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A NOVEL PASSIVITY BASED CONTROL WITHOUT CONVENTIONAL CROSS-FEEDBACK

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ABSTRACT

This paper discusses a passivity based control without conventional cross-feedback which is one of the nonlinearity canceling. The obtained controllers have NO canceling terms of the gyroscopic effect. First we discuss the modeling and clarify some important properties of the flywheel. Second, we discuss some PID (PID-type) controllers from the viewpoint of passivity. Finally we give some simulation and experimental results.

INTRODUCTION

This paper discusses a passivity based control without conventional cross-feedback. The cross-feedback is one of the (so called) nonlinearity canceling methodologies which make it possible to apply many fruitful results from linear systems and control theory.

On the other hand, the passivity based control is still the one of the recent trends of the nonlinear control. This approach utilizes the physical or structural properties for stabilization or tracking (as well as modeling) without or with less nonlinearity canceling. In our case, this nonlinearity is nothing else the gyroscopic effect.

In this paper, first we discuss the modeling and clarify some important properties of the flywheel. Second, we discuss some PID (PID-type) controllers from the viewpoint of passivity. These controllers have no canceling terms of the gyroscopic effect. Finally we give some simulation and experimental results and conclude this paper.

PRELIMINARY

In this section, we give some definitions of basic concepts, such as passivity, port-Hamiltonian systems and their properties.

Passivity

The concept of passivity is not quite new and the passivity based control can be traced back at least to the work of Takegaki and Arimoto [1]. However, this methodology is recently developed in a new frame work and more fruitful results are obtained.

Let us consider a finite-dimensional linear space U and let the output space Y be the *dual space* U^*. Denote the duality product (*power*) between U and $U^* = Y$ by $< y|u >$ for $y \in U^*$ and $u \in U$. $< y|u >$ is the linear function $y : U \to R$ evaluated in $u \in U$.

Furthermore, take any linear space of functions $u : \mathbf{R}^+ \to U$ denoted by $L(U)$, and any linear space of functions $y : \mathbf{R}^+ \to Y = U^*$, denoted by $L(U^*)$. Define a duality pairing (*supplied energy*)

$$< y|u >_T = \int_0^{\mathrm{T}} < y(t)|u(t) > dt \quad (1)$$

between $L_e(U)$ and $L_e(U^*)$ by defining for $u \in L_e(U)$ (i.e. u_T, the trancation of u to the interval $[0, T]$, exists in $L(U)$ for all $T \geq 0$), $y \in L_e(U^*)$, assuming that integral on the right-hand side exists.

Definition 1 Let $G : L(U) \to L(U^*)$. Then G is passive if there exists some constant β such that

$$< G(u)|u >_T \geq -\beta \quad (2)$$

for any $u \in L(U)$ and any $T \geq 0$, where it is assumed that the left-hand side is well-defined. Intuitively, the maximally extractable energy is bounded by a finite constant β. G is passive iff only a finite amount of energy can be extracted from the system G.

Port-Hamiltonian systems

This section refers to the port-Hamiltonian systems [5] and the generalized canonical transformation (g.c.t.) [4],[3].

Definition 2 A (simplified version) port-Hamiltonian system with a Hamiltonian $H(x) \in \mathbf{R}$ is a system described by

$$\begin{cases} \dot{x} &= J(x)\dfrac{\partial H(x)}{\partial x}^{\mathrm{T}} + g(x)u \\ y &= g(x)^{\mathrm{T}}\dfrac{\partial H(x)}{\partial x}^{\mathrm{T}} \end{cases} \tag{3}$$

with $u, y \in \mathbf{R}^m$, $x \in \mathbf{R}^n$ and a skew symmetric matrix $J(x)$, i.e. $-J(x) = J(x)^{\mathrm{T}}$ holds. The following property of such systems is known.

Lemma 1 [5] Consider the port-Hamiltonian system (3). Suppose the Hamiltonian $H(x)$ satisfies $H(x) \geq H(0) = 0$. Then the input-output mapping $u \mapsto y$ of the system is passive with respect to the storage function H, and the feedback

$$u = -C(x)\ y \tag{4}$$

with a matrix $C(x) \geq \varepsilon I > 0 \in \mathbf{R}^{m\times m}$ renders $(u, y) \to 0$. Furthermore if $H(x)$ is positive definite and if the system is zero-state detectable, then the feedback (4) renders the origin asymptotically stable.

