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ROBUSTNESS OF RECURSIVE AUTO-TUNING STRATEGY IN FLEXIBLE ROTOR AND MAGNETIC BEARING SYSTEMS

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ABSTRACT

Active magnetic bearings offer the capability to control rotor vibration and transmitted forces through the design of adaptive controllers. These controllers can take into account the changing operating conditions and variations in the system dynamics. A Recursive Open-Loop Adaptive Control (ROLAC) algorithm was developed, which minimizes the sum of squares of the measurements in frequency domain. ROLAC is used in addition to local PID controllers, which ensure the stability.

Magnetic bearing dynamics depend on the eccentricity of the rotor with respect to magnetic poles. Non-concentric rotation of the rotor within the magnetic bearings causes changes in the bearing dynamic properties. Apart from system faults, non-concentric operation of the rotor can occur due to misalignment of the auxiliary bearings with respect to magnetic poles, changes in sensor calibration, and misalignment of bearings in systems where there are three or more bearings. This misalignment leads to changes in the magnetic bearing characteristics thereby affecting the overall system dynamics. This in turn may necessitate re-tuning of the ROLAC. The paper studies the effect of such misalignments on the controller performance and the robustness of the auto-tuning process.

INTRODUCTION

Magnetic bearings possess attractive characteristics for industrial and manufacturing applications due to their ability to support high-speed rotating loads with no friction, no lubrication and under extreme environmental conditions [1]. Since magnetic bearings can be actively controlled, they offer potential advantages compared with conventional bearings including vibration control, automatic balancing, condition monitoring and fault diagnostics [2]. However, due to their limited force capacity, active magnetic bearings have to incorporate retainer bearings to protect the laminations when rotor vibrations reach the clearance level. The future of magnetic bearing applications relies on addressing the safety and reliability issues in critical applications [3].

A range of control techniques have been developed to control active magnetic bearings under normal operation, to minimize rotor vibrations, and also to minimize transmitted forces [4–8]. An effective method of controlling synchronous vibration under varying operating conditions has been introduced by Burrows and Sahinkaya [4, 9, 10]. This open-loop adaptive control (OLAC) strategy, also referred as automatic balancing, can be extended to attenuate multi-frequency vibrations of the rotor [11]. However, because it depends on the steady state response and a Fourier transform of measured displacements, it may not be fast enough to prevent rotor contact with auxiliary bearings in response to a sudden change of unbalance. Therefore a recursive version of the algorithm has been developed, which updates the optimum force amplitude and phase at each sampling interval. This technique has improved the reaction speed of the controller to transient changes. It has also been shown that the ROLAC can prevent rotor contact with an

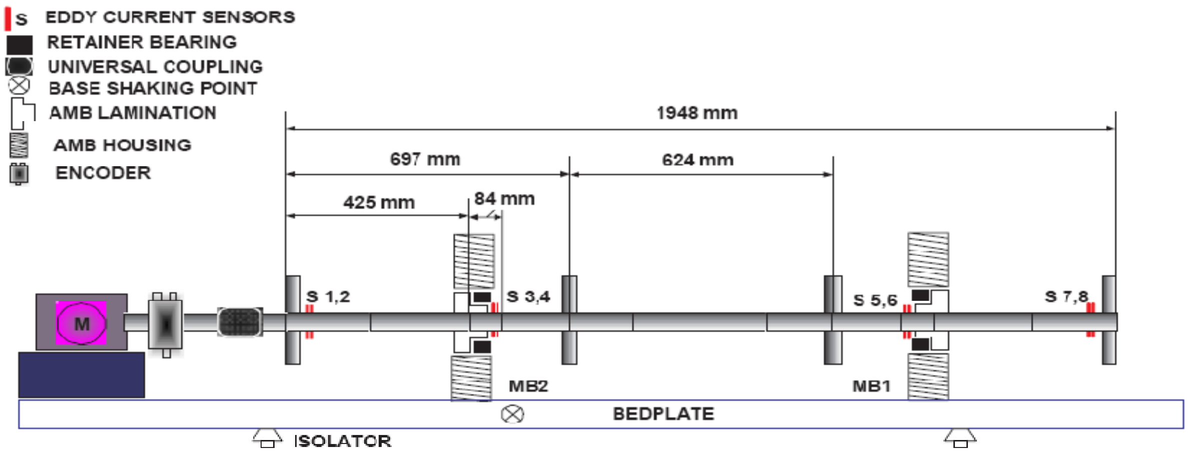

FIGURE 1: A schematic view of the experimental flexible rotor/bearing system.

auxiliary bearing when there is a sudden change in unbalance distribution. The controller can also minimise the contact forces and hence the damage if contact does occur and it can recover the rotor position [12, 13].

