Control and Dynamics of a 2-Degree Rigid Rotor – Magnetic Bearing System with Time Delay

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***ABSTRACT*-The introduction of digital control into AMB systems brings us not only many benefits but also the time delay in computing control output, so called computing time delay. In this paper, a 2-degree rigid rotor magnetic system is employed and the negative effect of computing time delay on this system is discussed. Then an attempt to improve control strategies on compensation for time delay is made and a state predictive module is introduced into closed-loop control system. Simulation results show the control performance degrades owning to computing time delay, even causing instability when the value of time delay exceeds the upper bound as we discussed. And via the addition of time delay compensating module, the control performance is well improved, indicating the effectiveness of the proposed method.**

***Index Terms* – time delay, stability, control.**

I. INTORDUCTION

Significant improvements have been in the control system of active magnetic bearing in recent years. Many advanced approaches, for instance, H_∞ control[1], fuzzy control[2] and sliding mode[3], are inducted in AMB system. Advances in control strategies have generally kept pace with the need for new instrument. Increasingly complicated algorithms have then stimulated the transformation of controller's hardware, from conventional analog controller to digital one.

Digital control of magnetic bearing systems has attracted great interest.[4]-[6]Compared to analog control, digital control offers many benefits, for instance, less difficulty of changing controller component, and can satisfy the demand for high speed and high control precision[7]. However, many new problems arise when digital control is introduced in AMB system.

One of these problems is time taken by CPU to process data and execute programs for implementing control algorithms, that is, computing time delay [8] or computational time delay[9].Hence, when we consider the reliability of a digital controller for the AMB system, we will find it depends not only on the reliability of the hardware and software used, but also on the time delay in computing the control output. Shin and Cui [8] have discussed the effect of computing time delay on general real-time control systems, but analysis will be more complicated for AMB systems due to its uncertainty, highly nonlinerity and instability.

Computing time delay is intrinsic for digital control. It can be reduced by upgrading CPU's operation rate, but cannot be eliminated. Although this kind of time delay usually is very small in numerical value, its negative effect still cannot be overlooked, especially for high-speed rotating machines like magnetic bearing. Thus, it is necessary for controller designers to improve strategies and algorithms on compensation for time delay.

In the following section, we discuss the effects of time delay in digital control on system stability and dynamics, a 2-degree rigid rotor magnetic bearing system is employed for a simple case. The third section makes an attempt to improve control strategies on compensation for time delay. Section four provides the simulation results. Concluding remark is given in section five.

II. EFFECTS OF TIME DELAY ON MAGNETIC BEARING SYSTEM

A dynamic mathematical model of a 2-degree rigid rotor magnetic bearing system with time delay can be established as follow:

$$\ddot{V}(t) = U_c(t-\tau) + \omega^2 Q(t) \tag{1}$$

Where $V(t) \equiv \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, and $x(t)$, $y(t)$ is the vibration of the rotor in x, y directions on the time interval $[0,+\infty)$,, respectively. U_c is a 2×1 vector of the control force generated by magnetic actuators. Q is a 2×1 vector of the uncontrolled force normalized by the square of the rotor speed, ω . τ is the computing time delay that is introduced into feedback control loop.

Suppose that the cross stiffness coefficient of this magnetic bearing system is 0. Then we can design the controller $U_c(t)$ as follows:

$$U_c(t) = R(t) - (K_d\dot{V}(t) + K_p V(t)) \tag{2}$$

Where $R(t)$ is a 2×1 vector of the desired position,

and $K_d \equiv \begin{bmatrix} K_{dx} & 0 \\ 0 & K_{dy} \end{bmatrix}$, $K_p \equiv \begin{bmatrix} K_{px} & 0 \\ 0 & K_{py} \end{bmatrix}$ are the matrix of controller gains. One may choose proper K_{dx}, K_{dy}, K_{px} and K_{py}, such that the close-loop system is stable in condition that time delay $\tau = 0$.
Plugging (2) into (1), we derive:

$$\ddot{V}(t) = R(t) + \omega^2 Q(t) - [K_d \dot{V}(t-\tau) + K_p V(t-\tau)] \tag{3}$$