The zero-state detectability and the positive definiteness of the Hamiltonian assumed in Lemma 1 do not always hold for general port-Hamiltonian systems. In such a case, the generalized canonical transformation is useful. A generalized canonical transformation (g.c.t.)[4] is a set of coordinate transformations and feedback transformations

$$\begin{aligned} \bar{x} &= \Phi(x) \\ \bar{H} &= H(x) + U(x) \\ \bar{y} &= y + \alpha(x) \\ \bar{u} &= u + \beta(x) \end{aligned} \tag{5}$$

which preserves the structure of the port-Hamiltonian system given in (3). $\bar{x}$, $\bar{H}$, $\bar{y}$ and $\bar{u}$ denote the new state, the new Hamiltonian, the new output and the new input respectively. The properties of such transformations and how to utilize them for stabilization are summarized as follows:

Lemma 2 (g.c.t.) [4] (i) Consider the port-Hamiltonian system (3). For any functions $U(x) \in \mathbf{R}$ and $\beta(x) \in \mathbf{R}^m$, there exists a pair of functions $\Phi(x) \in \mathbf{R}^n$ and $\alpha(x) \in \mathbf{R}^m$ such that the set (5) yields a generalized canonical transformation. Any coordinate transformation $\Phi(x)$ yields a generalized canonical transformation if and only if

$$J(x)\frac{\partial U}{\partial x}^{\mathrm{T}} + K(x)\frac{\partial H + U}{\partial x}^{\mathrm{T}} + g(x)\beta(x) = 0 \tag{6}$$

holds with an arbitrary skew-symmetric matrix $K(x) \in \mathbf{R}^{n\times n}$. Further the change of output α and the matrices $\bar{J}$ and $\bar{g}$ are given by

$$\alpha(x) = g(x)^{\mathrm{T}}\frac{\partial U(x)}{\partial x}^{\mathrm{T}} \tag{7}$$

$$\bar{J}(\bar{x}) = \left.\frac{\partial \Phi(x)}{\partial x}(J(x) + K(x))\frac{\partial \Phi(x)}{\partial x}^{\mathrm{T}}\right|_{x=\Phi^{-1}(\bar{x})} \tag{8}$$

$$\bar{g}(\bar{x}) = \left.\frac{\partial \Phi(x)}{\partial x}g(x)\right|_{x=\Phi^{-1}(\bar{x})}. \tag{9}$$

and the new input-output mapping $\bar{u} \mapsto \bar{y}$ is passive with respect to the new storage function $\bar{H}(\bar{x}) \geq 0$.

Using the generalized canonical transformation, we can change the property of the system without changing the inherent generalized Hamiltonian structure with passivity, and we can convert the system into several convenient forms.

Theorem 1 (A relation between Casimir function and g.c.t.) *Suppose that the port-Hamiltonian system has Casimir function with respect to J, that is,*

$$\frac{\partial C(x)}{\partial x}J = 0 \tag{10}$$

holds for the input $u = 0$. Then, the following transformation

$$U(x) = H_c(C(x)), \alpha = \beta = 0 \tag{11}$$

satisfies the condition (6) in Lemma 2 for any function H_c.

Proof of Theorem 1 The dynamics after the transformation is

$$\begin{aligned} \dot{x} &= J(x)\frac{\partial (H + H_c(C))}{\partial x} \\ &= J(x)\frac{\partial H}{\partial x}^{\mathrm{T}} + \left(-\frac{\partial C}{\partial x}J(x)\right)^{\mathrm{T}}\frac{\partial H_c(C)}{\partial C}^{\mathrm{T}} \qquad (12) \\ &= J(x)\frac{\partial H}{\partial x}^{\mathrm{T}} \qquad (13) \end{aligned}$$

and equivalent to the original one. There is no need to apply feedback transformation. (Q.E.D.)

