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Adaptive Control for Attenuating Vibrations in AMB based systems using Multirate and Nonuniform Sampling

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Abstract

Different adaptive control algorithms have been developed for attenuating vibrations on AMB systems. The main aim of this work is to introduce the use of multirate and nonuniform sampling methods reducing the mean sampling rate, with the objective of minimizing the execution time of such algorithms. In particular, the proposed one is a feedforward adaptive vibration controller which generates sinusoidal signals whose amplitudes and phases are updated by the adaptation algorithm. This update is at different sampling times in the multirate version and at randomized sampling times, for the nonuniform sampling version. Such signals are applied to an AMB system obtaining, as expected, a minimization of the vibrations. Experimental results show the effectiveness of the adaptive control approach, allowing an important increment in the mean sampling period and leading to a reduced calculation time.

Keywords- Adaptive control, AMB, vibration elimination, nonuniform sampling

1 Introduction

A large number of electromechanical systems are affected by repetitive disturbances with sinusoidal form as is the case of active magnetic bearings (AMB) based rotating machines. The reduction of the effects of such vibrations can be crucial since those vibrations can damage internal parts of the devices or lead to the reduction of the mechanical strength, not allowing a proper operation, for instance in relation to the precision. In order to minimize those sinusoidal perturbations different techniques have been developed using passive and active elements, being remarkable the active noise control techniques based on digital signal processing, [1, 2] and the use of schemes based on adaptive filtering methods, [2, 3, 4, 5].

In particular, different adaptive control algorithms, various of LMS-type, have been developed for attenuating vibrations on AMB based electromechanical devices. Such adaptive methods are able to compensate large vibrations near critical resonance points, but also during non stationary states, [6, 7, 8]. The adaptive vibration controller has to generate signals at the synchronous frequency, which is related with the perturbation signals, in order to compensate their effects.

On the other hand, the use of digital signal processing techniques for control purposes is increasing thanks to the higher calculation power and bandwidth of the modern digital signal processors and related hardware as reconfigurable hardware (FPGA). However, the increasing complexity of the task performed for the different control systems, as communication or fault detection, makes interesting the reduction of the sampling rate for allowing real-time performance of the overall system. In this sense, the use of multirate systems permits a minimization of the needed calculation time, reducing the sampling rate when it is possible and, on the other hand, randomized sampling techniques have led to increase the working bandwidth without decrementing the sampling time, since its main advantage is the elimination of the aliasing.

The last techniques are known as digital alias-free signal processing (DASP), and their advantages have shown their special interest in the analysis of high frequency signals. In particular, the use of pseudorandom sampling techniques can improve the useful bandwidth with high sampling times in the mean, [9, 10]. These techniques add, logically, some drawbacks related directly to the stochastic nature of the resulting dynamics.

The main purpose of this work is the implementation of multirate and DASP techniques to an adaptive feedforward vibration control scheme applied to an AMB system, showing that it is possible to improve the calculation time maintaining the reduction of the vibrations. The proposed scheme is an extension of the adaptive feedforward vibration controller (AFVC) presented in [11] and the main characteristic is the application of a multirate and a randomized adaptive algorithm for obtaining the estimated parameter vector, leading to a nonuniform sampling based adaptive feedforward vibration controller (NAFVC). Although the proposed approaches are applicable to different schemes, the one considered here is an AFVC which generates periodic compensating signals ideally having adequate frequency, magnitudes and phases respect to the synchronous disturbances in order to minimize the vibrations. The adaptive algorithm is of least-squares type, [3], and this class of algorithms have proven their effectiveness in different real situations.

The approach has been experimentally applied to a laboratory AMB system which is based on the *MBC500 Rotor Dynamics* from Launchpoint Technologies [12], obtaining simulation and experimental results which validate the two proposed approaches. The testbed uses a real-time controller powered by a FPGA and implemented using Labview.

The paper is organized as follows. First, section 2 describes the problem statement and, then, section 3 explains the scheme of the modified adaptive feedforward control strategy to reduce the sinusoidal perturbations. Experimental results which illustrate the effectiveness of the presented approach to reduce the vibrations in the system are presented in section 4. In section 5, conclusions and future research perspectives end the paper. Finally, in order to clarify DASP concepts, the basic properties of the randomized signal processing are briefly described in the appendix.

