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A Diffusion Model for Active Magnetic Bearings in Large Turbomachinery

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Abstract

The interest in AMB technology spans over two decades of application to large turbomachinery yet there is not a suitable mathematical model used in this industry that describes the penetration of the technology into the domain of conventional oil lubricated bearings. This paper adapts previous work in product diffusion models to this purpose.

Introduction

The diffusion of new technologies into industrial and consumer goods market has been studied extensively over the past four decades. This effort has given rise to several useful mathematical model formulations that describe the diffusion process as a process by which users will substitute an older established technology with a newer one on the basis of how efficiently communications about the newer technology's attributes, risks, and costs are enabled.

The well established conformance of a wide variety of durable goods, everything from mainframe computers to machine tools to color televisions, to these various models brings into question how well they would suffice in predicting the penetration of active magnetic bearing (AMB) technology into the domain of oil lubricated bearings in large rotating machinery, specifically centrifugal compressors as used in the oil and gas industry.

The drivers for this penetration have been promoted extensively. They are reduced power losses, reduced maintenance costs, wider operating speed and temperature range, better rotordynamic control, lower fire risk, less expensive foundations, etc. while achieving reliability and availability figures that are at least as good as oil lubricated bearing systems, and with a justifiable price premium. These positive attributes can therefore be expected to facilitate the penetration of AMB technology into this class of machinery in a systematic way that can be described accurately by a mathematical model. This would allow the prediction of the rate at which the technology would be adopted as well as what fraction of the population of ultimate adopters will adopt at what time.

History of AMB Adoptions in Large Turbomachinery

The first generation of AMB technology as applied to large turbomachinery appeared in the mid 1980's. It was characterized by analog electronics, relatively low power output amplifiers that drive current to the magnetic bearing windings, and simple stator mounted ball bearings that served as the machine protection in the event of loss of stability or internal system failures. A significant number of applications were undertaken with this technology in large turbomachinery, especially compressors, in Europe (including Russia) and North America.

Important applications to other large rotating equipment, including turboexpanders, turbine generators, and pumps have also been made, but the applications to compressors, whether driven by gas turbines or electric motors, is chosen for this study because the large number of applications to this class of machinery provides a statistical basis for the study of AMB diffusion process characteristics. Moreover, compressors typify the application of AMB technology to other large turbomachines. A typical example is shown in Figure 1.

Note that the salient figure of merit when examining adoptions of the new technology is not the number of adoptions by the original equipment manufacturers that use bearings in their compressor designs. Rather, the important figure of merit is the number of adoptions by end users to whom the financial and operational benefits accrue. Figure 2 displays the cumulative number of applications during this time period (pre 1995 error band: ±1 year) by end users in large compressors above 2 MW power rating. An impressive upswing in new applications begins in

2004, especially in Russia. Like the other plots that follow, this chart includes the known applications of the major AMB suppliers. Eliminating multiple and repeat adoptions by the end users from this data yields Figure 3, a more modest view of the penetration of AMB technology because these are first adoptions only. Although a prediction of the total number of applications would certainly be of interest, no mathematical model would be capable of this type of forecasting with good certainty.

Sustained adoptions of AMB technology did not occur until the mid 1990's with the advent of the second technology generation that addressed the teething problems that had caused the early decline in new adoptions of the first generation technology. The second generation technology introduced several significant improvements in AMB controllers. Most importantly, digital electronics were introduced enabling more flexible and faster implementation of control algorithms. The control algorithms were, in turn, derived from newly developed fully integrated rotor-bearing system models that allowed a complete definition of system response and stability. In addition, higher (by more than twofold) output power rating of the amplifiers used to drive current to the AMB coils were also used for the first time.

This group of technology features allowed the robust control of fully levitated machine trains. Evolution of electrical designs coupled with improved thermal management assured higher reliability comparable or superior to oil lubricated systems. The digital technology employed with the electronics assured shorter commissioning of new machines while enabling remote communications that enhanced the operability of the system. In turn, the latter allowed reductions in operating and maintenance personnel to be considered thereby lowering operating costs.

