The Use of Wavelet Analysis for the Closed Loop Control of Vibration in Magnetic Bearing Systems

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***Abstract*— Wavelet analysis provides a method of identifying localised time-frequency components within a vibratory signal. It also has direct links with digital signal processing. This paper focuses on the use of wavelet analysis within rotor/active magnetic bearing systems for fault detection, system identification and closed loop transient control. The closed loop controller is formulated in the wavelet coefficient domain using real-time evaluation of the wavelet transform, which involves a signal processing delay. The controller is designed to mimimise the wavelet coefficients of measured rotor vibration signals. Hence, this implies that the rotor vibration will be attenuated by the control action. Experimental results are presented to show how control in the wavelet coefficient domain is effective in attenuating transient rotor vibration arising from step synchronous input forces.**

***Index Terms*— Wavelet analysis, Transient response, Rotor vibration attenuation, Fault detection.**

I. INTRODUCTION

Wavelet analysis has been used in a wide variety of applications exploiting its localisation characteristics. These include image processing [1], [2], neural networks [3], ultrasound [4], de-noising [5], [6], communication theory [7] and vibration analysis [8], [9], [10]. The use of wavelet analysis in signal de-noising is in many ways similar to the open-loop control techniques used for synchronous rotor vibration control. The initial signal is decomposed into its frequency elements, or wavelet coefficients, upon which a decision is made in order to achieve a desired output signal. Threshold techniques of de-noising are among the most common used. Here a signal is decomposed using a discrete wavelet transform, then the smaller noise related coefficients are removed according to a threshold criterion (Frodor *et al.* [6] and Donoho [5]).

Newland [8], [11] proposes a method for analysing a recorded vibratory signal with wavelets leading to an analysis technique involving wavelet maps. It was indicated that the wavelet transform would allow for the detection of small details in a waveform that may otherwise go unnoticed. Wavelet maps of the analysed signal allow for immediate identification of any signal perturbations, while the Fourier transform of the same signal may not. Wavelet analysis is therefore a significant tool for the analysis of time varying signals. However, the methods presented by Newland are limited to post event analysis rather than for use in real-time applications. It is also worth noting that the harmonic wavelets developed have a magnitude over all time. Harmonic wavelets are also useful because of their close relation to the Fast Fourier Transform algorithm.

Harmonic wavelets were also used by Chancey *et al.* [9], [12] as a means of studying and characterising rotor dynamic vibrations. The technique uses the frequency banding capabilities of the harmonic wavelet. This forms an analysis providing good representation of a specific octave dependent frequency. If the frequency band contains only a single significant frequency then the growth function of that particular frequency can be extracted. Chancey and Flowers [9] propose that, from the growth function identification of the modal damping, the characterisation of faults is possible from transient data.

Staszewski [10], [13] makes use of the continuous wavelet transform in identifying the damping characteristics of multi-degree-of-freedom systems. Three techniques are outlined. The simplest is a cross-section of the continuous wavelet transform (CWT) along a fixed pseudo frequency. This provides a measure of the damping from the change in magnitude of a wavelet coefficient. An impulse response recovery method and the "Ridges and Skeleton" method are also presented. This is used further by Staszewski in the identification of non-linear systems [13].

Wavelet analysis has further been used to identify fault conditions in a variety of rotating machinery components including rolling element bearings [14], gear boxes [15], cracked [16] and misaligned [17] rotors. Lin and Qu [18] use the Morlet wavelet to extract features from the measured vibration of a gear-box to identify the magnitude and position of gear damage. Boulahbal *et al.* [19] identify cracks in geared systems from measured vibrations and off-line analysis using amplitude and phase wavelet maps.

