CROSS FEEDBACK CONTROL OF MAGNETIC BEARING BASED ON ROOT LOCUS

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ABSTRACT

The design and analysis method on cross feedback controller is presented for the flat outer-rotor flywheel. Two sets of characteristic equations of rotation are discussed on different bearing condition. Stability analysis is discussed by the comparison between conventional PD controller and controller with cross feedback. And in the end, relationship between controller's parameters and motion of nutation and precession is investigated by means of root locus analysis. The strong gyroscopic effects are overcome in the after experiments by the implementing of cross feedback control.

INTRODUCTION

For Magnetic Suspending Flywheel Systems (MSFS), classical decentralized PID controller is of high effective in many applications, however, the performance of controller is depended on system decoupling[1]. For the systems of notably nonlinearities, it is very hard to decouple the system model, even if some details of the control plant are ignored and the model is decoupled, the performance of the controller would not be good enough.

To overcome this problem, an excellent method to improve the traditional PID controller is to add cross feedback to compensate gyroscopic effects. Ahrens presented that operation performance of magnetic bearing can be improved by additional velocity cross feedback control[2]. Displacement cross feedback is discussed by Zhao L to restrain rotor's motion of precession[3]. In the aspect of Kai Zhang, the whirl motion of rotor is composed of precession and nutation, positive displacement feedback and negative velocity feedback are introduced to damp precession and nutation, and decrease gyroscopic effect remarkably[4].

Our laboratory has developed MSFS with flat flywheels suspended by hybrid magnetic bearing, when the rotation speed reaches to a certain level, the gyroscopic effects would be a dominant influence to the stability of the flywheel. To compensate the precession and nutation brought by gyroscopic effects, radial active control with cross feedback is taken in 4 radial axes (direction), besides, 1 axes passive control is designed to provide thrust in axial.

This paper discusses our work on the analysis, simulation and design of cross feedback controller. For conventional PD controller and controller with additional cross feedback, rotor's rotation equations are discussed on different bearing condition. After the corresponding characteristic equations deduced, stability analysis is studied by comparing the two sets of equations. Furthermore, relationship between controller's parameters and motion of nutation and precession is discussed by means of root loci.

ROTOR EQUATIONS OF GYRODYNAMICS

Gyro Equation without Cross Feedback

Here, the traditional bearing condition is, in another word, the control forces generated by coils in the magnetic bearing. The following diagram illustrates the stator of 4 axis active controlled MSFS developed in our laboratory, takes traditional bearing condition for consideration, the transfer function matrix for the rotor's rotation equations can be written in the form

$$\begin{bmatrix}\alpha(s)\\ \beta(s)\end{bmatrix}=\begin{bmatrix}\dfrac{G(s)}{\left[G(s)\right]^2+\left(J_p\Omega s\right)^2} & -\dfrac{J_p\Omega s}{\left[G(s)\right]^2+\left(J_p\Omega s\right)^2}\\ \dfrac{J_p\Omega s}{\left[G(s)\right]^2+\left(J_p\Omega s\right)^2} & \dfrac{G(s)}{\left[G(s)\right]^2+\left(J_p\Omega s\right)^2}\end{bmatrix}\begin{bmatrix}M_x(s)\\ M_y(s)\end{bmatrix} \tag{1}$$

As it is shown in FIGURE 1, sensor are distributed quartered in the upper and down rings, usually, the 4 axial active displacement controllers are the same with each other, and power amplifiers for these 4 channels are also the same. Denotes the controller by transfer function $G_c(s)$, and define $K_0 \triangleq l^2 K_s K_i$, where l is sensors' distance between upper and down rings, for