In addition to the above, designs for direct immersion of the AMBs and auxiliary bearings in the process fluid were introduced thereby reducing bearing spans, eliminating seals and realizing much simpler and more reliable designs. This capability is especially significant in those cases where contamination and corrosion are an issue. Lastly, another significant enhancement of the technology was the introduction of advanced auxiliary bearing designs that not only extended the service life before replacement but had a much greater capability for protection of machine internals under arduous field conditions.

These new features and capabilities allowed resurgence in the adoption of AMB technology by more end users as shown in the right side of Figure 3. Reviewing Figure 3 it can be seen that the data comprises a fairly smooth curve except for the "kink" in the mid 1990's at the end of life for the first generation systems and before the second generation designs were adopted.

New AMB Adoption Forecasting

The usual approach to prediction of future trends is to employ time series forecasting based on the historical data. The best fit for the Figure 3 second generation data is with double exponential smoothing that yields a 0.92 root mean square error for the cumulative adoptions, but only 30 total new adoptions would be predicted at the 20 year point, a modest penetration by most any measure.

This trend is symptomatic of a very slow adoption rate compared to similar industrial equipment and, if true, it can only be described as a result of the perception of the risk/reward paradigm for AMB technology over the time period in question. This paradigm should be expected to change over time as positive experiences with the technology are communicated including the financial benefits.

The problem with time series forecasting is that it does not recognize the mechanisms of substitution and diffusion of one technology over that of another. This is where various diffusion models have been shown to accurately predict the first adoption of new technologies by new adopters across a broad spectrum of products and markets.

Most diffusion models are based on sigmoid functions (that yield "S" shaped curves) like the Gompertz function that describes a diffusion process whereby new adoptions start off slowly from a "floor" at zero, gains strength and then, after a peak in new adoptions, starts to taper off as market saturation builds near the "ceiling" because the number of potential adopters whom have not adopted diminishes. Diffusion data is accordingly usually displayed as cumulative adoptions vs. time to illustrate the classic "S" curve.

No diffusion model has seen more widespread study and acceptance as the Bass model introduced in 1969 [1], another sigmoid type function. In fact, the Bass model has been identified as among the top ten developments in management science and the only one dealing with the study of marketing. As originally conceived, the Bass model was applied to identify the rate at which first time buyers make single purchases. Variations of this model have been developed to address different circumstances including repeat sales and multiple unit sales.

The Bass model and most of its variations consider that new technologies diffuse into the marketplace by mass media and a word-of-mouth effect as new adopters communicate their experiences both internally and externally. These communications serve to encourage more adoptions internally as well as to build "pressure" on other potential users to adopt to take advantage of technology attributes and remain competitive with adopters. The basic Bass model describes the probability that an adoption will occur at time t given that it has not yet occurred, [2]:

$$\frac{f(t)}{1-F(t)} = p + qF(t) \quad (1)$$

where $f(t)$ is the density function of time to adoption
$F(t)$ is the cumulative fraction of adopters at time t
p is the coefficient of internal influence, aka the "coefficient of innovation"
q is the coefficient of external influence, aka the "coefficient of imitation"

The basic premise underlying the Bass model states that the conditional probability of adoption at time t is increasing in the fraction of the population that has already adopted. Part of the adoption influence depends on imitation or learning and part of it is independent of previous adoption.

Defining

$$n(t) = \frac{dN(t)}{dt}$$
$$mf(t) = n(t)$$
$$mF(t) = N(t)$$

where $n(t)$ is the density function of time to adoption
$N(t)$ is the cumulative fraction of adopters at time t
m is the total population of ultimate adopters

and substituting in Equation 1 yields

$$\frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)] \quad (2)$$

The first term in Equation 2 represents new adoptions by potential users who are not influenced in the timing of their adoption by the number of previous adopters. Bass referred to this group as "innovators". The second term in Equation 2 represents adoptions by potential users who are influenced by the number of previous adopters. Bass referred to this group as "imitators". Note in Equation 1 that at time $t = 0$, $n(t) = pm$.