Misalignment in rotor/bearing systems is a common fault condition and has been investigated by many researchers [14-16]. This paper considers a flexible rotor/magnetic bearing system, which exhibits critical speeds in the running speed range including rotor flexure modes. The primary objective is to assess the ROLAC controller performance under misalignment conditions which may suddenly occur in practice through a physical change of the housing position or through steady-state offset errors in the sensor measurement signals.

SYSTEM MODELLING

An existing experimental system is simulated consisting of a flexible steel rotor of length 2 m and mass of 100 kg including 4 discs as shown in Figure 1. The rotor is supported by two active magnetic bearings, each of which has a dynamic force capacity of 1.75 kN. A total of 8 eddy current displacement transducers are positioned at 4 planes along the rotor to measure the displacements along two orthogonal directions. The magnetic bearings have an air gap of 1.2 mm and each is protected by a retainer bearing having a 0.75 mm radial clearance. Critical speeds were measured experimentally as 10 Hz, 17 Hz, and 28 Hz. The rotor magnetic bearing system stability is ensured through local PID controllers.

The rotor is modelled using finite elements utilising 13 nodal planes. Each node has 4 degrees of freedom corresponding to two orthogonal linear displacements x_i and y_i and two angular deflections θ_i and ϕ_i indicating the rotations around the x_i and y_i axes respectively. Thus the displacement vector $\mathbf{q}$ is given by:

$$\mathbf{q} = [x_1, \cdots, x_N, y_1, \cdots, y_N, \theta_1, \cdots, \theta_N, \varphi_1, \cdots, \varphi_N]^T \quad (1)$$

The linearised equation of motion for the rotor/bearing system is written as:

$$\mathbf{M\ddot{q}} + (\mathbf{C} + \Omega\mathbf{D})\mathbf{\dot{q}} + \mathbf{K} \quad (2)$$

where $\mathbf{M}$ is the mass and inertia matrix; $\mathbf{K}$ and $\mathbf{C}$ are the stiffness and damping associated with the rotor and the bearings; $\mathbf{D}$ is the gyroscopic matrix; and $\mathbf{f}$ is the vector of control forces, moments and external forces [17,18].

MAGNETIC BEARING MODELLING

Active magnetic bearing forces are generated by applying control current to the coils. The configuration of two opposing poles makes it possible to generate positive and negative forces. In the differential mode, the control current is added and subtracted from bias current to generate the driving control current.

Defining x as the position of the rotor centre from the centre of two opposing magnetic poles, and g_o the radial clearance between the rotor and the magnetic coils (g_o = 1.2x10^{-3} m), the force generated in the x-direction can be written as:

$$f_b = B\left[\frac{(i_b + i_c)^2}{(g_o - x)^2} - \frac{(i_b - i_c)^2}{(g_o + x)^2}\right] \quad (3)$$

The bearing constant B is a function of the permeability of free space μ_o, the pole area A_p, the number of windings N_a, and the half angle γ between double pole faces.

$$B = \frac{\mu_o A_p N_a^2}{4} \cos\gamma \qquad (4)$$

The magnetic forces can be linearized for small variations around a static eccentricity of ε, and bias current of i_b as follows:

$$f_b = K_x x + K_c i_c \qquad (5)$$

The current gain coefficient K_c and the inherent negative bearing stiffness K_x are given by:

$$\mathrm{K_c} = \left.\frac{\partial f}{\partial i_c}\right|_{\substack{x=\varepsilon \\ i_c=0}} = 4Bi_b\left[\frac{g_o^2+\varepsilon^2}{(g_o^2-\varepsilon^2)^2}\right] \qquad (6)$$

and

$$\mathrm{K_x} = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\varepsilon \\ i_c=0}} = 4Bi_b^2 g_o\left[\frac{g_o^2+3\varepsilon^2}{(g_o^2-\varepsilon^2)^3}\right] \qquad (7)$$