Let $U(t) \equiv R(t) + \omega^2 Q(t)$, taking Laplace transform, we get:

$$s^2 V(s) + sK_d V(s)e^{-\tau s} + K_p V(s)e^{-\tau s} = U(s) \tag{4}$$

For digital processors in use at present, τ is very small. Thus it can be approximated that:

$$e^{-\tau s} \approx 1 - \tau s \tag{5}$$

Then (4) becomes

$$s^2 (I - K_d \tau)V(s) + s(K_d - K_p \tau)V(s) + K_p V(s) = U(s) \tag{6}$$

From Routh-Hurwitz Stability Criterion, with respect to the system stability, τ should satisfy:

$$\tau < \min\left(\frac{k_{dx}}{k_{px}}, \frac{k_{dy}}{k_{py}}, \frac{1}{k_{dx}}, \frac{1}{k_{dy}}\right) \tag{7}$$

III. CONTROL STRATEGIES ON COMPENSATING TIME DELAY

Conventional controllers in AMB systems are designed without considering the computing time delay. Naturally an attempt to improve control strategies on compensation for time delay is made. One solution is proposed by Smith in 1957[10].The desired system stability and control performance can be achieved via the addition of a predictor for single-input/single-output(SISO) systems. When one takes over the idea of Smith predictor and extend it to multiple-input/multiple-output(MIMO) systems, the theories like state predictive control[11] and process model control[12] are built. Now we try to design a state predictive observer and controller as a solution for computing time delay problem in AMB digital control systems.

The design of controller consists of two parts, a state predictive observer and a displacement controller. The displacement controller is designed in term of the system without time delay, while the predictive observer compensates for the effect of time delay.

Consider the 2 –degree rigid rotor magnetic bearing system with time delay described in (1).Let $X \equiv \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$, and convert system in (1) into the following equivalent state-space form:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t-\tau) \\ \quad t > 0, X(0) = X_0 \\ \quad y(t) = Cx(t) \end{cases} \tag{8}$$

Where $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$,

$$u(t-\tau) = \begin{bmatrix} 0 \\ 0 \\ -\mathrm{u}_{cx}(\mathrm{t}-\tau) + \omega^2 q_x(t) \\ -\mathrm{u}_{cy}(\mathrm{t}-\tau) + \omega^2 q_y(t) \end{bmatrix}$$

$u(t)$ includes both controlled and uncontrolled inputs. u_{cx}, u_{cy} are controlled magnetic force in x, y directions, respectively. q_x, q_y is a the uncontrolled force in x, y directions ,respectively ,which are normalized by the square of the rotor speed, ω . .
We adopt a simple PD controller

Then an observer estimating $X(t)$ is constructed. When a full-order observer is used, the equation of
The observer is given by

$$\begin{cases} \dot{\hat{X}}(t) = (A - KC)\hat{X}(t) + Bu(t-\tau) + Ky(t) \\ \qquad t > 0 \\ \qquad \hat{X}(0) = X_0 \end{cases} \tag{9}$$

Where $\hat{X}$ is the estimated state of X .Chose the appropriate K to assign the characteristic values of $(A - KC)$ in the left half part of the complex plane.
Based on the system model in (1), the predictive model is given by

$$\begin{cases} \dot{\tilde{X}}(t+\tau) = A\tilde{X}(t+\tau) + Bu(t) \\ \qquad t > 0, \tilde{x}(0) = \tilde{x}_0 \\ \tilde{y}(t) = C\tilde{X}(t+\tau) + F[y(t) - C\tilde{X}(t)] \end{cases} \tag{10}$$

Where $\tilde{X}$ is the predictive state of X, and F is the weighted matrix for correction. $\tilde{y}(t)$ is the final predictive state serving to be reference input for the displacement controller. The state observer and the predictive model constitute the compensating module, in which the state observer offer the information of the system, and the predictive model make use of the information to predict the action of the rotor.

