Sliding Mode Control for Active Magnetic Bearings

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I. INTRODUCTION

The paper evaluates the sliding mode control methodology as a control law for an active magnetic bearing.

Developments in power electronics has made it possible to implement a modern control theory to various kinds of electrical and mechanical systems. This paper describes sliding mode control of a single-axis magnetic bearing's actuator.

Sliding mode control (SMC) offers several advantages over the other control methods as follows:

- robustness;
- good dynamic response;
- simple implementation.

Sliding mode control is a non-linear, model-based control method supported by the Lyapunov stability theory. It has been proved to be an efficient technique to provide good tracking performance, applied to non-linear systems with external disturbances. It also allows the plant model to be imprecise, for example a model that has been simplified due to difficulties in representing system dynamics. Among applications where sliding mode control has been applied successfully there are: submarines, ground vehicles and aeroplanes.

II. THEORY OF SMC

Systems such as active magnetic bearings are second or third order systems. The basic idea is to replace an n^{th} order system $(n \geq 1)$ by a first order system to facilitate the control. Instead of getting an *n*-dimensional state vector to track an *n*-dimensional desired time-varying state vector, the method reduces the problem to the tracking of a scalar.

The mathematical approach to the sliding mode control can be described as follows. Consider the single input non-linear system (active magnetic bearing) described by the following equation:

$$y^{(n)} = f(x) + b(x) \cdot u \tag{1}$$

where:

u – the control input;

$x = [y,\ \dot{y},\ \ldots,\ y^{(n-1)}]^T$ – the state vector.

The function $f(x)$ and control gain $b(x)$ are normally non-linear and not exactly known, but the extent of uncertainty is upper bounded by a continuous function of x. The control gain $b(x)$ is of known sign. The desired state vector:

$$x_d = \left[y_d,\ \dot{y}_d,\ \ldots,\ y_d^{(n-1)} \right]^T \tag{2}$$

is to be tracked. A tracking error vector is:

$$\tilde{x} = \left[\tilde{y},\ \dot{\tilde{y}},\ \ldots,\ \tilde{y}^{(n-1)} \right]^T = x - x_d \tag{3}$$

and represents the error of each state. These state errors are then weighted individually and together define a function $s(x, t)$, which is the weighted sum of errors:

$$s(x,t) = \sum_{i=1}^{n} \left(\frac{d}{dt} + \lambda \right)^{n-1} \cdot \tilde{y} \tag{4}$$

where λ is a design parameter. We can find error $s(x, t)$ for any n^{th} order system, for instance:

$$\begin{aligned} n=1 \quad &\Rightarrow \quad s = \tilde{y} = y - y_d \\ n=2 \quad &\Rightarrow \quad s = \dot{\tilde{y}} + \lambda \tilde{y} = \left(\dot{y} - \dot{y}_d \right) + \lambda \left(y - y_d \right) \\ n=3 \quad &\Rightarrow \quad s = \ddot{\tilde{y}} + 2\lambda \dot{\tilde{y}} + \lambda^2 \tilde{y} \end{aligned} \tag{5}$$

We see, for example, that the second-order mechanical system is replaced by a first order system.

A time-varying surface $S(t)$, called sliding surface, is defined by the scalar-equation $s(x, t) = 0$. The sliding surface of a second order system becomes a line and is presented graphically in a phase-plane *(Fig. 1).*

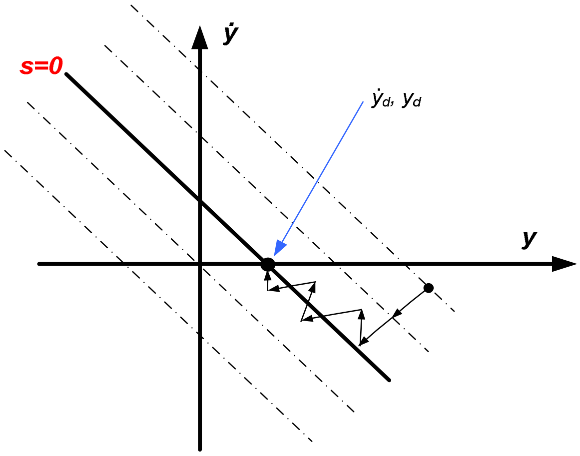

Fig. 1. The sliding surface is described by s = 0.