MODELING

In this section, we discuss modeling of the flywheel system and its some properties. The state-space

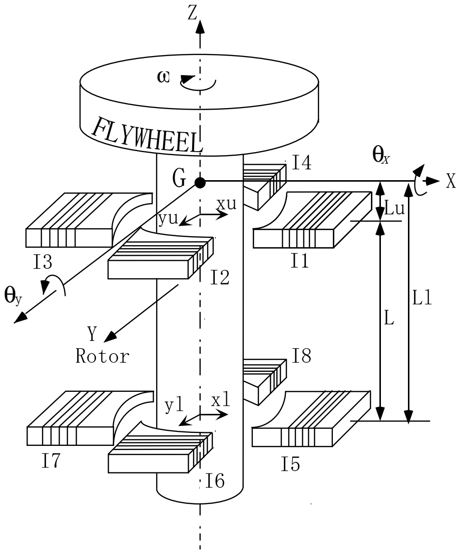

Figure 1: Flywheel.

equation of the flywheel is usually given as

$$m\ddot{x} = u_1 \tag{14}$$
$$m\ddot{y} = u_2 \tag{15}$$
$$I_d\ddot{\theta}_x = +\omega_3 I_p \dot{\theta}_y + u_3 \tag{16}$$
$$I_d\ddot{\theta}_y = -\omega_3 I_p \dot{\theta}_x + u_4 \tag{17}$$

where m is the rotor mass, I_d, I_p is the rotor inertia, x, y are the rotor (the center of gravity) displacements, θ_x, θ_y are the rotor angles, $u_i (i = 1, ..., 4)$ is the control inputs. Fig. 1 shows the (normal) coordinates of the flywheel.

Remark The third and the forth equations (16), (17) are the approximation in the following sense at least. First, the equations are not the rigid body which should be discussed not on $\mathbf{R}^3$ but on SO(3) or should be discussed as the form

$$\begin{bmatrix} \dot{\rho} \\ \dot{M_3\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(I_3 + S_3(\rho) + \rho\rho^{\mathrm{T}}) \\ -S_3(\omega)M_3\omega \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{18}$$

where $\rho = (\rho_1, \rho_2, \rho_3) \in \mathbf{R}^3$ is from Euler parameters, $\omega = (\omega_1, \omega_2, \omega_3) \in \mathbf{R}^3$,

$$S_3(\omega) = \begin{bmatrix} 0 & +\omega_3 & -\omega_2 \\ -\omega_3 & 0 & +\omega_1 \\ +\omega_2 & -\omega_1 & 0 \end{bmatrix}, M_3 = \mathrm{diag}(I_1, I_2, I_3) \tag{19}$$

and $u = (u_3, u_4)$.

Second the input u_3 and u_4 depend on the flywheel orientation since they are the torques in the body coordinate frame and the above equation (18) has

$$u = R(\rho_1, 0, 0)R(0, \rho_2, 0)R(0, 0, \rho_3)\ {}^{E}u \tag{20}$$

where ${}^{E}u \in \mathbf{R}^3$ is the torque (calculated by AMB forces) in the inertia coordinate frame, $R(\bullet) \in \mathbf{R}^{3\times 3}$ is some orientation matrix. This means that $\dot{\omega}_3$ always depends on input u through equation (20) and ω_3-dynamics should not be reduced as long as the rotor orientation is vertical exactly.

Theorem 2 (modeling) Consider the coordinate transformation

$$\left\{ \begin{array}{lcl} q & = & \theta \\ p & = & M\dot{\theta} \end{array} \right. \tag{21}$$

where $\theta = (\theta_x, \theta_y) \in \mathbf{R}^2$ and $M = \mathrm{diag}(I_d, I_d) \in \mathbf{R}^{2\times 2}$ and take the output ω for the system.

Then, equations (16) (17) are modeled in the port-Hamiltonian system

$$\Sigma_{fw} \left\{ \begin{array}{lcl} \dot{x} & = & \begin{bmatrix} 0 & I_2 \\ -I_2 & S_2 \end{bmatrix} \frac{\partial H}{\partial x}^{\mathrm{T}} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \\ y & = & \frac{\partial H}{\partial p}^{\mathrm{T}} \end{array} \right. \tag{22}$$

where $x = (q, p) \in \mathbf{R}^4$, $H = (1/2)p^{\mathrm{T}}M^{-1}p$ and

$$S_2(\omega_3) = \begin{bmatrix} 0 & +\omega_3 I_p \\ -\omega_3 I_p & 0 \end{bmatrix}. \tag{23}$$

Proof of Theorem 2 Equations (22) can be derived by a direct calculation. (Q.E.D.)