Wavelets are not only limited to signal processing in off-line techniques, they are increasingly being considered for real-time systems. The use of wavelets as basis functions in a network is presented by Zhang and Benveniste [20]. This is further utilised by Lin and Huang [21] who propose an on-line approach to the control of a servomotor by realising wavelet based network-learning. The disturbance rejection of repetitive and rapidly varying features is a natural basis for wavelets and is utilised by the learning network. This method may exploit many of the attributes of wavelet analysis, however, it takes no account of the

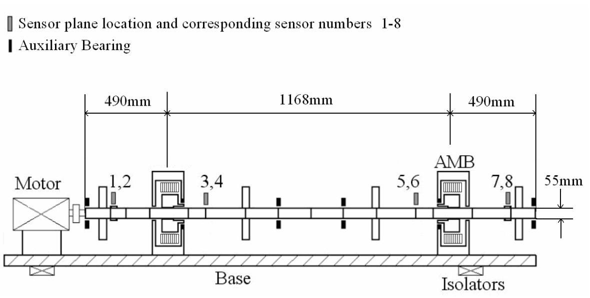

Fig. 1. Schematic diagram of experimental flexible rotor/active magnetic bearing rig.

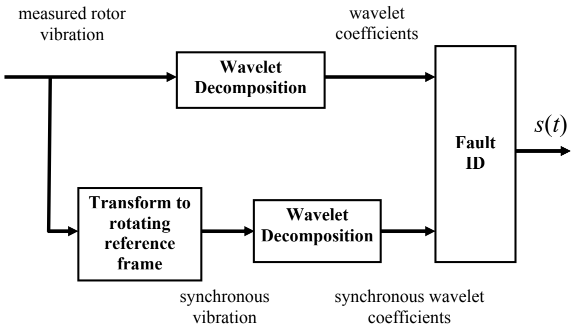

Fig. 2. Block diagram showing fault identification procedure.

digital implementation and signal processing required.

A method of control acting directly on the wavelet coefficients as gains is proposed by Parvez and Goa [22]. A generalised proportional-integral-differential (PID) controller is derived from feedback of scaled wavelet coefficients. The multiple tuneable parameters (each for a wavelet coefficient level) offer advantages over PID control, however, steady state error will always be present since a wavelet of infinite time duration would be required. Zhou *et al.* [23] use a method of wavelet weighted residuals to determine a control signal for piezoelectric sensor plates.

This paper presents a overview of research undertaken to utilise wavelet analysis in the closed loop control of rotor/active magnetic bearing systems. This includes fault detection, system dynamics and closed loop control in the wavelet coefficient domain.

II. WAVELET ANALYSIS

A mother wavelet $\psi(t)$ has zero mean and compact support (finite non-zero range). Given a time signal, $f(t)$, wavelet analysis allows a multi-resolution time-frequency decomposition in the form

$$c(a,b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt \qquad (1)$$

where the variables a and b correspond to the translation and dilation of the wavelet respectively.

Strang and Nguyen [24] show how discrete wavelet coefficients can be obtained from discrete time signal by passing through filterbanks consisting of appropriate high and low pass filters. The wavelet coefficients follow from a downsampling process while the original signal may be reconstructed from the sampled coefficients through an upsampling process.

III. ROTOR/ACTIVE MAGNETIC BEARING RIG

To validate proposed methods of fault detection and closed loop control a flexible rotor/active magnetic bearing rig was used. The system (figure 1) consists of a 2m long shaft upon which four moveable disks are positioned. The rotor is supported by two active radial magnetic bearings. Each magnetic bearing has 8 poles forming four coil pairs. These are configured as two orthogonal opposing pairs and are arranged at $\pm 45^0$ to the vertical in order to maximise static load capacity. Auxiliary rolling element bearings are positioned inside the active magnetic bearings in order to prevent rotor contact with the lamination stacks. The radial clearance between rotor and auxiliary bearings is nominally 750μm. Two further bronze bush bearings are positioned at the ends of the rotor with a radial clearance of 900μm. Rotor displacement is measured using eight eddy current transducers arranged into four orthogonal pairs. Transducer pairs are positioned at the rotor ends and next to the active magnetic bearings on the inner side. Initial rotor levitation is achieved using proportional-integral-differential (PID) feedback local to the magnetic bearings. A separate motor is used to drive the system, through a flexible coupling. The motor can run at speeds up to 100Hz (6000rpm).