In a control design – the main problem is to get $x = x_d$, which is equivalent to keep system described by function *s(t)* on the sliding surface.

III. Plant of Active Magnetic Bearing

Dynamics of active magnetic bearing can be described by following equations [3]:

- mechanical part:

$$m\ddot{x} = 2k_i i + 2k_x x + F_z \tag{6}$$

- electrical part:

$$\frac{di_1}{dt} = \frac{u_1}{L_o + L_S} - \frac{Ri_1}{L_o + L_S} - \frac{k_i}{L_o + L_S}\frac{dx}{dt} \tag{7}$$

$$\frac{di_2}{dt} = \frac{u_2}{L_o + L_S} - \frac{Ri_2}{L_o + L_S} + \frac{k_i}{L_o + L_S}\frac{dx}{dt} \tag{8}$$

where:

m – mass of rotor;
k_i – current stiffness;
k_x – displacement stiffness;
x – displacement of rotor;
$i_{1,2}$ – currents in electromagnetic coils;
F_z – external force;
$u_{1,2}$ – voltages;
R – resistance of coil;
$L_{o,S}$ – inductances.

The model represented by above equations is the one used in the simualtions.

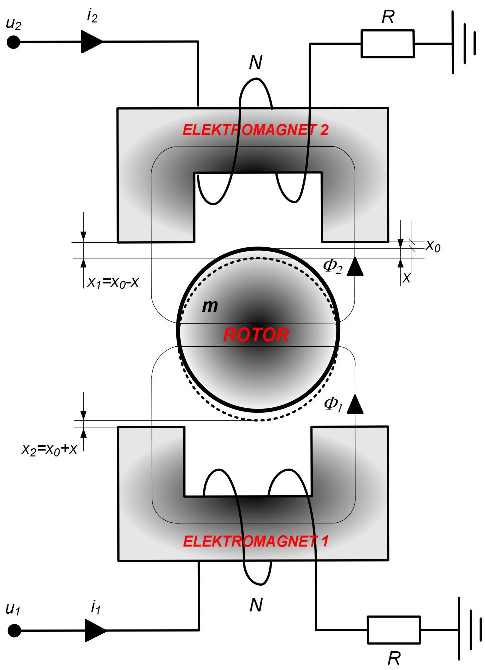

Fig. 2. A magnetic bearing system.

Figure 2 shows the scheme of a typical magnetic bearing system. The system consists of two electromagnets and of an object to be levitated. In the figure: i_1 and i_2 represent the coil currents input to the upper and lower electromagnets, respectively; x_0 denotes the nominal air gap, and x is the displacement. It is also assumed that both electromagnets have the same pole area A and identical number of turns N in the coil.

IV. Design of Controller

Classical and modern control theory bases on linear systems and provides very good design solutions. To get the advantage of linear control theory it is necessary to linearize the system at some operation point. For systems like AMB it is possible to find a feedback law for a controller so that the closed-loop system satisfies predefined specifications [6].

Due to the system nonlinearity and the constrained imposed on the dynamic response facing the variation of operational conditions, it is impossible to obtain better results with linear controller design and this fact leads us to design and use of nonlinear controllers. For stabilize the AMB system, it requires a feedback control. In order to control the rotor response, nonlinear control strategies are to be used. One such control strategy is the sliding mode control.

The mathematical model of open-loop system and controller has been implemented using Matlab-Simulink development environment, and the optimization of the controller was done by simulation aiming to improve the step response of the system.

The sliding mode controller consists of two components, one that keeps the error at zero and one that guarantees the error reaches zero. The sliding mode control law is as follows:

$$u = u_1 + k(x) \cdot sign(s) \qquad (9)$$

where:

u_1 – is an equivalent control, a level-keeping component. It is obtained from equation $\overset{\circ}{s} = 0$, which keeps the $s(x, t)$ on the sliding surface, once it is reached;

$k(x)sign(s)$ - the striking component that makes sure that the weighted sum of error s, reaches zero. It also forces the error back to zero if it becomes non-zero due to external disturbances or model uncertainties.