The inherent negative magnetic stiffness causes bearing instability, and position feedback is required to generate a control current:

$$I_c = -K_p x \qquad (8)$$

giving the following bearing dynamics:

$$f_b = -K_b x = -(K_p K_c - K_x)x \qquad (9)$$

The proportional feedback gain k_p should be large enough to produce a positive effective bearing stiffness k_b. A local PID controller is designed for a concentric operation of the magnetic bearing (ε = 0) with a proportional feedback gain set to K_p = 5.88x10^3 NA/m. The current gain coefficient K_c = 510.2 A. The inherent negative magnetic bearing stiffness is calculated as K_x = 1.96x10^6 N/m. This gives an effective bearing stiffness of K_b=1.04x10^6 N/m. The derivative action is tuned to give a damping of 5000Ns/m. Integral action is introduced to centralise the rotor, and balance the weight of the rotor.

Any deviation of the rotor static centre from the bearing centre will change the dynamic characteristics K_c and K_x as described in equations (6) and (7), respectively. The displacement transducers are normally calibrated with respect to the auxiliary bearings. Therefore, any misalignment between auxiliary bearings and the magnetic poles would result in non-concentric operation of the magnetic bearing (mainly due to integral action). This is examined later.

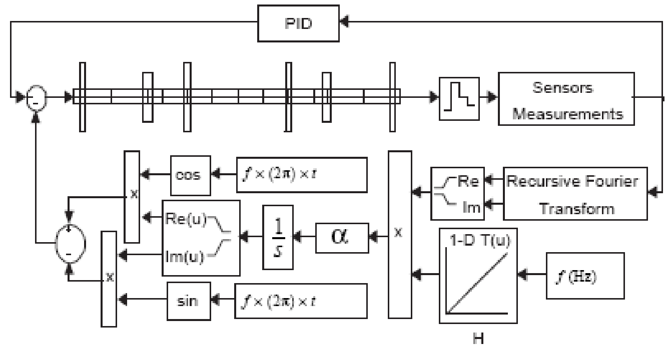

FIGURE 2: Block diagram illustrates the recursive open-loop adaptive controller (ROLAC).

RECURSIVE OPEN-LOOP ADAPTIVE CONTROLLER (ROLAC)

The open-loop adaptive control (OLAC) algorithm [4] minimizes the sum of squares of measured vibrations along the rotor and calculates the optimum change in the control force by a Least Square Estimator as follows:

$$\begin{aligned}\Delta\mathbf{U}(j\Omega) &= [\mathbf{R}(j\Omega)^T\mathbf{R}(j\Omega)]^{-1}\mathbf{R}(j\Omega)^T\mathbf{Q}(j\Omega) \\ &= \mathbf{H}(j\Omega)\mathbf{Q}(j\Omega)\end{aligned} \qquad (10)$$

where $\mathbf{Q}(j\omega)$ is the frequency response vector containing all eight measurement locations, $\mathbf{H}(j\omega)$ is the complex control gain matrix, and $\mathbf{R}(j\omega)$ is the receptance matrix between the measured and control signals. The control force in the time domain can be constructed as follows:

$$\mathbf{u}(\mathrm{t}) = \mathrm{Re}[\mathbf{U}]\cos(\Omega t) - \mathrm{Im}[\mathbf{U}]\sin(\Omega t) \qquad (11)$$

The matrix $\mathbf{H}(j\omega)$ is a function of the system dynamics and does not depend on the external excitation. The control matrix $\mathbf{H}(j\omega)$ can be determined either from the system model, or measured experimentally. The ability to estimate the complex partial receptance matrix $\mathbf{R}(j\omega)$ and hence the $\mathbf{H}(j\omega)$ matrix in-situ (self-tuning property) makes the OLAC applicable to systems where there are no reliable mathematical models. If there are multiple frequency excitations, then parallel controllers can be designed for each frequency component.

The recursive version of the open-loop adaptive control algorithm updates the optimum control force at every sampling interval through an integrator as shown in Figure 2 with the following control force expression:

$$\mathbf{U}(j\Omega, \mathrm{t}) = \alpha \int \mathbf{H}(j\Omega)\mathbf{Q}(j\Omega, \mathrm{t})\, dt \qquad (12)$$

where $\mathbf{Q}(j\Omega, t)$ is the recursive Fourier transform of the displacements and can be calculated as follows:

$$\mathbf{Q}(j\Omega, \mathrm{t}) = \frac{\Omega}{\pi} \int_{\tau=\mathrm{t}-2\pi/\Omega}^{\mathrm{t}} \mathrm{q}(\tau) \mathrm{e}^{-j\Omega\tau}\, \mathrm{d}\tau \qquad (13)$$

where $\mathbf{q}(t)$ is the time domain measurement vector.