IV. FAULT DETECTION

During variation of the disturbance inputs acting on the rotor, localised transients may occur. Wavelet analysis has been used in a variety of applications to identify and measure transient events. A digital signal processing method was presented by Cade *et al.* [25] identifying localised transient rotor responses due to sudden variation in rotor synchronous forcing as caused by rotor mass-loss, and rotor/auxiliary bearing contact. This method focused on using the Haar wavelet and considered the transient and steady state responses in fixed and rotating reference frames. Figure 2 shows a schematic block diagram of the fault identification process. The measured rotor vibration signal $\mathbf{q}(t)$ is converted into fixed and synchronous rotating reference frame vectors, $\mathbf{q}_f(t)$ and $\mathbf{q}_r(t)$, respectively. Following wavelet decomposition the wavelet coefficients are evaluated using logic to produce an output signal $s(t)$.

Experimental validation was performed using mass-loss tests. Two experiments (cases 1 and 2) were performed using two different sized mass-losses from an the same indeterminate initial state of rotor balance. In case 1 the system was excited into a synchronous orbit. In case 2 the system was excited into a synchronous orbit with repetitive rotor/auxiliary bearing contact. Figures 3 (a) and (b) show the rotor vibratory responses at the non-driven disk end due to the two disturbances. The contacts for case 2 were audible and coincide with vibration amplitudes exceeding

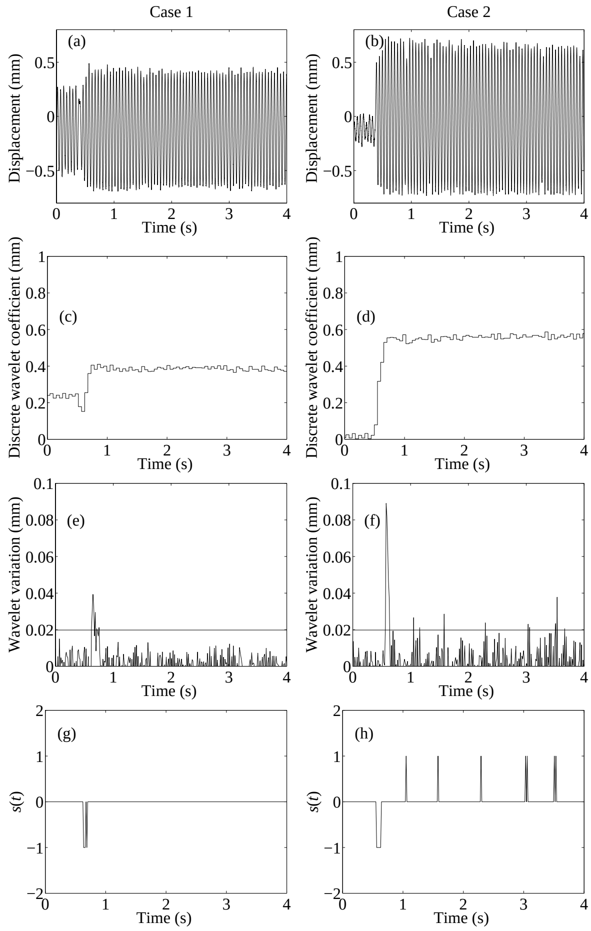

Fig. 3. (a) and (b) show measured rotor displacement in the x-axis at right hand rotor disk due to case 1 and case 2. (c) and (d) show the fixed frame wavelet coefficient variations. (e) and (f) show the rotation frame wavelet coefficients variations. (g) and (h) show the identified fault signal.

the nominal clearance. Figures 3 (c) and (d) shows the responses of the synchronous measured wavelet coefficients observed in a fixed reference frame for both cases. Figures 3 (e) and (f) show the variation in synchronously rotating frame wavelet coefficients corresponding to $4\times$ the synchronous frequency of the rotor in order to identify the sudden rotor/bearing contacts. In order to identify faults a thresholding method may be applied. Figures 3 (g) and (h) show a possible fault identification signal evaluated from wavelet coefficient thresholding and a comparison of the wavelet coefficients present in each reference frame. Negative logic indicates sudden rotor unbalance and positive logic indicates rotor/auxiliary bearing contact.

