# A NOVEL FORCE FEEDBACK CONTROLLER FOR ACTIVE MAGNETIC BEARINGS

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# ABSTRACT

A novel force feedback controller is presented to overcome disturbing forces acting on magnetic suspended rotors of Active or Hybrid Magnetic Bearings (AMB or HMB). As there is linear relationship between disturbing forces and changes of magnetic field on bearings' operating point, the force feedback is implemented by measuring bearings' magnetic flux density by hall components. Detailed analysis is discussed also, the dynamic stiffness and damping of the controlled plant are increased when force feedback added into the controller; meanwhile, dynamic response of power amplifier is improved as the novel controller provides a zero point that counteract with the pole point of the power amplifier. A novel controller is developed based on conventional PID leading correcting network. Both simulation and experiment indicate that this novel controller can effectively decrease the influence of disturbing forces, and thus improves the controller's performance.

# **INTRODUCTION**

From the point of controller design, Active and Hybrid Magnetic Bearings (AMB and HMB) are typical systems of nonlinearity, especially rotors in high speed or with remarkable gyroscopic effects<sup>1</sup>. Many advanced control methods have been developed to achieve higher performance<sup>1,2</sup>, these advanced control method would usually include H- $\infty$ , Sliding role, Fuzzy logic, Neural Networks, etc.. However, PID controllers as classical ones have been broadly applied, and at the same time, improvements have been made to satisfy those new demands of higher operation performance.<sup>3-6</sup>

Our work is to improve conventional PID controller with direct force feedback, and the motivation is to compensate outer disturbing forces which acting on the suspended rotor.

On knowing that, there is linear relationship between

disturbing forces and changes of bearings' magnetic field, so called force feedback is able to be developed by measuring the changes of bearings' magnetic flux density by hall components.

When outer disturbing forces acting on the rotor, changes on magnetic field are measured and outputted in voltage signal by hall components. Then, after signal conditioning, the voltage signal is processed by force feedback circuit; finally, control value is calculated by controller and outputted in voltage or current signals.

Theoretical analysis is presented to reveal the role of force feedback also. As force feedback can be configured in proportional element or with integral gain, the pole of power amplifier can be precisely balanced by properly selected coefficients of force feedback. Thus the power amplifier is improved with its dynamic response performance. Moreover, the stiffness and damping of the controlled plant is increased due to direct feedback of disturbing forces.

Furthermore, rotor's acceleration of disturbed translation is obtained in the form of disturbing forces, and the corresponding velocity can be derived from acceleration's integral, this may provide an idea to utilize velocity and acceleration of disturbed translation.

Improvement on conventional PID controller has been made by additional direct force feedback. And comparing this novel controller to conventional ones, the proposed novel controller has certain advantages proved by simulation and experiment.

# PRINCIPLE

Note that the rotor is suspended by magnetic bearings, non-contacted sensors are needed to measure the displacement and velocity if possible, and other related information such as shaft's inclination angle can also be measured or calculated from displacement data. Indeed, no matter what types the controllers are, the input information is mostly displacement data, and thus can be viewed as position control.

The design for most bearings controllers is based on the following equation, the magnetic force generated by bearings  $as^1$ 

$$f = K_i i + K_x x \tag{1}$$

Where f is the magnetic force act on rotor, i is the current in the bearing coils, x is the displacement of rotor's translation,  $K_i$  is factor of current stiffness and  $K_y$  is factor displacement stiffness.

If take current control policy, the controller is, for example, a PID controller, the control equation would be written in the form of

$$i(t) = Px(t) + D\dot{x}(t) + I \int_{0}^{t} x dt$$
 (2)

Here in Eq.(2), *P* is proportional gain, *D* is differential gain, *I* is integral gain, and  $\dot{x}(t)$  is the velocity of translation.

Generally speaking, displacement x is easily to be measured by sensors, while the velocity  $\dot{x}$  is relatively hard to be acquired. Yet,  $\dot{x}$  can be obtained by the differential of displacement x, however, noises introduced by differentiator are great trouble, make the differential method unable to be implemented in the design of controllers and actual application.