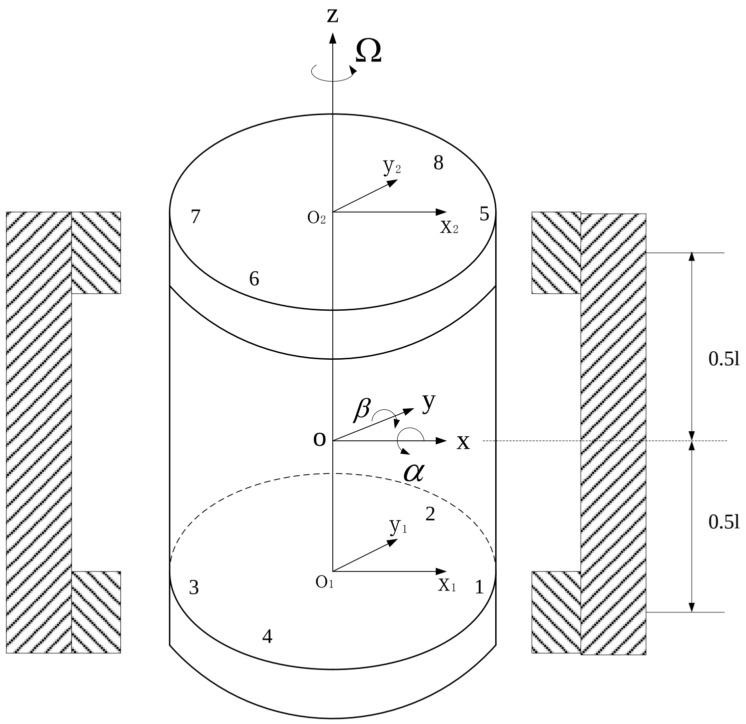

FIGURE 1: Coordinates and Sensors Place for Flywheel of 4 axis HMB

example, the distance between sensor 4 and sensor 6; K_s denoted sensitivity of the sensors, K_i is current stiffness.

Here in Eq.(1), denotes $G(s)$ as $G(s) = J_d s^2 + K_o G_c(s) G_p(s)$, J_d is rotor's radial moment of inertia, and J_p is polar moment of inertia, Ω is rotor's rotation speed in Z direction, M_x is outside moment applied to the rotor in X direction, M_y is outside moment to the Y direction.

Gyro Equation with Cross Feedback

FIGURE 2 illustrates cross feedback in the traditional decentralized PID controller, the gyro equation with cross feedback is written as

$$\begin{aligned} J_d\ddot{\alpha} + J_p\Omega\dot{\beta} &= M_x + M_x^c(\alpha) + C_x^c(\beta) \\ J_d\ddot{\beta} - J_p\Omega\dot{\alpha} &= M_y + M_y^c(\beta) + C_y^c(\alpha) \end{aligned} \quad (2)$$

Here in eq.(2), $M_x^c(\alpha)$ denotes the control moment in X direction generated by translation controller to compensate the change of α ; $C_x^c(\beta)$ denotes the control moment in X direction generated by cross feedback controller to compensate the change of β ; $M_y^c(\beta)$ denotes the control moment in Y direction generated by translation controller to compensate the change of β ; $C_y^c(\alpha)$ denotes the control moment in Y direction generated by cross feedback controller to compensate the change of α .

Cross feedback signal could be constructed after flywheel's rotation speed measured, and then, the transfer function matrix for the rotor's motion equation with cross feedback will be

$$\begin{bmatrix} \alpha(s) \\ \beta(s) \end{bmatrix} = \begin{bmatrix} \dfrac{G(s)}{[G(s)]^2 + [C_r(s)]^2} & -\dfrac{C_r(s)}{[G(s)]^2 + [C_r(s)]^2} \\ \dfrac{C_r(s)}{[G(s)]^2 + [C_r(s)]^2} & \dfrac{G(s)}{[G(s)]^2 + [C_r(s)]^2} \end{bmatrix} \begin{bmatrix} M_x(s) \\ M_y(s) \end{bmatrix} \quad (3)$$

In Eq.(3), denotes $C_r(s)$ as

$$C_r(s) = J_p\Omega s + K'_\Omega K_0 \Omega K_m(s) C(s) G_p(s)$$

defines K'_Ω as the coefficient of Frequency-Voltage conversion, K_m is the proportional coefficient for the multiplier, and $C(s)$ is transfer function of cross feedback controller.

The main task of the controller is to keep flywheel's stability when suspended by magnetic bearing and driven by motor, especially in high rotational speed operation. From Eq.(3), it is clearly to see that the 4 components' pole points are the same in the matrix.

