership of the input variables as shown in Fig 3.

It was found that in the case of rotational speed $\Omega = 600 rad/s$ and $c_e = 0.001in$, the close-loop stability is guaranteed by the control law in [19] even when delay $\Delta T = 3ms$, which indicates that the effect of delay on system stability can be neglected when rotation speed is low. However, in the case of $\Omega = 1100 rad/s$, the rotor lose stability when the delay is larger than $0.12ms$,about 2.2% of rotor vibration period. Moreover, with increase of rotation speed, the stability limit of time delay decreases dramatically. This emphasizes the necessity of taking a consideration of time delay when designing a controller for a high-speed AMB system.

To furthermore demonstrate the advantage of the fuzzy model proposed in this paper, we apply the delay-dependent criterions proposed in Section III to fuzzy model (30) as a comparison.

Setting $A_{is} \equiv \tilde{A}_i$, $B_{is} \equiv \tilde{B}_i$, $K_{js} \equiv \tilde{K}_j$ and solving the inequalities (15) and (16) with $\Omega_0 = 209.4 rad/s$, $\mu = 0.3$, $\tau_0 = \sigma = 0.0523$ (the maximum delay determined by *lemma 1* is $(\tau_0+\sigma)/\Omega_0 = 0.5ms$), yields a fuzzy controller based on model (30):

$$\tilde{u}(t) = \sum_{j=1}^{5} g_j(\mu(t))\tilde{K}_j X(t) \tag{32}$$

with dimensionless gains

$$\tilde{K}_1 = [7.4456 \quad 4.1840], \tilde{K}_2 = [7.3495 \quad 4.1276], \tilde{K}_3 = [8.3809 \quad 4.6622],$$
$$\tilde{K}_4 = [12.5559 \quad 6.9962], \tilde{K}_5 = [16.3971 \quad 9.2713]$$

Finally by using *Proposition 1* with $\mu = 0.3$, $\rho_1 = 0.1884$, $\rho_2 = 0.3726$, $\tau_0 = \sigma = 0.052$ (the maximum delay is also $(\tau_0+\sigma)/\Omega_0 = 0.5ms$) ,we obtain the control gain and observer gain for controller (12):

$$K_1 = [3.2716 \quad 1.9677], \quad K_2 = [14.2761 \quad 8.0413] \quad l = 32.3 \ .$$

The simulation is carried out with the initial conditions $q_0 = 0.0002in$. Fig.4 shows the transient responses of rotor trajectories in the case of $\Delta T = 0.12ms$ under three controllers mentioned above: controllers proposed in [19] without considering time delay, delay-dependent controller given in (32) based on fuzzy model (30)and output-feedback controller (12). It is obvious that controllers in both (32) and (12) can stabilize the system when the stability limit of time delay is exceeded for the controller presented in [19]. Furthermore, based on the new fuzzy model (7), controller (12) indicates a better performance from not only the overshot but also the set time.

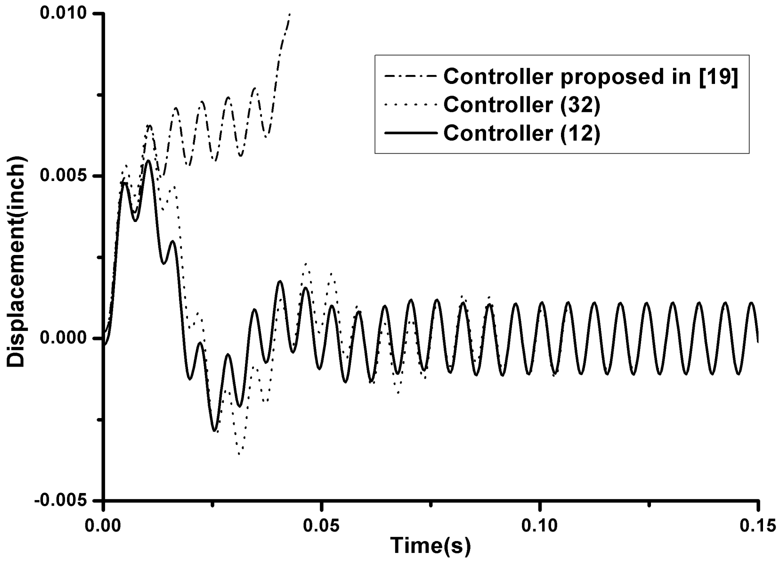

Fig. 4: The trajectories of the rotor motion for three controllers

It has been found that for controller (32), although delay-dependent criterion is applied, the system becomes unstable when delay reaches 0.43ms, less than the designed maximum delay. However, the close-loop stability is guaranteed by the controller (12) for any delay $\Delta T \le 0.5ms$. The responses of two control laws for the case $\Delta T = 0.5ms$ are shown in Fig.5.

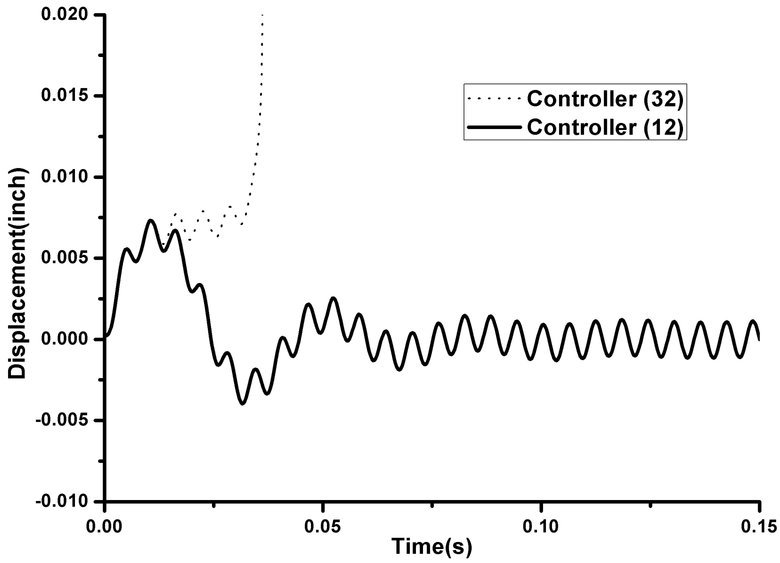

Fig. 5: The trajectories of the rotor motion in the case of $\Delta T = 0.5ms$

Conclusion

In this paper, the time delay problem in a nonlinear AMB system is proposed, and a stabilizing fuzzy controller is designed to decrease the negative effect of the delay. The nonlinear magnetic bearing is represented by a new Takagi-Sugeno fuzzy model, which is structured by a set of linear parameter-varying systems and provides a good approximation on stiffness functions. Then a fuzzy-model-based PDC controller is designed in terms of a proposed delay-dependent stabilization criterion which guarantees the asymptotic stability of the fuzzy model. The simulation results show that the designed controller provides not only a wider range of stability boundary but also better performances. This indicates the effectiveness of the proposed method.

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