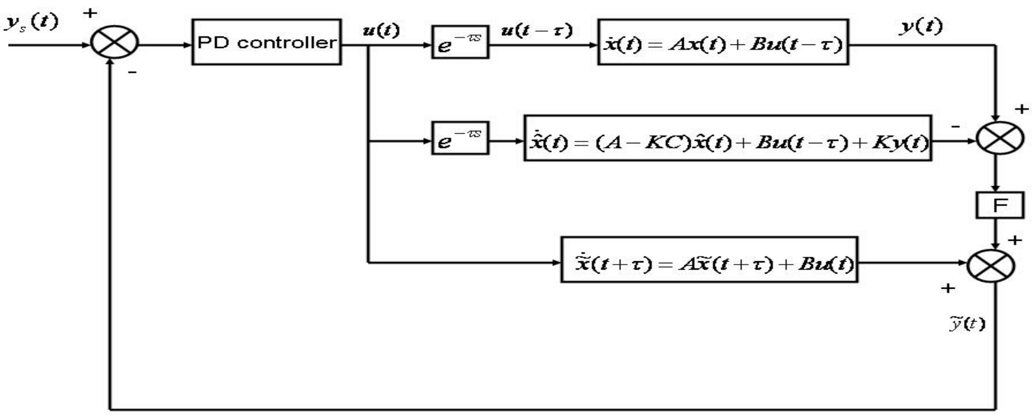


Fig.1.The AMB system in presence of computing time delay with addition of a state predictive observer

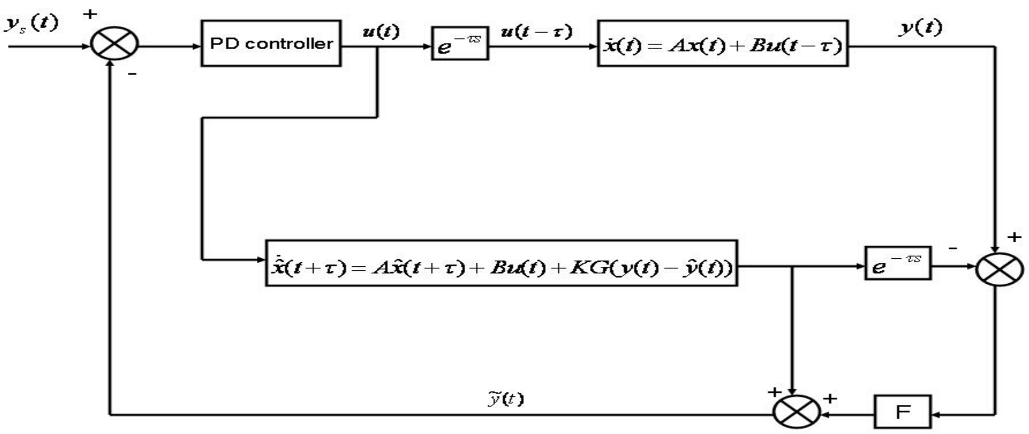


Fig.2. The AMB system in presence of computing time delay with addition of simplified compensating module

Fig (1) shows the structure of the model in (1) with addition of compensating model. When we combine the observer and predictive model into one part, the structure can be simplified into one showed in Fig. (2).

Similar to Smith predictor[10], the compensating module transfers the time delay to the outside of close-loop structure of control system to eliminate the delay elements from the characteristic equations. Thus, this method can solve the problem perfectly in theory on condition that the model parameters we derived are accurate.

VI. SIMULATION

Simulation were performed using two-degree freedom AMB with a simple PD controller model, in which the computing time delay is considered as pure time delay existing between the digital controller and AMB system. Fig. (3) Shows the structure of the integrated 2-degree rigid rotor magnetic bearing system developed in Simulink of Matlab.