However, on the other hand, integrator is better than differentiator in actual instance. Considering Newton's equation of

$$f = m\ddot{x} \tag{3}$$

Here in Eq.(3), *m* is the mass of an object and  $\ddot{x}$  is the acceleration of the object. It is not hard to understand from Eq.(3), if  $\ddot{x}$  can be measured in some way,  $\dot{x}$  would be integrated by integrator and is suitable for the controller design.

Suppose the magnetic bearing is operated on operating point, given a differential excited electromagnets pair, the magnetic force obtained from magnetic circuit calculation is

$$f = \frac{B_{a1}^2 A}{\mu_0} - \frac{B_{a2}^2 A}{\mu_0}$$
(4)

Where, f is the magnetic force,  $B_{a1}$  is the magnetic flux density in air gap 1 between bearing and flywheel,  $B_{a2}$  is the magnetic flux density in air gap 2 on another side of the bearing opposite to air gap 1, A is the section area in the air gap for magnetic circuit, and  $\mu_0$  is magnetic permeability in vacuum. For rotor's initial equilibrium position, gap 1 and gap2 usually designed as  $B_{a1} = B_{a2}$ , and denotes as  $B_a$ .

The relationship between force and magnetic flux density can be set up by Eq.(3) and Eq.(4), fortunately, changes of magnetic field on magnetic flux density can

be measured by hall components. If changes on magnetic field is measured, changes of f could be derived, and it can be utilized as a feedback signal in the controllers of AMB and HMB, so the question left to the possibility of force feedback.

Supposes outer disturbing force  $\Delta f$  imposing on the rotor, the air gap on both sides changes, are  $\Delta B_{a1}$  and  $\Delta B_{a2}$ , and assumes *A* doesn't change, we have

$$f + \Delta f = \frac{A}{\mu_0} \Big[ (B_{a1} + \Delta B_{a1})^2 - (B_{a2} - \Delta B_{a2})^2 \Big]$$
(5)

Eq.(5) minus Eq.(4), we get

$$\Delta f = \frac{A}{\mu_0} \Big( \Delta B_{a1}^2 + \Delta B_{a2}^2 + 2B_{a1} \Delta B_{a1} + 2B_{a2} \Delta B_{a2} \Big) \quad (6)$$

Assumes  $\Delta B_{a1}$  and  $\Delta B_{a2}$  are little values as they can't dramatically change in a short time, we can simplify Eq.(6) by ignoring high order little values, and rewritten Eq.(6) as

$$\Delta f = \frac{2AB_a}{\mu_0} \left( \Delta B_{a1} + \Delta B_{a2} \right) \tag{7}$$

Finally, we could draw a conclusion form Eq.(7), the outer disturbance force  $\Delta f$  has a linear relationship with the changes on magnetic flux density  $\Delta B_{a1}$  and  $\Delta B_{a2}$  of magnetic bearings.

So, the force feedback is feasible and reasonable in the application of AMB or HMB's controller design.

In general, possible advantages of force feedback are:

1), adds force feedback into the controller, and would make it able to response the disturbing forces directly and rapidly;

2), force feedback can acquire acceleration information, thus velocity of disturbed translation could be derived by integrating.

## MODELING

## **Conventional Controller**

Generally, a complete AMB or HMB system at least has 4 parts, they are sensors, the controller, the power amplifier and the control plant<sup>1,3</sup>. Displacement and other information are measured by sensors, then the measured data is calculated in the controller in respect of reference information which usually be zero, and current and voltage signals as the controller's output are amplified by power amplifier afterward, finally, bearing's coils are driven by power amplifier to control the rotor. The system architecture in transfer function is illustrated in FIGURE 1.

Here in FIGURE 1, i is the control current signal, X(s) is rotors displacement information on translation, and R(s) is the reference value, usually zero.