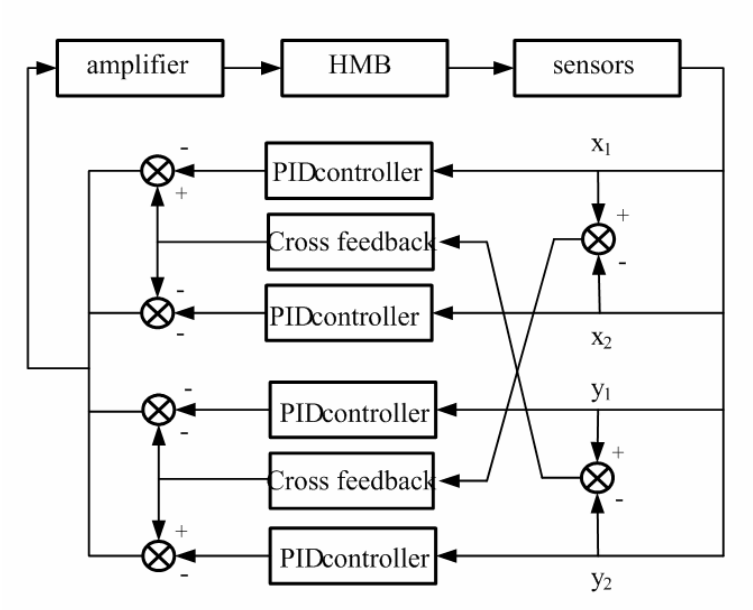

FIGURE 2: System Architecture of Cross Feedback Controller

Thus, the system's pole points can be derived by

$$\left[G(s)\right]^2 + \left[C_r(s)\right]^2 = 0$$

Or in conjugated form

$$G(s) \pm iC_r(s) = 0$$

STABLITY ANALYSIS BY ROOT LOCUS[5]

Model without Cross Feedback

The implemented displacement controllers are composed of 2 level PD controllers, view the power amplifier as 1st order sector, the transfer function of the controller can be written as

$$G_c(s) = K_c \frac{(1+\alpha Ts)^2}{(1+Ts)^2}$$

and power amplifier as

$$G_p(s) = \frac{K_p}{1+\tau_p s}$$

Here, assumes $C(s) = 0$, then we get

$$G(s) = J_d s^2 + K \frac{(1+\alpha Ts)^2}{(1+Ts)^2(1+\tau_p s)} \qquad (4)$$

while $K = K_0 K_c K_p$. And defines $C_r(s) = J_p \Omega s$, we obtain

$$G(s) + iC_r(s) = 0$$

Form the equation $G(s) - iC_r(s) = 0$ we can derive the set

$$\{p_i, p_i\} \quad (i=1,2,\cdots 5)$$

it is the roots of the characteristic equation when flywheel system operated at the rotational speed $\Omega = 0$, and the set $\{z_j, z_j\}$ $(j=1,2,\cdots 4)$ is the root for the characteristic equation when operated at the rotational speed $\Omega = \infty$.

Thus the locus equation could be written as

$$G_0(s) = \frac{K' \prod_{j=1}^{4} (s - z_j)^2}{\prod_{i=1}^{5} (s - p_i)^2} = -1 \qquad (5)$$

Here,

$$K' = \left(\frac{J_p}{J_d} \Omega \right)^2$$

The root locus of Eq.(5) is the same as system characteristic equation $G(s) \pm iC_r(s) = 0$, when K' varied from 0 to infinite.