2 Problem statement

This work presents the implementation of an adaptive controller to attenuate the vibrations in an AMB system, following the basic scheme displayed in Fig. 1, where the goal is to obtain $x(t) = 0$. The proposed approach is based on a special version of the adaptive scheme of this figure with a nonregular sampling time, in order to minimize the calculation power required. In particular, two versions of an adaptive filter based on the LMS algorithm for attenuating vibrations are presented, the first one using multirate sampling techniques and the second one using nonuniform DASP techniques. It must be remarked that the main objective is not the evaluation of an adaptive scheme in particular but the application of multirate and DASP techniques for vibration reduction.

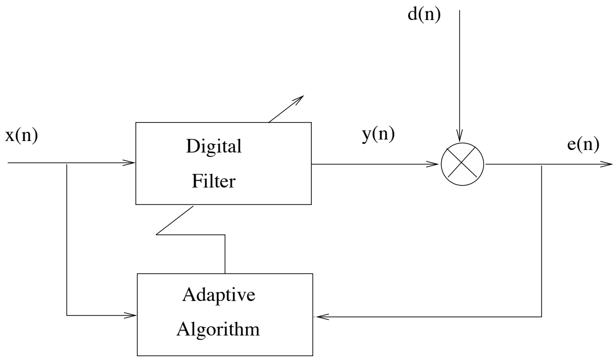

Figure 1: Block diagram of an adaptive filter

The original scheme used has been successfully applied by means of standard periodic sampling techniques, with an unique sampling period T. In this case, a regular and sufficiently small sampling period is considered for the sinusoidal compensation signal, while the adaptation process for the gains is performed based on the scheme proposed in [3, 11] and following two different ways:

- *Use of a randomized sampling period.* This can lead to increase the working bandwidth without increasing the sampling time in the mean. Its main advantage is the elimination of the aliasing leading to DASP, adding some issues related directly to the stochastic nature of the resulting dynamics, which must be taken into account. In practice, the implementation is based on pseudorandom techniques facilitating the implementation of the algorithm, which forces an aliasing related to the minimal difference between the sampling times chosen in a particular application, [9, 11]. Note that this parameter is freely selected by the designer.

- *Use of multirate sampling period.* In this case, two different regular sampling rates are used, T and T_l. The adaptive algorithm is carried out with a sampling period T_l, which must be not harmonically related with the frequency of the perturbation signal, since in this case, the aliased frequency corresponds to the DC case and the vibration is only partially minimized.

The proposed scheme can be observed in figure 2.

3 Estimation algorithm of the vibration controller

The nonuniform adaptive feedforward vibration controller (NAFVC) used in this paper is based on the randomized AFVC (RAFVC) presented in [11], which is

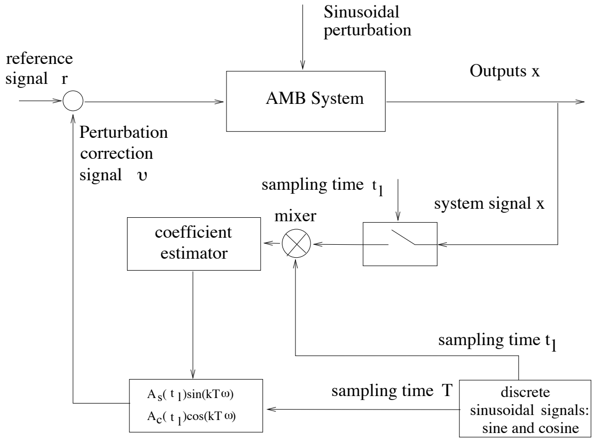

Figure 2: Vibration minimization scheme. The sampling time t_l can be selected as random or regular sampling.

considered as a direct adaptive method, [6]. The scheme is more detailed in Fig. 3. There are two different inputs for the controller: the signal $y(t)$ where the perturbation effect acts and the frequency of the sinusoidal perturbation ω, which is considered known in this paper since it is synchronous with the rotating speed of the AMB systems. The perturbation signal (continuous) may be described as

$$d(t) = A_d \sin(\omega t + \varphi_d) = A_{d_1} \sin(\omega t) + A_{d_2} \cos(\omega t)$$

and the output of the NAFVC will be also a sinusoidal signal, defined by the next discrete expression for each measured signal i in the AMB system

$$\begin{aligned} \vartheta_i(kT) &= A_{\vartheta_i}(t_l)\sin(\omega kT + \varphi_{\vartheta_i}(t_l)) \\ &= A_{is}(t_l)\sin(\omega kT) + A_{ic}(t_l)\cos(\omega kT) \end{aligned} \tag{1}$$

where A_{is} and A_{ic} are time-varying parameters updated by the multirate or DASP adaptation algorithm in order to minimize the effect of the perturbation. T denotes the sampling period for the correction signal and t_l are random or periodic sampling-time instants, depending of the case considered.