FIGURE 1: Magnetic bearings' architecture in transfer function

In FIGURE 1,  $G_C(s)$  is the controller's transfer function,  $G_P(s)$  is the power amplifier's transfer function,  $G_B(s)$  is the transfer function of the controlled plant, and  $G_S(s)$  is the transfer function of sensor element.

The transfer function of power amplifier is depended on detailed circuit design, however, in most cases,  $G_P(s)$  can be viewed as first order inertia element, that is

$$G_P(s) = \frac{K_P}{1 + T_P s} \tag{8}$$

Here,  $K_p$  is the gain and  $T_p$  is time constant corresponding to dynamic response.

 $G_s(s)$  is considered as proportional device usually, that is  $G_s(s) = K_s$ .

If take current control policy,  $G_B(s)$  can be written in the form

$$G_B(s) = \frac{K_i}{ms^2 - K_x} \tag{9}$$

Where in Eq.(9), m is the mass of rotor.

#### **Force Feedback Controller**

As an improvement to the conventional position control, and most importantly, to overcome disturbing forces acting on the suspended rotor, the method force feedback is discussed.

Note that FIGURE 1 contains simple position controller, for the outer disturbing forces acting on rotors, force feedback control is added into the controller, and transforms system architecture into FIGURE 2. Here, df stands for outer disturbing forces, and  $G_F(s)$  is force feedback element. Comparing to FIGURE 1's position control, FIGURE 2 processes more information of the rotor and bearings.

FIGURE 2 can be further transformed into following FIGURE 3, where we denotes  $K'_x(s)$  in the dashed line box by

$$K'_{x}(s) = K_{x} - K_{i}ms^{2}G_{P}(s)G_{F}(s)$$
 (10)

Lets  $G_0(s)$  be the transfer function in FIGURE 2's dashed line box from i(s) to X(s). And we get

$$G_0(s) = \frac{1}{ms^2 - K'_x(s)}$$

$$-\frac{1+T_{p}s}{ms^{2}(1+T_{p}s)+K_{i}K_{p}ms^{2}G_{F}(s)-K_{x}(1+T_{p}s)}$$
(11)

Assume a proportional force feedback, let  $G_F(s) = K_F$ . So there will be

$$G_0(s) = \frac{1 + T_P s}{T_P m s^3 + (1 + K_i K_P K_F) m s^2 - K_x T_P s - K_x}$$
(12)

Compares to FIGURE 1, function's order for FIGURE 3 has changed from 2 to 3, and a zero point is obtained at the same time. Considering the power amplifier's transfer function, the zero point in Eq.(8) can be exactly counteracted by  $G_p(s)$ 's pole point. Multiply Eq.(12) with Eq.(8), we get

$$G_{p}(s)G_{0}(s) = \frac{K_{p}}{T_{p}ms^{3} + (1 + K_{i}K_{p}K_{F})ms^{2} - K_{x}T_{p}s - K_{x}}$$
(13)

Here, let equivalent power amplifier

$$G_{p}(s) = K_{p} \tag{14}$$

From Eq.(14), it is easy to understand the dynamic response of power amplifier is improved.

Yet, the above analysis on force feedback is based on proportional feedback, as the corresponding velocity information can be obtained by the integral of acceleration, so, adds an integral gain in the force feedback element, that is, replace  $K_F$  by

$$K_{FP} + \frac{K_{FI}}{S}$$

Where  $K_{FP}$  and  $K_{FI}$  are proportional and integral gains, Eq.(12) will be rewritten as

$$G_0(s) = \frac{1 + T_P s}{T_P m s^3 + (1 + K_i K_P K_{FP}) m s^2 + (K_i K_P K_{FI} m - K_x T_P) s - K_x}$$
(15)

Compares Eq.(15) with Eq.(12), the system order remains the same, the zero point can still be counteracted by the pole point of power amplifier. There is not so much difference between Eq.(12) and Eq.(15), indicates that  $K_{FI}$  does little influence to the controller.