Once the system has reached the sliding surface, the switching component is zero.

The striking control $k(x)$ is chosen to fit the Lyapunov condition:

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\eta \cdot |s| \qquad (10)$$

where:

$\frac{1}{2}\frac{d}{dt}s^2$ contains $k(x)$, which makes sure that the function reaches the sliding surface in a finite time;

η – is a design parameter that decides about response time of the system. Greater η means that the sliding surface is reached quicker, but it also brings higher control activity (energy).

The sign function sets striking direction:

$$sign(s) = \begin{cases} -1 & for \quad s < 0 \\ 0 & for \quad s = 0 \\ 1 & for \quad s > 0 \end{cases} \qquad (11)$$

The striking and the level-keeping component cooperates to reach the desired state x_d *(Fig. 3)*.

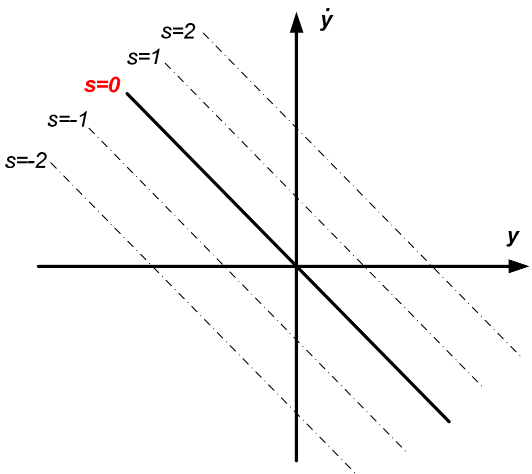

Fig. 3. Realization of SMC for second order system using sliding mode controller.

The sliding mode control is based on the model structure presented in section III.

We Introduce as a new state-space variable the following displacement error:

$$e(t) = x_1 = x_0(t) - x \qquad (12)$$

where $x_0(t)$ is a reference input. On the base dynamic equations and displacement error we get state-space model in form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{2k_i}{m}u + \dfrac{2k_x}{m}x_1 + \dfrac{F_z}{m} \end{cases} \qquad (13)$$

where:

- $k_x = \dfrac{K}{2}\dfrac{i_0^2}{x_0^3}$ - displacement stiffness;
- $k_i = \dfrac{K}{2}\dfrac{i_0}{x_0^2}$ - current stiffness.

For discontinuous control [5]:

$$u = u_0 \cdot sign(s) \qquad (14)$$

with following switching function in form:

$$s = c \cdot x_1 + x_2 \qquad (15)$$

where c, u_0 are constant, the error x_1 decays exponentially after reaching of the switching line $s = 0$ since its equation:

$$c \cdot x_1 + \dot{x}_1 = 0 \qquad (16)$$

Taking above equations into account and introducing to control law we can obtain the following equation:

$$\dot{s} = c \cdot x_2 + \frac{2k_i}{m}u + \frac{2k_x}{m}x_1 + \frac{F_z}{m} \qquad (17)$$

$$\dot{s} = c \cdot x_2 + \frac{2k_i}{m}u_0 \cdot sign(s) + \frac{2k_x}{m}x_1 + \frac{F_z}{m} \qquad (18)$$

It can be noticed that the values of switching function s and their time derivative $\acute{s}$ have opposite signs when control signal fulfil the following condition:

$$\frac{2k_i}{m}u_0 > \left| -c \cdot x_2 - \frac{2k_x}{m}x_1 - \frac{F_z}{m} \right| \qquad (19)$$

As a result of above the state of plant reaches the sliding surface $\acute{s} = 0$ in finite time. It means, the above equation indicates minimal value of control voltage which causes the error slides to zero.

In Figure 4 there is a Simulink model of the active magnetic bearing with a sliding mode controller.