Analyzing this signal, two different discrete times are observed: a regular and periodic discrete time denoted by kT and a second discrete time represented by the sequence index l. The first regular kT time represent the actualization time of the perturbation correction signal $\vartheta_i(kT)$ while the index l represents the actualization of the adaptive parameters of the controller at time t_l, which occurs at randomized sampling times or at another periodic sampling time denoted by lT_l. In any case, $A_{ic}(t_l)$ and $A_{is}(t_l)$ are adjusted to minimize the effect of the synchronous disturbance. Note that in the original adaptive scheme, the adaptation instants t_l and the uniform sampling-instants $t_k = kT$ are coincident.

Normally, the desired position in a rotating AMB system is the origin and, then, the desired output signal is considered null, $r(l) = 0$, and the performance measure is given by

$$\xi(l) = E\left\{\varepsilon^2(l)\right\} = E\left\{y^2(l)\right\} \tag{2}$$

3.1 Adaptive algorithm

The system outputs denoted by means of $y(t) = \left[\begin{array}{cccc} y_1 & y_2 & \ldots & y_4 \end{array}\right]^T$ are the input signals of the adaptive controller, being 4 the number of outputs of a typical AMB system with two radial bearings. These signals are sampled at multirate or randomized sampling instants t_l, obtaining $y(t_l)$ in discrete time. The adaptive controller provides a discrete signal vector $\vartheta(k) = \left[\begin{array}{cccc} \vartheta_1 & \vartheta_2 & \ldots & \vartheta_4 \end{array}\right]^T$ which tries to compensate the vibrations and each of the signals ϑ_i follows the expression given in eq. (1). Note that the controller output is defined for regular sampling instant $t_k = kT$.

In addition, the adaptive parameters in eq. (1) are grouped in the vector

$$\begin{aligned} \hat{\theta}_c(t_l) = [\ & A_{1s}(t_l) \quad A_{1c}(t_l) \quad A_{2s}(t_l) \quad A_{2c}(t_l) \\ & \ldots \quad A_{4s}(t_l) \quad A_{4c}(t_l)\ ]^T \end{aligned} \tag{3}$$

From Fig. 3 the output signal can be represented as

$$y(t) = G_d(s)d(t) + G_{cl}(s)\vartheta(t) \tag{4}$$

where the transfer functions $G_d(s)$ and $G_{cl}(s)$ describe the influence of the disturbance and the input over the output $y(t)$, respectively, and $\vartheta(t)$ is the continuous version of the NAFVC controller output, obtained with a zero order hold.

The design objective is the obtaining of the NAFVC parameter vector θ_c which minimizes the performance measure of eq. (2). As is shown in the block diagram, the filtered-x variant of the LMS algorithm is considered, [3]. This objective is fulfilled adjusting the vector θ_c along the gradient direction Δ. Since this gradient is unavailable, the next unbiased estimate of the gradient is used, [3]

$$\hat{\Delta}(t_l) = 2y(t_l)G_{cl}(q_l^{-1})\left[\begin{array}{c} \sin\omega t_l \\ \cos\omega t_l \end{array}\right] \tag{5}$$

where $G_{cl}(q_l^{-1})$ represents the discrete version of $G_{cl}(s)$, being a transfer function whose parameters changes with t_l. In the randomized version the resulting system can be considered as a jump discrete system, [13]. Choosing the parameter adaptation rate much lower than the closed-loop dynamics described by $G_{cl}(s)$, this transfer function is replaced by a constant $\lambda/2$ and the eq. (5) is rewritten:

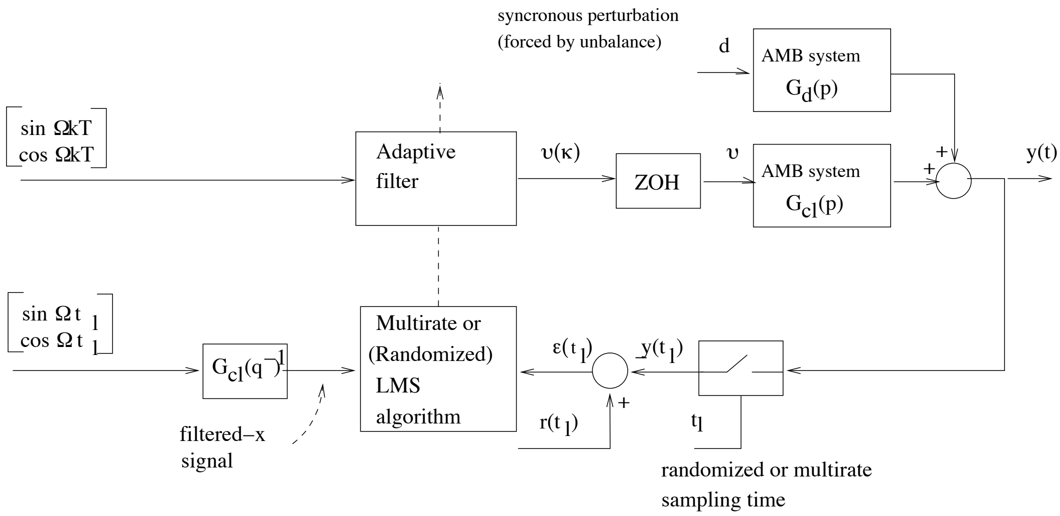

Figure 3: Block diagram of the vibration controller including the NAFVC.

$$\hat{\Delta}(t_l) = \lambda y(t_l) \begin{bmatrix} \sin \omega t_l \\ \cos \omega t_l \end{bmatrix} \tag{6}$$

Defining

$$\varphi(t_l) = \begin{bmatrix} s_1 & c_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & s_1 & c_1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & s_1 & c_1 \end{bmatrix}$$

with $s_1 = \sin \omega t_l$ and $c_1 = \cos \omega t_l$ and $\varphi(t_l)$ is a $n \times 2n$ matrix, the adaptive algorithm becomes

$$\theta_c(t_l) = \theta_c(t_{l-1}) + \frac{P\,\varphi(t_l)^T x(t_l)}{1 + \gamma\, tr\,[\varphi(t_l) P \varphi(t_l)^T]} \tag{7}$$

the normalized least-squares algorithm with constant covariance matrix P, where tr denotes the trace of a matrix, $\varphi(t_l)$ the input matrix and γ a real constant. Both must be chosen such that the estimation process is much slower than the closed-loop dynamics of the AMB system, described by $G_{cl}(s)$, [6].

An important characteristic of the proposed scheme, observed in Fig. 3, derives from the nonuniform nature of the adaptation process respect to the compensation signal itself.

About the stability of the algorithm, the performance of the standard LMS algorithm is well-known, [3, 2]. The randomized version is considered in [11] where the requisites for stability are resumed into two conditions: the different random processes are stationary and independent and the adaptation process is sufficiently slow. Similar conditions can be considered for the multirate version. However, in this case, the aliasing must be considered and, then, the selected adaptation sampling time must not be harmonically related to the frequency of the perturbation to minimize.

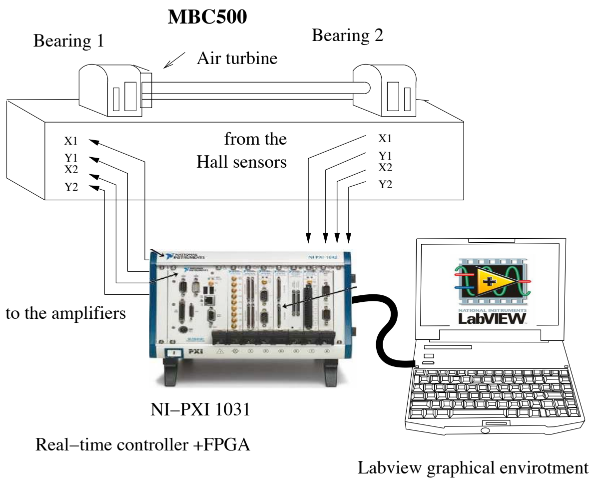

Figure 4: Experimental testbed based on the MCB500 and a real-time controller implemented using Labview.