Note that the function H is bounded from below and a positive definite function. S_2 is a skew-symmetric matrix, that is, $S_2^{\mathrm{T}} = -S_2$ holds for any ω_3.

From Lemma 1, the input-output mapping $u \to y$ of the system (4) is passive (lossless) with respect to the storage function H, that is,

$$\dot{H} = y^{\mathrm{T}}u. \tag{24}$$

Furthermore, the feedback

$$u = -C(x)y \tag{25}$$

with a positive definite matrix $C(x)(> 0)$ renders $(u, y) \to 0$.

As the rigid body system, the following (important) result holds.

Theorem 3 (zero-state observability) *The input-output map of the system (22) is zero-state observable.*

Proof of Theorem 3 With the input $u = 0$,
$y \equiv 0 \Leftrightarrow \omega \equiv 0 \Rightarrow \dot{\omega} \equiv 0 \Rightarrow q \equiv p \equiv 0$. (Q.E.D.)

As for the first and the second equations (14) (15), the same procedure can be applied straightforwardly. In this paper, the control problem of (14) (15) is not stated since they are decoupled from equations (16) (17) and easier (i.e. just corresponding to the case of $S_2 \equiv 0$)

CONTROL

In this section, we discuss some PID controls without conventional cross-feedback based on the previous sections.

PD Control without cross-feedback against the gyroscopic effect

First, as the simplest case, we discuss the global stability of PD control for the flywheel. No that there is no cross feedback in the controllers.

Lemma 5 Consider the system Σ_{fw} and the following (nonlinear) controller

$$u = -\frac{\partial U(q)}{\partial q} - Cy. \tag{26}$$

with a (radially unbounded) positive definite function U and any positive definite matrix $C > 0$. Then the equilibrium set of the closed-loop system contains only the origin and it is (globally) asymptotically stable.

Remark Note that there is NO closs-feedback term as

$$u = -\frac{\partial U(q)}{\partial q} - Cy - \begin{bmatrix} 0 & +\omega_3 I_p \\ -\omega_3 I_p & 0 \end{bmatrix} \begin{bmatrix} \dot\theta_x \\ \dot\theta_y \end{bmatrix} \tag{27}$$

in equation (26), that is, the gyroscopic effect is not canceled any more. The common (joint-independent) PD controller is a special case of the above controller. Indeed, If $U = (1/2)q^{\mathrm T}K_q q$ and $C = \bar{C}M$ where $K_q, \bar{C}$ are (positive definite) diagonal matrices, then

$$u = -\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} - \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \dot\theta_x \\ \dot\theta_y \end{bmatrix} \tag{28}$$

is derived directly. In equation (28), the gain parameters do not have to be constant and can depend on ω_3.

Proof of Lemma 5 Since the first term satisfies the condition in Lemma 2, the system is again port-Hamiltonian system. By applying the first term of the controller, the original system is converted to

$$\begin{cases} \begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0 & I_2 \\ -I_2 & S_2 \end{bmatrix} \begin{bmatrix} \frac{\partial(H+U)}{\partial q}^{\mathrm T} \\ \frac{\partial(H+U)}{\partial p}^{\mathrm T} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \bar u \\ \bar y = \frac{\partial(H+U)}{\partial p}^{\mathrm T} \end{cases} \tag{29}$$

where $H = (1/2)p^{\mathrm T}M^{-1}p + U(q)$ is the total energy with virtual potential energy U. By applying the second term as in Lemma 1, the state converses to the input-output nilling space

$$\Omega_0 \equiv \{ \bar y = \bar u = 0\} \tag{30}$$

From Theorem 2, the state also converses to $\Omega_1 \equiv \{q \equiv 0\}$, that is, the origin since $y = \bar y$. It is easily shown that this origin is globally asymptotic stable if U is radially unbounded. (Q.E.D.)

PID Control (local stability)

In this section, we discuss the local stability of a PID control without any cross-feedback All gain matrices are diagonal but must satisfy an assumptions below.