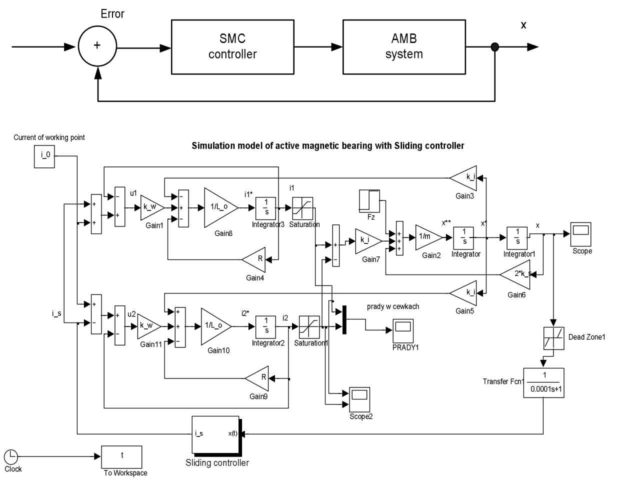

Fig. 4. Overview of the model of active magnetic bearing system.

V. Computer Simulation

Control systems with Sliding Mode Controllers (SMC) were investigated in the computer simulation.

The dynamic properties of close-loop system with SMC controllers based on the following control laws:

$$u = u_0 \cdot sign(s) \qquad (20)$$

$$u = u_0 \cdot sign(e) \qquad (21)$$

and PID controller are shown in Fig.5. The transient responses were obtained during start phase of the system.

The result of responses is that the order-reduction does not cause incorrect work of control system. But time response of the model with SMC controller given by control law:

$$u = u_0 \cdot sign(s) \qquad (22)$$

has smaller amplitude of oscilations.

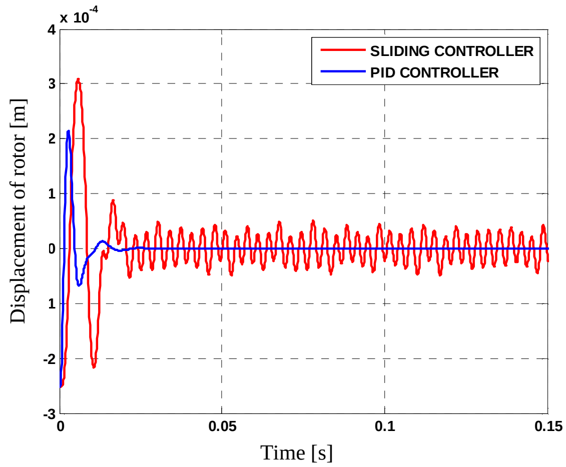

Fig. 5. Time responses of systems with SMC control rules: $s = cx_1 + x_2$ and PID controller (lifting of rotor).

The control system is characterized by slide of state-space modes on switching line called sliding surface, as it is seen in Fig. 6.

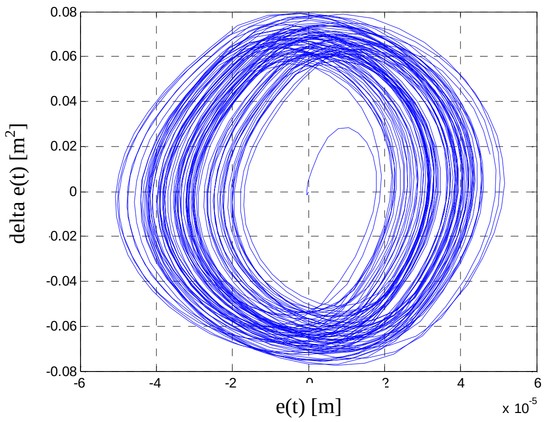

Fig. 6. Getting the sliding line by state-space variables: $x_1 = e$ and $x_2 = \dot{e}$.

The systems with SMC controllers are characterized by the changes of control signal with great frequency, it is shown in Fig. 7.

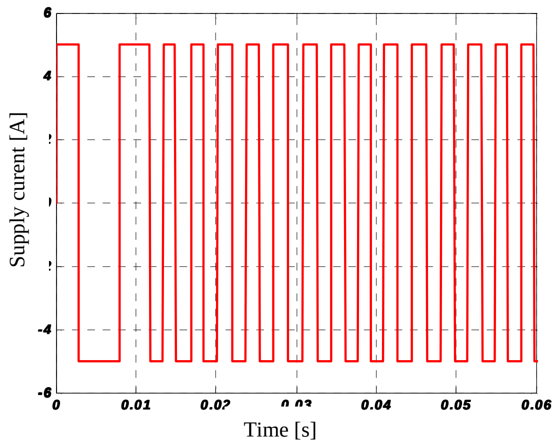

Fig. 7. Supply current during start phase.