V. SYSTEM DYNAMICS IN THE WAVELET COEFFICIENT DOMAIN

When designing a system or controller it is important to have an accurate system model. Therefore, when considering the system in the wavelet coefficient domain it is important to have a dynamic model for the system wavelet coefficients. In standard notation the system transfer function can be represented in the Laplace transform domain as

$$\mathbf{Q}(s) = \mathbf{G}(s)\mathbf{F}(s) \tag{2}$$

where $\mathbf{Q}(s)$, $\mathbf{F}(s)$ and $\mathbf{G}(s)$ represent the system response, disturbance and plant respectively. When considering the wavelet coefficient domain a similar expression can be derived in the form

$$\mathbf{Q}(a,s) = \mathbf{G}(a,s)\mathbf{F}(a,s) \tag{3}$$

where equation (3) is specific to a wavelet dilation, a. However, in order to implement a wavelet based algorithm the system dynamics need to be expressed in the Z-transform domain. The discrete time dynamics of the system can be expressed as

$$\mathbf{Q}_{p,q}(z) = \mathbf{G}^0_{p,q}(z)\mathbf{F}^0(z) - \sum_{a=0}^{\infty}\sum_{b=0}^{2^a-1} \mathbf{G}_{p,q,a,b}(z)\mathbf{F}_{a,b}(z) \tag{4}$$

where a, b, p and q are the wavelet coefficient dilations and translations corresponding to the system disturbance and response. $\mathbf{G}^0_{p,q}(z)$ and $\mathbf{G}_{p,q,a,b}(z)$ represent the system transfer functions corresponding to the average of the disturbance and its corresponding wavelet coefficients and are given by

$$\begin{aligned} \mathbf{G}^0_{p,q}(z) &= \sum_{l=1}^{2N}(z+1)\mathbf{V}_l\left(\mathbf{V}^{-1}\mathbf{B}\right)_l G_1(z,\lambda_l,p,q) \\ \mathbf{G}_{p,q,a,b}(z) & \\ &= \sum_{l=1}^{2N}(z+1)\mathbf{V}_l\left(\mathbf{V}^{-1}\mathbf{B}\right)_l G_2(z,\lambda_l,T,a,b,p,q) \end{aligned} \tag{5}$$

where λ_l is the l^{th} system eigenvalue, $\mathbf{V}$ is the eigenvector matrix, and $\mathbf{V}_l$ is the l^{th} row of $\mathbf{V}$, $\left(\mathbf{V}^{-1}\mathbf{B}\right)_l$ is the l^{th} column of $\left(\mathbf{V}^{-1}\mathbf{B}\right)_l$, $\mathbf{B}$ is a force distribution matrix and T represents a time period over which the wavelet decomposition is expanded. $G_1(z,\lambda_l,p,q)$ and $G_2(z,\lambda_l,T,a,b,p,q)$ represent the characteristics of the wavelet used. A theoretical framework has been developed by Cade [26] to identify analytical expressions for the system dynamics in the wavelet coefficient domain. However, for practical applications a direct measurement approach may be undertaken. From consideration of the measured response of the system to step changes in the wavelet coefficient disturbance applied through individual magnetic bearing control axis, the corresponding system model can be identified in the wavelet coefficient domain. Figure 4 shows the measured system response at sensor 1, $x^{(1)}$, to a step change in the applied disturbance Haar wavelet coefficients applied through the non-driven end active magnetic bearing in the x-direction at 11Hz and 18Hz.

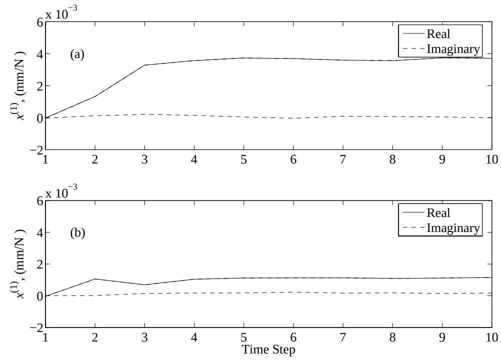

Fig. 4. (a) and (b) show measured system wavelet coefficient response to a step change in the disturbance wavelet coefficient corresponding to 11Hz and 18Hz respectively.