The integral constant α can be selected by trial and error either by using numerical tests on a simulated system or from physical measurement. Higher values give faster but oscillatory transient response, whereas low values make the controller slow-acting.

The controller was initially tested by using a computer model of the system [12]. The contact dynamics are simulated through a constrained Lagrangian modelling approach developed earlier [18], which is computationally efficient and does not require modelling of the contact forces. The controller was then applied in real time to the experimental flexible rotor supported by two AMBs [13]. The behaviour of the controller under non-concentric operations is studied in the next section.

SIMULATION RESULTS

Computer simulations were carried out to asses the effect of misalignment, i.e. non-concentric operation of the bearing, on the performance of the ROLAC. As stated earlier, the static eccentricity ratio, ε/g_0, alters the dynamics of the magnetic bearing in accordance with equations (6) and (7). This effect is shown in Figure 3 for the simulated system. The amplitude of the negative stiffness coefficient increases exponentially with the static eccentricity. This would reduce the effective bearing stiffness and have a significant effect on the system stability. However, the static eccentricity also increases the current stiffness, which increases the effective bearing stiffness and damping. When these two factors are combined in the bottom graph of Figure 3, the overall effect of eccentricity is to reduce the bearing stiffness. More importantly, the effective bearing stiffness becomes negative for $\varepsilon/g_0>0.4$ making the system unstable. It is important to note that the ROLAC is an open-loop control strategy that does not alter the stability. Therefore, if such a significant misalignment is suspected, either the proportional gain should be increased, or the misalignment problem should be addressed. However, any misalignment below the instability threshold can be

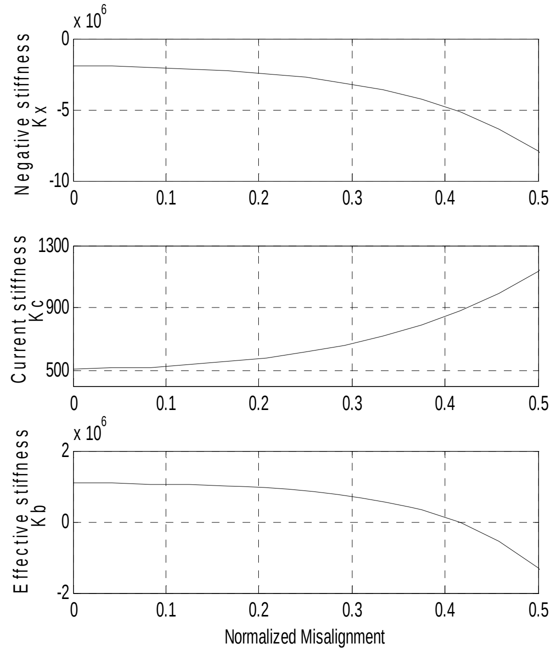

FIGURE 3: Effect of static eccentricity on the linearised magnetic bearing characteristics

accommodated by ROLAC through the auto-tuning facility, i.e. re-identification of the **H** matrix.

If there is a sudden change in the misalignment of the magnetic bearing, there may be delays before identifying the problem and re-estimating the **H** matrix, which involves injection of test signals at each force channel one at a time and waiting for the steady state responses at each stage. Therefore, the ROLAC has to work with the original **H** matrix during this period. The following simulation study was undertaken to assess the performance of the controller under this fault condition.

The simulations were run at a constant speed $\mathbf{\Omega}$ = 20 Hz with an unbalance of 10 gm attached at the non-driven end disk of the rotor with no misalignment at the bearings. Sudden rotor misalignment in the x direction of ε/g_o = 0.333 at the MB1 location was introduced after the 10^{th} cycle. Figure 4 shows the normalized radial displacements at both bearing locations. The ROLAC controller recovered the rotor position by adjusting the optimum control forces as shown in Figure 5. The complex force vector in the x direction for MB1 was (143.25 + j122.68) N before the misalignment. This was updated to (78.076+j132.44) N by ROLAC after the fault. No changes were observed in the y-direction, or in MB2. Therefore, the controller worked well despite the fact that