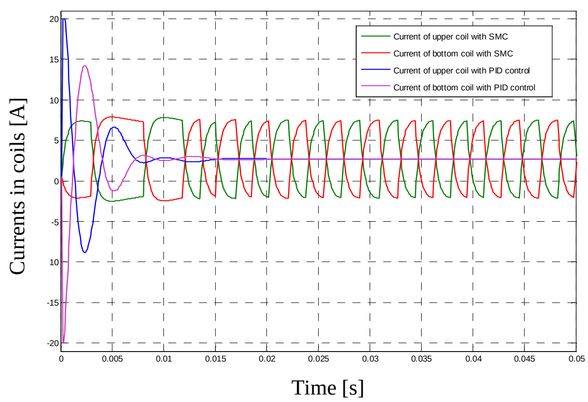

Fig. 8. Currents in electromagnetic coils during start phase.

In order to eliminate non-zero steady state error, an integral compensator was added of error to switching function in the following form:

$$s = c \cdot x_1 + x_2 + d \cdot \int_0^t x_1 dt \qquad (23)$$

where:

c and *d* were founded experimentally on the base of computer simulation. As we can see in Fig.9 the external disturbance does not shift the rotor from it neutral position.

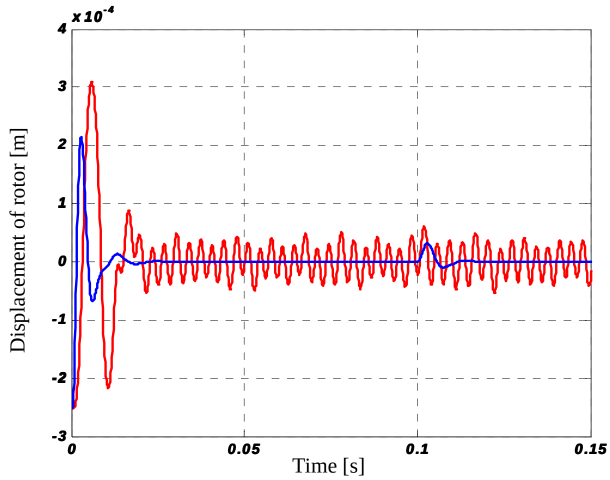

Fig. 9. Comparison of responses for systems with PID controller and SMC controller for system excited by F = 400 N.

The Bode diagram of closed-loop system is presented below:

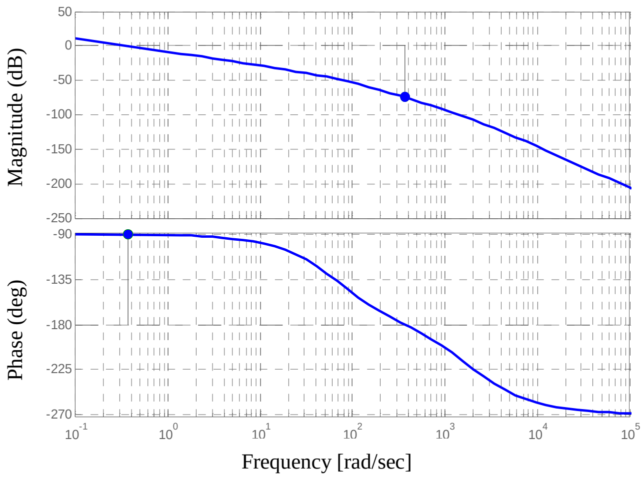

Fig. 10. Bode diagram of active magnetic bearing system.

VI. Conclusion

The paper shows possibilities of using of the sliding mode control to regulate the displacement of active magnetic bearing system.

To achieve a better performance of the system in face of the dynamic excitations the sliding mode control strategy was tuned for given application.

In sliding mode control, there are two parameters that affect the outcome of the response. In this paper these parameters were obtained on the simulation stage.

Idea of sliding mode controller which is presented above will be implemented and investigated in the lab stand with an active magnetic bearing.

VII. References

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