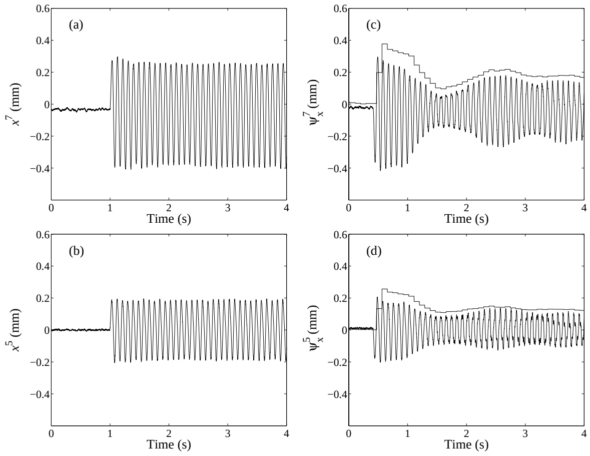

Fig. 5. Measured step synchronous force response in the *x*-axis at a rotational speed of 11Hz, showing uncontrolled responses and controlled response with wavelet coefficients.

VI. CLOSED LOOP WAVELET COEFFICIENT TRANSIENT CONTROL

Closed loop control of a rotor/active magnetic bearing system may be achieved using methods derived by Cade *et al.* [27] with control signals evaluated in the wavelet coefficient domain. The controller may be designed to suppress the transient response of the wavelet coefficient in a finite settling time. Furthermore, a prescribed transient response may be specified at the design stage to dictate the specific rate at which the wavelet coefficients are attenuated. Control force wavelet coefficients can be specified by

$$\mathbf{U}_{p,q}(z) = -\mathbf{L}_{p,q}(z)\mathbf{Q}_{p,q}(z) \tag{6}$$

where $\mathbf{L}_{p,q}(z)$ represents a wavelet coefficient controller. The closed loop transfer function of the system can be specified by expanding equation (4) to incorporate disturbance and control force wavelet coefficients as

$$\begin{aligned}(\mathbf{I} - \mathbf{G}_{p,q,p,q}(z)\mathbf{B}_u\mathbf{L}_{p,q}(z))\,\mathbf{Q}_{p,q}(z) &= \\ \mathbf{G}^0_{p,q}\mathbf{f}^0_0 - \sum_{a=0}^{\infty}\sum_{b=0}^{2^a-1}\mathbf{G}_{p,q,a,b}(z)\mathbf{F}_{a,b}(z)\end{aligned} \tag{7}$$

For a controller designed to minimise the transient response of the system at a given wavelet coefficient level and provide optimal steady state performance,

$$\mathbf{L}_{p,q}(z) = (\mathbf{G}_{p,q,p,q}(z))^* \left(\frac{1}{\alpha_{p,q}(z)(z-1)} - 1\right) \tag{8}$$

where $(.)^*$ indicated the pseudo-matrix inverse and $\alpha_{p,q}(z)$ specifies a prescribed transient response that can be expressed at each time step k as

$$\alpha_{p,q}(z) = \sum_{k=1}^{\infty}\frac{c_k}{z^k} \tag{9}$$

Experimental validation was performed using a flexible rotor/active magnetic bearing system (figure 1). Step synchronous force tests were performed by applying a sudden change in the synchronous disturbance force, 150N, acting through the non-driven end active magnetic bearing. Results are presented for step synchronous disturbance tests at 11Hz and 18Hz. These correspond to the first two nominally rigid body rotor modes of vibration. The transient wavelet controller was configured using the Haar mother wavelet to provide exponential decay to the system. However, this method allows for any prescribed transient response to be specified. Figures 5 and 7 show the rotor response at the non-driven end and non-driven end active magnetic bearing in the x-axis for the standard rotor-PID controlled case and rotor-PID system with additional transient wavelet control. Figures 6 and 8 show the total (PID plus wavelet) magnetic bearing control forces.

The controller used to achieve the results of figures 5-8 was based on an exponentional decay, with appropriate coefficients specified in equation(9). In fact, the series in equation (9) was truncated to five terms so that after five synchronous cycles the target transient response is zero. In reality, due to system model errors/uncertainties the actual rotor vibration has a residual steady state component. However, the transient response is effective. The question arises as to whether the target response specified by equation (9) could be reduced to achieve faster transient vibration attenuation. The problem that may occur is closed loop instability due to the modelling errors and also from the delays involved in evaluating the wavelet coefficients.