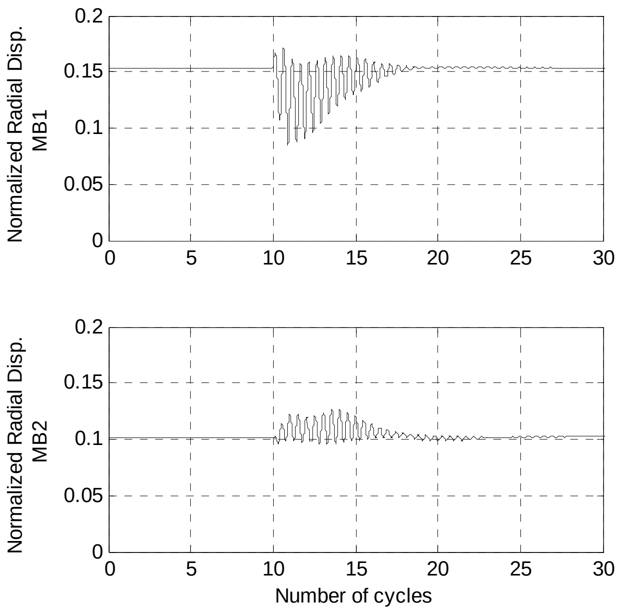

FIGURE 4: Normalised rotor radial displacements at both magnetic bearing locations following a sudden introduction of a misalignment at MB1

the **H** matrix was not updated. This was probably due to the low sensitivity of the **H** matrix and/or the optimum force with respect to changes in the bearing stiffness. The corresponding orbits at the MB1 location are shown in Figure 6, which confirms that the updated ROLAC control forces returned the rotor to its original position. Since the fault was introduced only in *x*-direction only, the orbit at MB1 became elliptic after the sudden introduction of the fault, but ROLAC returned it to a near-circular orbit.

Further simulations were carried out at other speeds (not shown here) with results consistent with those described here. Therefore it can be concluded that the re-tuning of the controller is not necessary.

CONCLUSIONS

A computer simulation of a horizontal flexible rotor and two radial magnetic bearings was undertaken to assess the capability and robustness and the effectiveness of the ROLAC when there is a misalignment in the magnetic bearings. The misalignment may be caused by the changes in the position and calibration of the displacement sensor signals used in the feedback, or due to misalignment between the auxiliary bearing and the magnetic poles. It was shown that such misalignments have the effect of increasing the magnitude of a magnetic

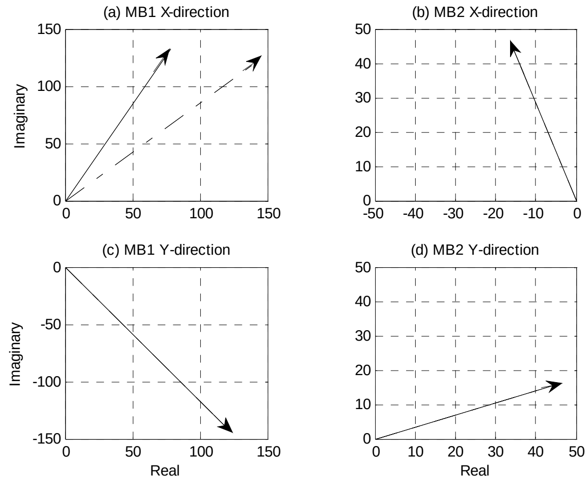

FIGURE 5: Optimum synchronous control force vector before (dashed) and after (solid) the introduction of a misalignment fault

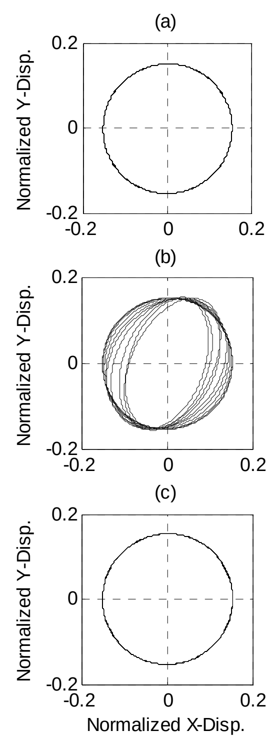

FIGURE 6: Rotor orbits at the MB1 location (a) before, (b) immediately after, and (c) after the introduction of a misalignment fault

bearing negative stiffness coefficient, and will lead to system instability at a critical eccentricity. Within the stable region of a misalignment, the ROLAC is shown to be effective for recovering the rotor position even without re-tuning the controller for the changed dynamics.

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