FIGURE 2: System architecture with force feedback element



FIGURE 3: Transformed architecture with force feedback element

## **Bearing Condition**

In Eq.(1) and Eq.(9),  $K_i$  and  $K_x$  are current stiffness and displacement stiffness, and are determined by electromagnetic design of AMB and HMB. However, like conventional bearings, AMB and HMB support the rotor with stiffness and damping also<sup>1,7,8</sup>, but, unlike conventional bearings, controller has a great effect on the bearing condition of AMB and HMB.

In FIGURE 2, the transfer function from F(s) to X(s) can be written as

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + K_x - K_i ms^2 G_P(s) G_F(s) + K_i G_S(s) G_C(s) G_P(s)}$$
(16)

Given  $K_e$  and  $D_e$  as equivalent stiffness and damping of AMB and HMB, we get dynamics equation

$$F(s) = ms^{2}X(s) + sD_{e}X(s) + K_{e}X(s)$$
 (17)

Replace s by  $j\omega$  in Eq.(16) and Eq.(17), here j is imaginary unit, we have equations in frequency domain

$$\frac{F(j\omega)}{X(j\omega)} = -m\omega^2 + K_x + K_i m\omega^2 G_P(j\omega) G_F(j\omega) + K_i K_s G_C(j\omega) G_P(j\omega)$$
(18)

and

$$\frac{F(j\omega)}{X(j\omega)} = -m\omega^2 + j\omega D_e + K_e$$
(19)

From Eq.(18) and Eq.(19), we get  

$$K_{e} = K_{i} \operatorname{Re} \left[ K_{s} G_{c}(j\omega) G_{p}(j\omega) + m\omega^{2} G_{p}(j\omega) G_{F}(j\omega) \right] + K_{x}$$

$$D_{e} = \frac{K_{i} \operatorname{Im} \left[ K_{s} G_{c}(j\omega) G_{p}(j\omega) + m\omega^{2} G_{p}(j\omega) G_{F}(j\omega) \right]}{\omega}$$
(20)

In Eq.(20),  $G_F(s) = 0$  indicates no force feedback in the control loop,  $G_F(s) = K_F$  stands for a proportional force feedback added in the controller. It is clearly that the equivalent stiffness and damping are increased for force feedback.

From Eq.(19), we can get the dynamic stiffness K and dynamic damping D in time domain, that is

$$K = \left| \frac{F(j\omega)}{X(j\omega)} \right| = \sqrt{\left(K_e - m\omega^2\right)^2 + \omega^2 D_e^2}$$
(21)

$$D = \left| \frac{F(j\omega)}{j\omega X(j\omega)} \right| = \sqrt{\left(\frac{K_e}{\omega} - m\omega\right)^2 + D_e^2}$$
(22)

It is obviously that both dynamic stiffness and damping are increased due to the introduction of force feedback in the controller.

# SIMULATION

The analog PID controller for AMB or HMB in our laboratory is actually leading correcting network, the transfer function is

$$G_{C}(s) = K_{C} \cdot \frac{1 + T_{1}s}{1 + T_{2}s} \cdot \frac{1 + T_{3}s}{1 + T_{4}s}$$
(23)

In Eq.(23), total gain  $K_c$ , time constant  $T_1$ ,  $T_2$ ,  $T_3$ and  $T_4$  are determined by controller circuit via adjustable resistance. According to actual measurement, we have  $K_c = 7.6$ ,  $T_1 = 0.00325$ ,  $T_2 = 0.00088$ ,  $T_3 = 0.0043$  and  $T_4 = 0.001$ . Notes that real system is different from theory model, we assume  $K_c = 3.2$  in the simulation as our other related studies do<sup>7,8</sup> (satisfy system stability in computer simulation while other parameters are exactly the measured data).

Other parameters related are: current stiffness  $K_i = 303$  N/A, displacement stiffness  $K_x = 390000$  N/m, sensor's proportional gain  $K_s = 5000$  V/m, power amplifier's time constant  $T_p = 0.00375$  s, gain  $K_p = 0.263$ , and rotors mass m = 4 kg.