Lemma 6 Consider the system Σ_{fw} and the following PID controller

$$\begin{cases} \dot r = q \\ u = -K_q q - K_I r - Cy \end{cases} \tag{31}$$

where K_q, K_I, C are positive definite matrix. Furthermore, suppose that there exits $a \in \mathbf{R}$ such that $(1/2)q^{\mathrm T}K_q q \geq a\|q\|^2$. Then the equilibrium set of the closed-loop system contains the only origin and it is (locally) asymptotically stable.

Proof of Lemma 6 The closed-loop system is described as

$$\begin{cases} \begin{bmatrix} \dot q \\ \dot p \\ \dot r \end{bmatrix} = \begin{bmatrix} 0 & I_2 & 0 \\ -I_2 & S_2 - C & 0 \\ K_q^{-1} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial(H+U+U_r)}{\partial x}^{\mathrm T} \\ \frac{\partial(H+U+U_r)}{\partial r}^{\mathrm T} \end{bmatrix} \\ \qquad - \begin{bmatrix} 0 \\ \frac{\partial(H+U+U_r)}{\partial r}^{\mathrm T} \\ 0 \end{bmatrix} \end{cases} \tag{32}$$

where $U = (1/2)q^{\mathrm T}K_q q$ and $U_r = (1/2)r^{\mathrm T}K_I r$ are the virtual potential energy. Consider the following scalar function

$$V = H + U + a(q^{\mathrm T}p + \frac{1}{2}(q^{\mathrm T}CM^{-1}q + r^{\mathrm T}K_I r)). \tag{33}$$

Then the time derivative of V (along the trajectory of the closed-loop system) is derived as

$$\begin{aligned} \dot V &= p^{\mathrm T}M^{-1}\dot p + q^{\mathrm T}K_q M^{-1}p \\ &\quad + a(q^{\mathrm T}\dot p + p^{\mathrm T}M^{-1}p + q^{\mathrm T}CM^{-1}\dot q + r^{\mathrm T}K_I q) \\ &= p^{\mathrm T}M^{-1}(-CM^{-1} + aM)M^{-1}p \\ &\quad - a(q^{\mathrm T}K_q q + q^{\mathrm T}SM^{-1}p + q^{\mathrm T}K_I q) \end{aligned} \tag{34}$$

from the condition of a, $\dot V \leq 0$.

On the other hand, since the inequality

$$\frac{1}{4}p^{\mathrm T}M^{-1}p + aq^{\mathrm T}p \leq -a^2 q^{\mathrm T}Mq \tag{35}$$

always holds from $M = (\sqrt{M})^2$,

$$V \geq \frac{1}{4}p^{\mathrm{T}}M^{-1}p + aq^{\mathrm{T}}q + \frac{1}{2}r^{\mathrm{T}}K_I r + \frac{a}{2}q^{\mathrm{T}}(CM^{-1} - 2aM)q \quad (36)$$

and V is bounded from below and a positive definite function of (x, r). Hence, if the initial state satisfy $V(0) < a\delta^2$, then $\|q(0)\| \leq \delta$ and $\dot{V} \leq 0$ holds because there exists the parameter a such that $(1/2)q^{\mathrm{T}}K_q q \geq a\|q\|^2$, $C \geq 2aI$ and $C \geq (a + \sqrt{a})I$. From LaSalle's invariance theorem, the local stability is proved. (Q.E.D.)

Remark Unlike with Lemma 5, Lemma 6 requires the assumptions for the gain parameters.

PID Control (global stability)

In this section, we discuss the global stability of a PID control without conventional cross-feedback. Similar to Lemma 5, the positive gain parameters do not have to satisfy assumptions any more.

We add the integrator $r \in \mathbf{R}^l$ to the system:

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -I & S_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q}^{\mathrm{T}} \\ \frac{\partial H}{\partial p}^{\mathrm{T}} \\ \frac{\partial H}{\partial r}^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} u \\ u_r \end{bmatrix} \\ \begin{bmatrix} y \\ y_r \end{bmatrix} = \begin{bmatrix} \frac{\partial H}{\partial p}^{\mathrm{T}} \\ \frac{\partial H}{\partial r}^{\mathrm{T}} \end{bmatrix} \end{cases} \quad (37)$$

Remark Here r is the state of the stabilizer. However, it is not connected to the system Σ_{fw} yet. Any function $C(r)$ is a Casimir function of the system with respect to J matrix. From Theorem 0, the Hamiltonian H can be replaced by new Hamiltonian $H + H_c(r)$ where $H_c(r)$ is any (lower bounded) function of r.