The choice of parameters p and q for Equation (2) has substantial grounding in all of the earlier studies of the application of the Bass equation to various durable good case studies across many markets. These studies demonstrate that the average value for p is 0.03, and the average value of q is 0.38, [3]. Moreover, as reported in [4], the sum of $p + q$ has been found to range from 0.3 to 0.7 across a broad spectrum of products.

One of the variations of the basic Bass model describes the adoption patterns of successive generations of high technology products: how successive generations will substitute for and displace previous generations, [5]. This would be particularly relevant for the study of magnetic bearing technology adoption except that certain teething problems in some high visibility applications of the first generation technology of one AMB supplier resulted in a period of substantial reduction of adoptions in the affected part of the world. Here, the word-of-mouth communication mechanism implicit in the Bass model served to diminish further adoptions of the first generation technology. Furthermore, introduction of the second generation designs had not yet occurred, and the financial case for AMBs [6] had not been convincingly made to assure that the number of ultimate adopters, m in Equation 2, would approach the total number of potential adopters.

The total number of potential adopters, m, is comprised of more than just the total number of end user companies in the world, but rather the total number of end user entities that operates independently by making their own decisions about use of new technologies. Thus, the number is certainly measured in the hundreds, if not more.

Referring to Figure 3, the question is whether the three parameter equation above can be curve fit to the historical data and used for forecasting future adoptions with better confidence than the time series modeling. Using nonlinear regression analysis, convergence to multiple solutions of Equation 3 is possible although these solutions involve negative parameters and therefore have no utility. Furthermore, research has shown that the Bass equation's usefulness in predicting future adoptions is not good unless the data from which the parameters have been derived includes the time of peak adoptions, [7, and 8]. This milestone has not been achieved.

Accordingly, the approach here is to utilize Monte Carlo simulation of the Bass model while independently varying the m parameter. Probability distributions for the p and q parameters are selected that encompass the second generation historical data from Figure 3 but are consistent with the results of the diffusion of other technologies. Monte Carlo simulation resolves the intrinsic uncertainly and allows the prediction of future adoptions within statistical bounds defined by the input probability distributions. The result itself is therefore a probability distribution because of the uncertainties introduced in the inputs.

Standard metrics for diffusion models include the elapsed time until peak adoption of the technology but the metric chosen here is the elapsed time at which 50% of all potential new adopters have adopted. Because of recurring multiple orders from the early adopters this milestone could easily account for well over 50% of all compressor sales in the year in question; early adopters or "innovators" tend to be the adopters with patterns of large annual orders. Indeed, one of the early adopters has stated publicly that they intend to purchase AMBs in well over 50% of their new compressor buys.

The Bass adoption spreadsheet model used to derive the results herein is similar to the one from [9] that uses Crystal Ball® software. The ultimate number of unknown adopters m is varied independently between 100 and 1000. The input probability distribution for p and q at each m is a uniform distribution with a minimum value defined by curve fitting of Figure 3 and a maximum value corresponding to the average value for other technologies. The values thus range from 0.002 to 0.03 for p, and 0.004 to 0.38 for q, a quite conservative representation of the adoption parameters by most measures.

The results obtained from 6000 simulations each are shown in Figure 4 for the range of the m values. Not shown are the standard deviation results which range from 0.3 years at m = 100, to 2.5 at m = 1000.