Builds a model in MATLAB SIMULINK, runs the simulation and plots the results in FIGURE 4, Here,  $K_F = 0.0147$  (actual measured), disturbing force is a 20N impulse signal lasting 200ms.

FIGURE 4 shows the great difference between the two controllers. Subplot (b)'s vibration is greater than that of (c)'s, indicates the bearing's stiffness increased with force feedback in the control loop. Subplot (c) can rapidly return to steady state while (b) will spend more time, shows the bearing's damping is increased also. FIGURE 4 tells us that the controller with force feedback can compensated disturbing forces rapidly.

So the result is, the performance of controller with force feedback element is better than the one without force feedback.

#### EXPERIMENT

Our laboratory has developed several types Reaction/Momentum Flywheels (RMF) based on HMB for satellites attitude control<sup>7,8</sup>. A flat outer flywheel is suspended on inner Hybrid Magnetic Bearings, the motor's permanent magnets are assembled inside the flywheel and driven by motor's stator, its rotation speed varies from -6500 round per minute (RPM) to 6500RPM. Decentralized PID controller is implemented and cross feedback control is used to balance gyroscopic effects.<sup>7,8</sup>

Actual application of force feedback element is implemented by hall components embedded in the stator of magnetic bearings, illustrated in FIGURE 5, where (a) is the bearing stator and (b) illustrates the position of sensors. In subplot (b), the eddy current displacement sensors are imbedded in two rings made of polysulfone, there are 8 sensors evenly distributed in the upper and down sensor rings. The hall components are embedded in the edge of magnetizer panels which are made of electrical steel (DT4), there are 8 hall components evenly distributed in the upper and down magnetizer panel.

Magnetic circuit is reshaped when disturbing forces acting on the flywheel, and magnetic flux density changes, then the changes are measured by those hall components. Voltage signals corresponding to magnetic flux density are outputted. It is important to notice that voltage signal is linked with outer force via the changing of magnetic flux density.

Then the problem left to circuit design. Signal conditioning is needed; a proportional controller circuit is developed to give the output control signal, however, this part can be coded into control program in digital controller.

For actual measured magnetic forces, for example, 29N, the hall component output value is 430mV, so we get  $K_F = 0.43/29 = 0.0147$  V/N, and this parameter is used in the above simulation.

Qualitative experiments are executed in the operation of RMF by knocking on the flywheel with a sinker, the flywheel keeps its stability as, and again, validates the above analysis and simulation.

## SUMMARY

The controller with force feedback is discussed to overcome the outer disturbing forces. By direct feedback of disturbing forces, the controller is able to response the disturbing forces immediately and effectively.

Theoretical analysis reveals that the pole of power amplifier can be precisely balanced by properly selected coefficients of force feedback. Thus the power amplifier is improved with its dynamic response performance. Moreover, the stiffness and damping of the controlled plant is increased due to direct feedback of disturbing forces.

Besides, the disturbing forces are corresponding to rotor's acceleration of disturbed translation, by measuring magnetic flux density in the bearings, the relationship between them is found. Hence, velocity of disturbed translation is obtained by integrating the acceleration. So the force feedback would be a reasonable way to utilize information of velocity and acceleration of disturbed translation.



FIGURE 4: Simulation results comparing controller with or without force feedback



(a) stator (b) location of sensors FIGURE 5: stator of Hybrid magnetic bearings and embedded hall components

In the final, improvement on conventional PID controller is made by additional direct force feedback. Better performance is achieved via qualitative experiments, proves the disturbing forces can be responded immediately and effectively due to the introduction of force feedback. Both simulation and experiments show advantages of the controller with force feedback over conventional PID ones.

Further developments on this novel controller in the future would be quantitative implemented experiments, moreover, velocity feedback would probably be studied and more works would be discussed and expected.

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