The following lemma connects the dynamics of the original system and that of r-integrator via a generalized canonical transformation.

Lemma 7 *Consider the system (37) with the Hamiltonian H. Then the transformation*

$$\begin{aligned} \bar{H} &= H(q,p) + \tfrac{1}{2}(K_q^{-1}p + r)^{\mathrm{T}}R(K_q^{-1}p + r) \\ \begin{bmatrix} \bar{y} \\ \bar{y}_r \end{bmatrix} &= \begin{bmatrix} \bar{y} \\ y_r \end{bmatrix} + \begin{bmatrix} 0 \\ R(K_q^{-1}p + r) \end{bmatrix} \\ \begin{bmatrix} \bar{u} \\ \bar{u}_r \end{bmatrix} &= \begin{bmatrix} u \\ u_r \end{bmatrix} + \begin{bmatrix} -\frac{\partial U}{\partial q}^{\mathrm{T}} + S_2\frac{\partial U}{\partial p}^{\mathrm{T}} \\ K_q^{-1}\frac{\partial U}{\partial q}^{\mathrm{T}} \end{bmatrix} \end{aligned} \quad (38)$$

converts the system into the following port-Hamiltonian system Σ_{fwi}

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & I_2 & -K_q^{\mathrm{T}} \\ -I_2 & S_2 & 0 \\ K_q^{-1} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial (H+U)}{\partial q}^{\mathrm{T}} \\ \frac{\partial (H+U)}{\partial p}^{\mathrm{T}} \\ \frac{\partial (H+U)}{\partial r}^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} 0 \\ u \\ u_c \end{bmatrix} \\ \begin{bmatrix} \bar{y} \\ \bar{y}_r \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{H}}{\partial \bar{p}}^{\mathrm{T}} \\ \frac{\partial \hat{H}}{\partial \bar{r}}^{\mathrm{T}} \end{bmatrix} \end{cases} \quad (39)$$

whose Hamiltonian is

$$\bar{H} = H + \frac{1}{2}q^{\mathrm{T}}K_q q + \frac{1}{2}(K_q^{-1}p + r)^{\mathrm{T}}R(K_q^{-1}p + r) \quad (40)$$

where $R > 0$ is any positive definite matrix.

Proof of Lemma 7 Since the transformation satisfies the condition (6), the transformation is a generalized canonical transformation and can be proved by a direct calculation. (Q.E.D.)

Theorem 4 (global stabilization) *The equilibrium set of the closed-loop system of Σ_{fw} and the following (PID-type) controller*

$$\begin{cases} \dot{r} = K_q^{-1}\frac{\partial (H+2U)}{\partial q}^{\mathrm{T}} \\ u = -\frac{\partial U}{\partial q}^{\mathrm{T}} + S_2\frac{\partial U}{\partial p}^{\mathrm{T}} - C\bar{y}_r. \end{cases} \quad (41)$$

only contains the origin and it is globally asymptotically stable.

Proof of Theorem 4 By applying the feedback

$$\begin{bmatrix} u \\ u_c \end{bmatrix} = C_c \begin{bmatrix} y \\ y_c \end{bmatrix} \quad (42)$$

with any positive definite matrix C_c, the state of the closed-loop system converses to the input-output nulling space

$$\Omega_0 = \{ \begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} y \\ y_c \end{bmatrix} = 0 \}. \quad (43)$$

Since this condition implies

$$\begin{cases} M^1 p + K_q^{-1}(K_q^{-1}p + r) = 0 \\ K_q^{-1}(K_q^{-1}p + r) = 0, \end{cases} \quad (44)$$

that is, the state converses to $\Omega = \{p = r = 0\}$ and the system Σ_{fw} is zero-state detectable from Lemma2, the system Σ_{fwi} is also zero-state detectable. The equilibrium set is the only origin and it is globally asymptotically stable. (Q.E.D.)

SIMULATION AND EXPERIMENT

Fig. 2 shows the closs-section of the flywheel in the experiment. The mass is 13.67 Kg and the diameter is 400 mm. The sampling time is 8 kHz. The

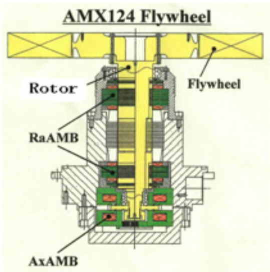

Figure 2: Cross-section of the flywheel system.