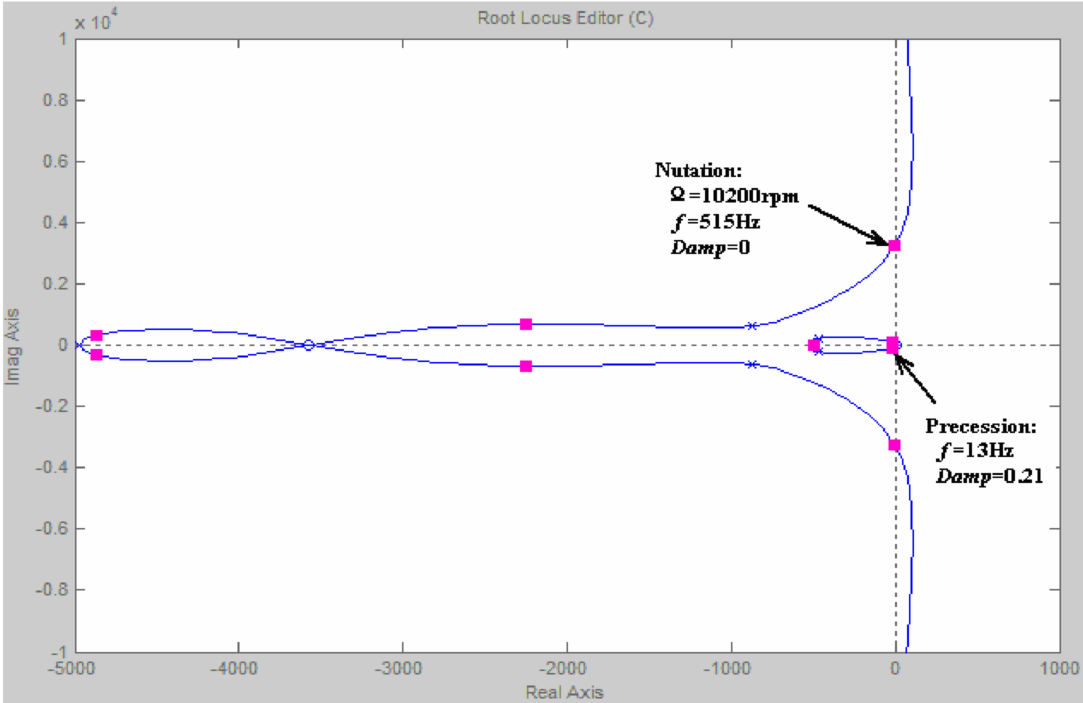

FIGURE 3: Root Locus of Magnetic Suspending Flywheel without Cross Feedback Controller

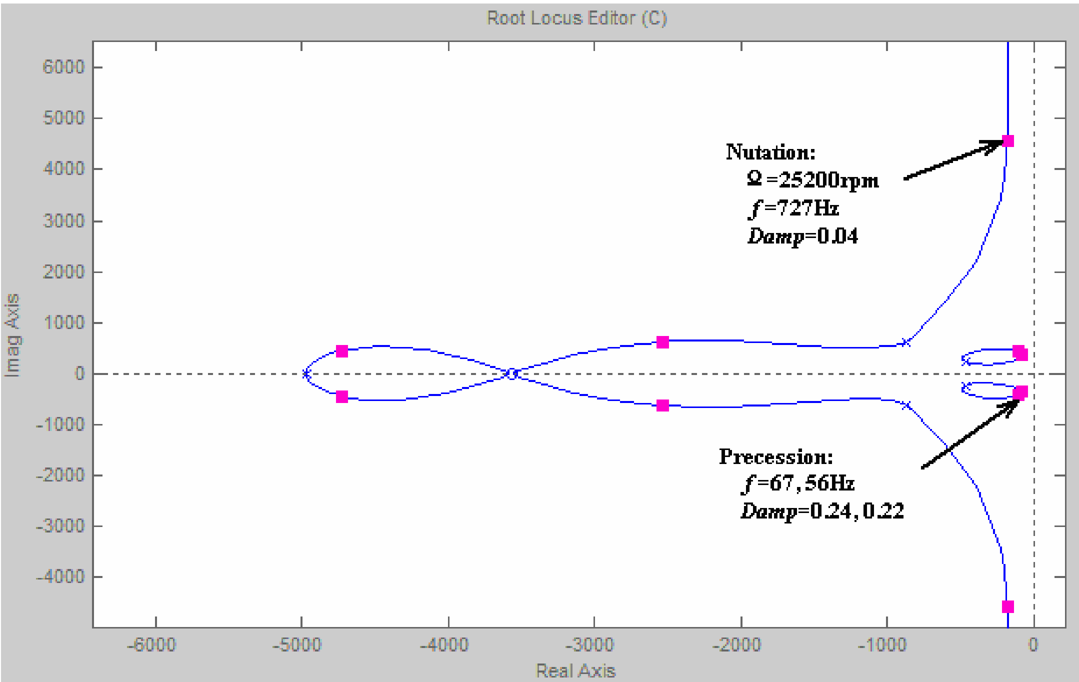

FIGURE 4: Root Locus of Magnetic Suspending Flywheel with Cross Feedback Controller