Repeating this analysis for the more typical values ascribed to p (0.02 to 0.04) and q (0.28 to 0.48) for new technologies [3] yields the results shown in the lower curve of Figure 4. This result suggests that as efficiencies in the word-of-mouth effect and mass media improve, the actual time to target of adoptions should move along a reversion path to the lower curve. For example, if the communications efficiencies take hold after one year of elapsed time, the total time to target in the event of a population of m = 200 new adopters would be reduced from 8 years (9-1) to 7 years (1+6); in the event of a population of m = 600 new adopters the time to target would be reduced from 13 years (14-1) to 8 years (1+7). This circumstance can be described therefore as the "Bass inefficiency index", the inability for communications about the attributes of AMBs including the financial case thereof, to reach all potential adopters.

The discussion about generational adoption also brings into question the next generation of AMB designs and how that will impact the adoption rate. Generation three will likely feature self tuning, advanced adaptive vibration control, advanced self diagnostics (expert systems), and transient overload capability for the auxiliary bearings. Some of these features have already been demonstrated by certain AMB suppliers. Rapid adoption should occur because of the carryover effect from the second generation.

Summary & Conclusions

Using Monte Carlo simulation to resolve intrinsic uncertainties, the Bass Diffusion Model can be applied to the diffusion of active magnetic bearing technology into the realm of oil lubricated bearings in large turbomachinery to describe the historical record, and to show at what rate the current second generation of this technology will be adopted with first purchases by new adopters. Despite the impressive total number of new AMB applications to compressors, the Bass model indicates a longer than typical time to adoption by new end users thus suggesting an inefficiency in the word-of-mouth communications mechanism believed by Bass to be fundamental to the sustained adoption of new technologies. As the lag in these communications is erased, the adoption rate will revert to the typical industrial trend and continue its substitution for oil lubricated bearing technology in large compressors.

FIGURE 1: AMB Equipped Motor Compressor

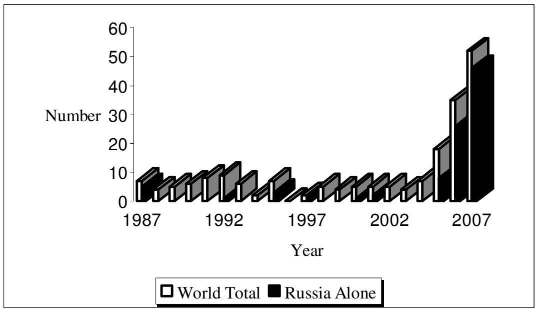

FIGURE 2: Total Number of Known Compressor Applications of AMB Technology by Year

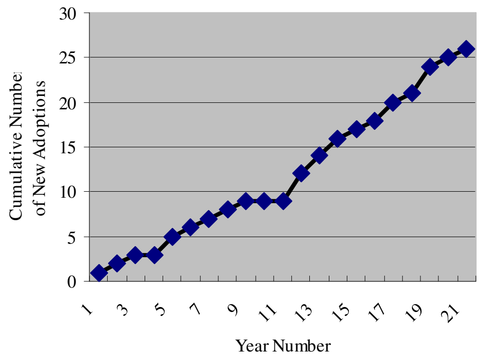

FIGURE 3: Cumulative Number of New AMB Adoptions over Two Generations (Year 1 = 1987)

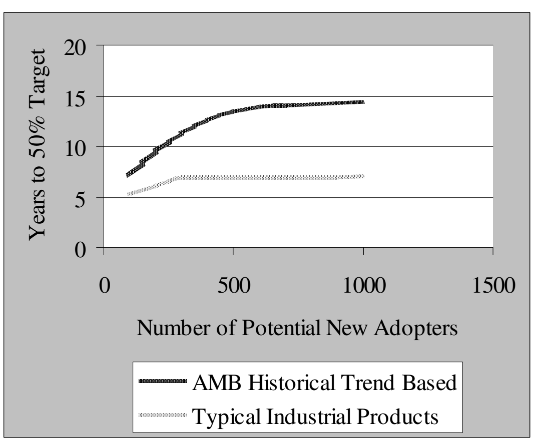

FIGURE 4: Years to 50% Adoption Target of New Adopters by Simulation Showing Inefficiency of Present Communications

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