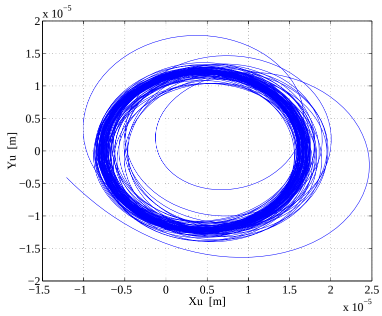

Figure 3: Simulation (210 Hz).

parameters are $I_p = 0.0179, I_d = 0.0180$ and the PID gains are shown in Table 1. These gains were scheduled according to ω_3 because not stability but control performance (i.e. overshoot) would depend on ω_3. One of the possible gain-tuning guidelines is discussed in [3].

Fig. 3 shows the result of simulation for 210 Hz. Fig. 4 and Fig. 5 show the results of experiment for 110 Hz and 210 Hz. For the contoller (41), $CM = k_d I_2$, $K_q = k_p I_2$, $K_I = k_i I_2$. We can find the small parameter a in Lemma 3. It is cofirmed that the rotor states are smoothly stablilized in both cases. There is no touch down and the position states are also smoothly stabilized.

Table 1: PID gains

ω_3 [Hz]	k_d	k_p	k_i
$0-20$	4×10^3	2×10^6	4×10^6
$20-45$	3.8×10^3	1.8×10^6	0.5×10^6
$45-100$	3.5×10^3	1.5×10^6	0.3×10^6
$100-150$	3×10^3	0.9×10^6	0.25×10^6
$150-200$	2.5×10^3	0.9×10^6	0.25×10^6
$200-240$	2.3×10^3	0.8×10^6	0.2×10^6
$240-300$	0.2×10^3	0.8×10^6	0.2×10^6

CONCLUSION

This paper discusses a passivity based control without conventional cross-feedback. These controllers have NO canceling terms of the gyroscopic effect. First we discuss the modeling and clarify some important properties of the flywheel. Second, we discuss some PID-type controllers from the viewpoint of passivity. Finally we give some simulation and experimental results.

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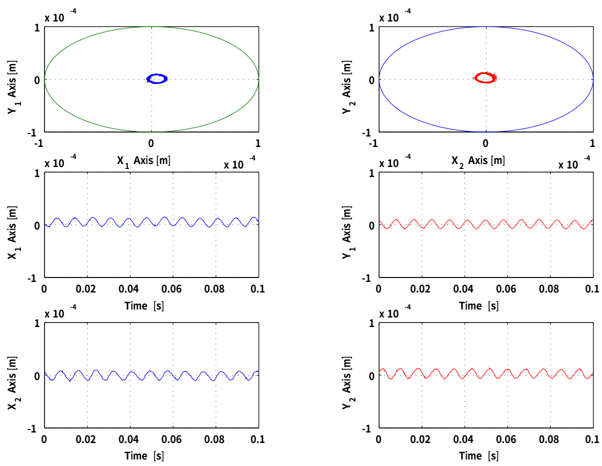

Figure 4: Experiment (110 Hz).

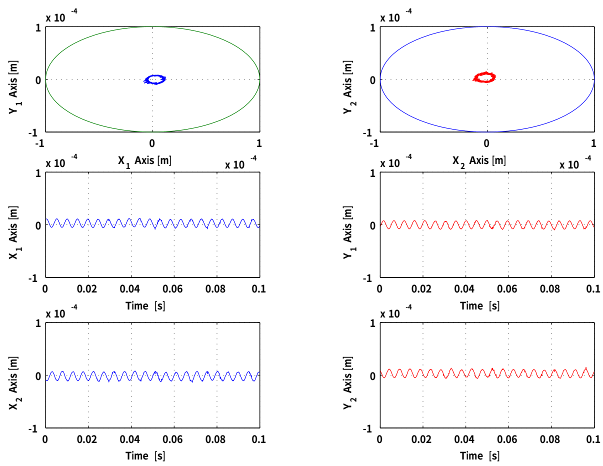

Figure 5: Experiment (210 Hz).