Model with Cross Feedback

Supposed the cross feedback controller $C(s)$ is described as

$$C(s) = K_1 - K_2 \frac{T_2 s}{1+T_2 s} = \frac{K_1(1-\gamma T_2 s)}{1+T_2 s}$$

Define

$$\gamma = \frac{K_2}{K_1} - 1$$

$$K_r \triangleq K'_\Omega K_0 K_m K_1 K_p$$

We obtain

$$\begin{aligned} & J_d s^2 (1+Ts)^2 (1+\tau_p s)(1+T_2 s) \\ & + K(1+\alpha Ts)^2 (1+T_2 s) \\ & \pm i\Big[J_p \Omega s (1+Ts)^2 (1+\tau_p s)(1+T_2 s) \\ & + K_r \Omega (1+Ts)^2 (1-\gamma T_2 s) \Big] = 0 \end{aligned} \quad (6)$$

The rest analysis is the same as Eq.(5) in the above part of model without cross feedback, and simulation model can be set up after the discussion of control parameters and system root locus.

SIMULATION AND DISCUSSION

One of the system's parameters are list in the following as

K_i=205, K_s=2500, l=0.04, K_c=7.7, α =9.2, T=0.00028, T_p=0.002, K_p=0.5, J_d=0.013, J_p=0.021.

According to Eq.(5), root locus can be generated by MATLAB, which is shown in FIGURE 3 and FIGURE 4.

In FIGURE 3, no cross feedback considered, parts of root locus are located in the right half of *s* plane, which implies the system is not stable. As marked in FIGURE 3, nutation would be contributed to the loss of stability as the rotation speed increasing, and the damping of nutation decreases to 0 when the flywheel operated at 10200 RPM. But in actual instance, the stability is lost at the rotation speed 4900RPM, while the corresponding damping of nutation in FIGURE 3 is 0.12 and frequency is 315Hz. Comparing to simulation results in FIGURE 3, actual damping must larger than certain value to keep flywheel's stability.

Defines parameters of cross feedback controller as

K_1=7.4, K_2=360, T_2=4.0e-5, K'_Ω =0.000416, K_m=5.5.

They are related to hardware design of the controller, and K_1, K_2 and T_2 should be designed as adjustable circuits in analog device, however, left program to choose in digital controller.

The root locus is shown in FIGURE 4 according to Eq.(6).

Compared FIGURE 4 with FIGURE 3, there are mainly two changes:

(1). Root locus in FIGURE 4 are all in the left half of *s* plane, theoretically, indicates that the rotor will never lost stability no matter what level the speed increased to. However, the rotor would lose its stability as the damping decreased to a certain value in actual operation;

(2). There are two precession frequencies, and the frequency of nutation is increased, a minimum 0.2 precession damping is noticed and will bring benefits to the control of whirl motion.

The improvement after adding cross feedback control shows advantages in MSFS operation, the rotation speed for the flywheel reaches as high as 13100RPM before the rotor lost its stability, the damping of nutation is 0.11 at this speed correspond to FIGURE 4 in the simulation. In fact, in order to increase stability margin, the damping of nutation and precession should be larger than 0.3.

SUMMARY

A new design and analysis method of cross feedback controller is discussed to restrain strong gyroscopic effect of flat outer-rotor flywheel. Comparing the two controller with or without cross feedback, rotation equations and corresponding characteristic equations of flywheel are studied to analyze rotor's stability. The relationship between controller's parameters and motion of nutation and precession is discussed by means of root locus analysis.

The gyroscopic effects are controlled effectively by the cross feedback controller, stability achieved while the flywheel suspending and rotating at the speed of 13100RPM.

REFERENCES

1. Gerhard Schweitzer, Hannes Bleuler, Alfons Traxler, Foundation, Performance and Application of Active Magnetic Bearings, Beijing, New Time Press, 1997

2. Ahrens M, Ladislav K, Larsonneur R. Performance of a Magnetically Suspended Flywheel Energy Storage Device. IEEE Transactions on Control Technology, 1996, 4（5）: 494～502

3. Zhao L, Zhang K, Zhu R S, etc. Experimental research on a momentum wheel suspended by active magnetic bearings. Proceedings of the 8th International Symposium on Magnetic Bearings. Mito, Japan, 2002: 605～609

4. Kai Zhang, Study on Magnetic Suspending Momentum Flywheel [D], Tsinghua University, Beijing, 2004

5. Kai Xiao, Control on Hybrid Magnetic Bearing and Flywheel [D] , National University of Defense Tech., Changsha, 2006