4 Experimental results

This section presents experimental results which show the effectiveness of the NAFVC adaptive controller developed to reduce the vibrations caused by sinusoidal perturbations in an AMB system. These results have been obtained using the laboratory testbed shown in figure 4, which is based in a *MBC500 Rotor Dynamics*. In the figure, three elements can be differentiated: the AMB system, a real-time controller where the stabilizing and adaptive controller is implemented using a FPGA and, finally, a host computer with the Labview graphical environment used for configuring and monitoring purposes. The stabilizing controller is a basic PID obtained after a complete modeling process, [14], with the same structure for each plain (x and y directions):

$$G = \begin{pmatrix} \frac{14(z-0.999)(z-0.9)}{(z-1)(z-0.45)} & 0 \\ 0 & \frac{14(z-0.999)(z-0.9)}{(z-1)(z-0.45)} \end{pmatrix}$$

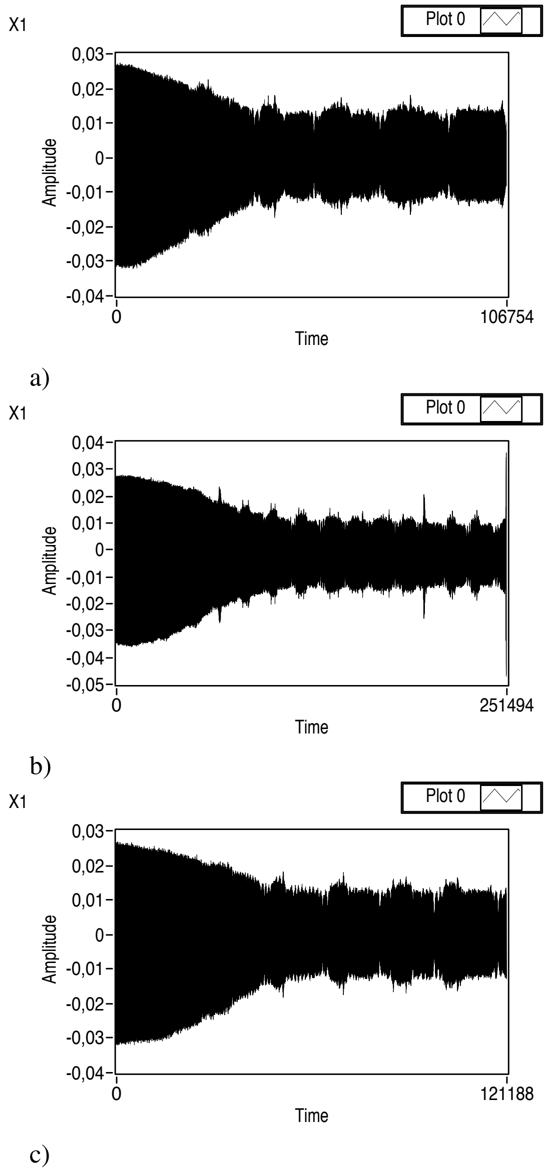

Figure 5: Elimination of the distortion using different versions of the AFVC. a) Standard version, b) Dasp version, c) Multirate version.

In the experiments, three different AFVC controllers have been implemented: the standard version as reference and the two NAFVC versions proposed in this paper. In all cases, the reference signal is zero and the objective is to reduce the vibrations around the equilibrium point. The results can be observed in the figure 5 where the important reduction of the distortion observed in this example is similar for the three adaptive algorithm, validating the proposed scheme. In particular, being the reference the period T used in the standard AFVC, the multirate version have a $3T$ period for the adaptation process and the DASP version have a random adaptation time varying from T to $5T$. Those time values can be incremented, effecting only to the increment of the time necessary to minimize the perturbation.

The result of the adaptation process for the DASP case is shown in Fig. 6, where it is displayed the time evolution of the parameters which define the amplitudes and phases of one compensation signal generated by the adaptive vibration controller. Such a figure shows the convergence of the parameters, which convergence rate is obtained using a constant $\gamma = 10000$ (approximately) in the adaptive algorithm. The other two cases are not shown since are very similar.

5 Conclusions

In this work the scheme of an adaptive vibration controller for AMB systems using nonuniform sampling-time in the estimation algorithm is presented. Two different versions are considered, the first one based on a multirate adaptation approach and the second one, in a randomized adaptation approach. The use of such controllers results in a reduction of the periodical perturbations observed in the system outputs thanks to the compensation signals generated by the proposed adaptive control scheme NAFVC, while the calculation power is optimized. This improvement is obtained since the controller updates the parameters which define the amplitudes and phases of the compensation signals in multirate or randomized time instants and those resulting signals effectively counteract the effects of the perturbations. On the other hand, the compensation signals are generated regularly in periodic time instants with more high rate.