VII. CONCLUSIONS

An investigation has been performed considering the use of wavelet analysis within rotor/active magnetic bearing systems.

Firstly, a method of fault detection using wavelet analysis was explained in order to distinguish between fault conditions arising from mass-loss tests and due to rotor/auxiliary bearing contact. This process is based around the ability of wavelet analysis to identify localised transients and to detect the onset of a fault condition by comparing them in different reference frames. Experimental validation was performed to assess the ability of this process to detect

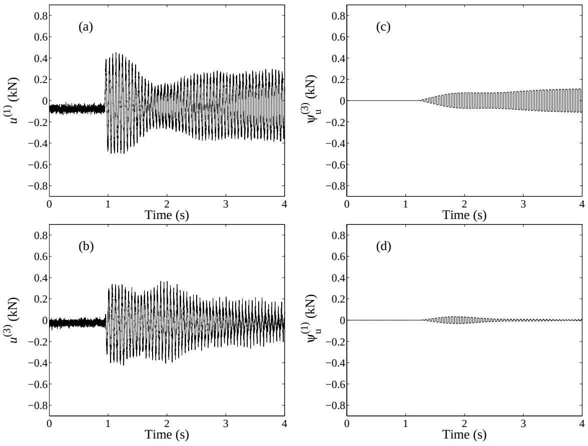

Fig. 6. (a) and (b) show the total control forces at non-driven end and driven end active magnetic bearings at a rotational speed of 11Hz. (c) and (d) show the wavelet coefficients of control force.

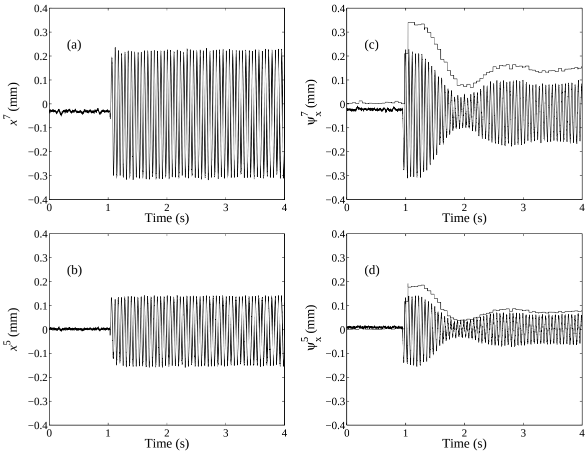

Fig. 7. Measured step synchronous force response in the x-axis at a rotational speed of 18Hz, showing uncontrolled responses and controlled response with wavelet coefficients.

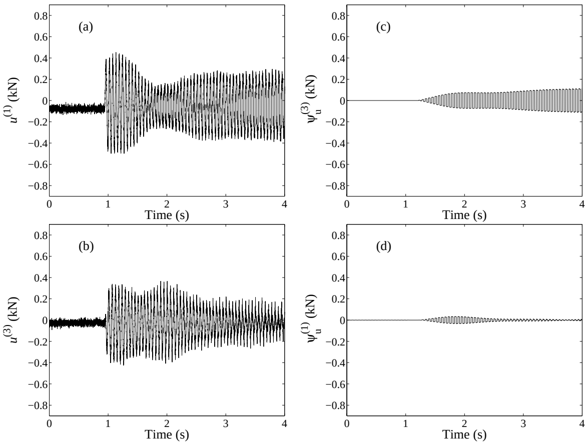

Fig. 8. (a) and (b) show the total control forces at non-driven end and driven end active magnetic bearings at a rotational speed of 18Hz. (c) and (d) show the wavelet coefficients of control force.

and identify these fault conditions and this was shown to be effective.

A closed loop control algorithm acting in the wavelet coefficient domain was evaluated and experimentally verified. The controller was designed to minimise wavelet coefficients with pseudo-frequencies matching the synchronous frequency of the rotor. The influences of modelling error and system delays in the control action mean that perfect control may not be possible with residual steady stae error and limitations on the achieveable transient vibration attenuation rates. Nonetheless, wavelet analysis is seen as a novel method for use within closed loop rotor/active magnetic bearing systems, with the potential for future improvements and applications.

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