The effectiveness of the developed adaptive controller has been shown by means of experimental results using a laboratory testbed, allowing an important reduction of the calculation time in the mean, while the vibration reduction is totally comparable with a standard scheme. In conclusion, this control strategy provides an interesting solution in high-speed AMB systems, or in other type of high-speed mechanical systems, resulting in a reduction of the vibrations observed in the shaft ends when it is rotating, improving the calculation time.

In future research works, different scheme alternatives can be analyzed and implemented in order to optimize the

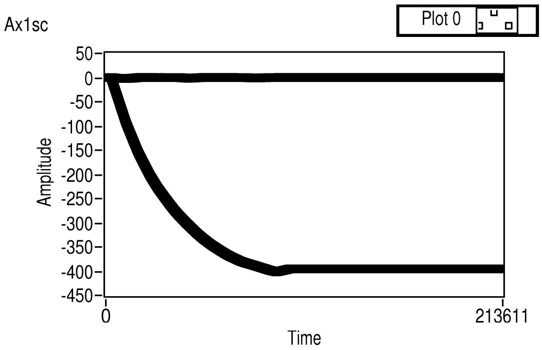

Figure 6: Evolution of the adaptive vibration controller parameters

results. On the other hand, quantitative results about the reduction of the calculation time are interesting too.

6 Acknowledgments

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Appendix- Randomized signal processing

Digital signal processing (DSP) techniques are based, in general, on the uniform and periodic sampling of signals, leading to the well-known problem of aliasing. This phenomenon appears similarly in multirate systems. In contrast, the aliasing problem disappears applying randomized sampling times, for any finite sampling-time in the mean, leading to DASP techniques, [9, 10].

However, the aliasing-free property is due to the theoretically possible zero (infinitesimally small) distance between two random sampling instants. In practice, this is not possible and a minimum distance must be defined. Although the alias-free condition is not fulfilled, this minimum time distance is directly related to the equivalent Nyquist frequency using DASP for any mean sampling-time, [9, 10]. In any case, using a nonregular sampling technique with a higher sampling period in the mean, the applicable frequency bandwidth can be higher than using a regular sampling time.

One of the main drawbacks of the DASP technique is that very extended DSP techniques, as the FFT algorithm, are not valid and, in general, specific algorithms must be

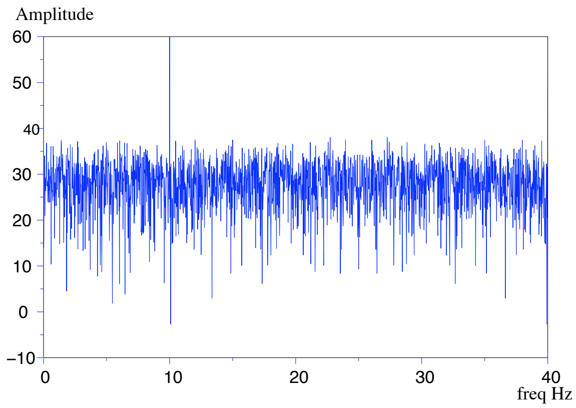

Figure 7: Spectrum of a one tone signal at 10Hz sampled at 1Hz (mean) using a randomized sampling technique

developed for using nonuniform sampling in any application. The discrete implementation of the Fourier transform for a signal $x(t)$ with randomized sampling times can be written as

$$\mathscr{X}(f) = \sum_{k=1}^{n} x(t_k) e^{-2\pi f t_k j} \quad (8)$$

with n the number of samples and t_k the randomized sampling instants, which presents a very similar implementation of an ordinary regular DFT, but there is not a fast version like the FFT algorithm. As example of the DFT transform (8), it is applied a randomized sampling time of mean $E[T_k] = 1$s., with $T_k = t_k - t_{k-1}$, to a pure sinusoid signal of frequency $f = 10Hz$ (Fig. 7). The minimum distance between samples is $\Delta_k = 1/50$s. and the equivalent Nyquist frequency is $25Hz$. The resultant signal representation in the frequency domain can be observed in Fig. 7. The minimum distance between samples is $\Delta_k = 1/50$s. and, then the equivalent Nyquist frequency is $25Hz$ (the first frequency alias is $40Hz$, not shown in the figure). Note that, if a regular sampling is utilized, the equivalent frequency spectrum is obtained by mean of a sampling time $E[T_k] = T = 0.05$s, that is, twenty times lower.

In Fig. 7, another characteristic derived from the randomized sampling can be observed: the increment of the noise floor at non existent frequency components in the original signal under analysis. Although this property can be a serious drawback in different applications, the use of specific reconstruction techniques can avoid this problem.