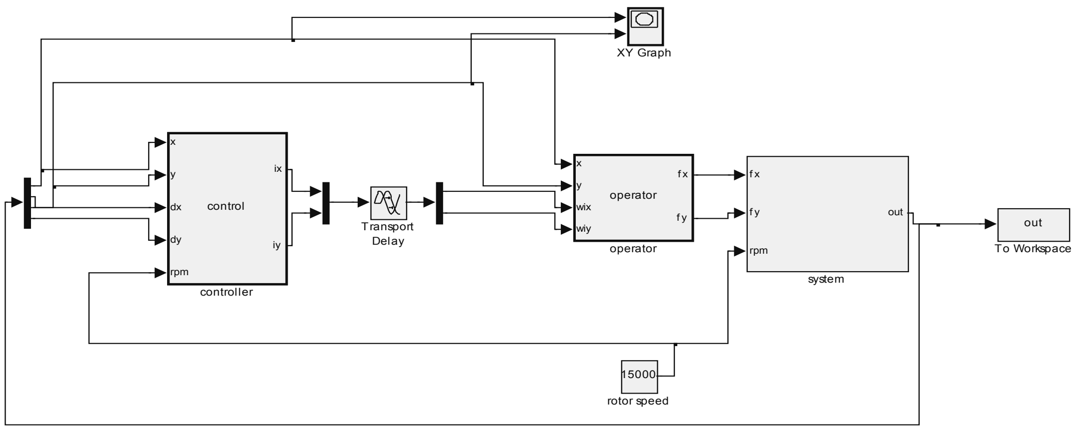


Fig.3. The 2-degree rigid rotor magnetic system with time delay in Simulink of Matlab

Firstly, we conducted control simulations to evaluate effect of computing time delay of different value on system stability and dynamic characteristics. As in (2), the control gain K_{px}, K_{py}, K_{dx} and K_{dy}, which is related to parameters of PD controller, were designed as follow:

$$K_{px} = 3.0716 \times 10^7 kg / s^2 ,$$

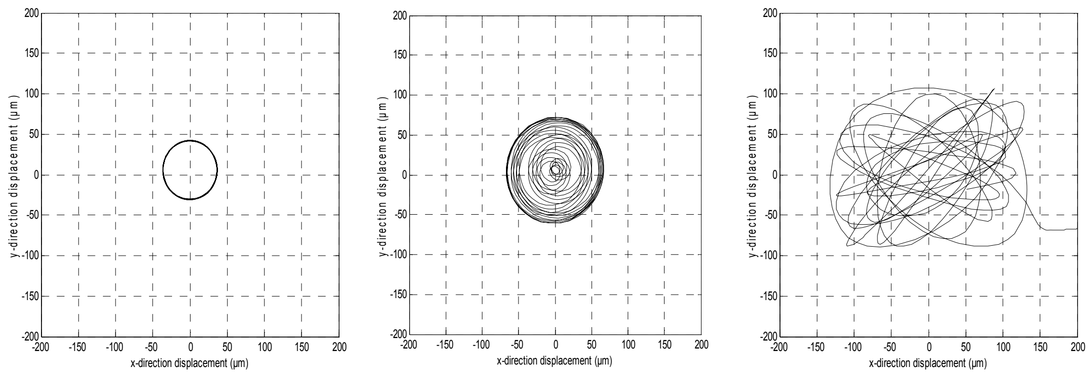
$$K_{dx} = 1.0292 \times 10^3 kg / s ,$$

$$K_{py} = 3.0716 \times 10^7 kg / s^2 ,$$

$$K_{dy} = 1.0292 \times 10^3 kg / s$$

. So from (7), time delay should satisfy

$$\tau < 3.3506 \times 10^{-5} s \quad \text{or} \quad \tau < 33.506 \mu s \tag{11}$$



(a) (b) (c)

Fig.4. The transient orbits of rotor systems with time delay τ

(a) $\tau = 0$ (b) $\tau = 32\mu s$ (c) $\tau = 34\mu s$

Fig.4(a) ,(b) and (c) show the transient orbits of rotor systems with time delay $\tau = 0$, $\tau = 32\mu s$ and $\tau = 34\mu s$,respectively. We can see that compared to the condition that there is no time delay in closed loop system as showed in Fig(a), the orbits of the rotor motion enlarge greatly and control performance degrades when $\tau = 32\mu s$ .Moreover, when τ reaches $34\mu s$,exceeding the upper bounds in (11), the system became instable.

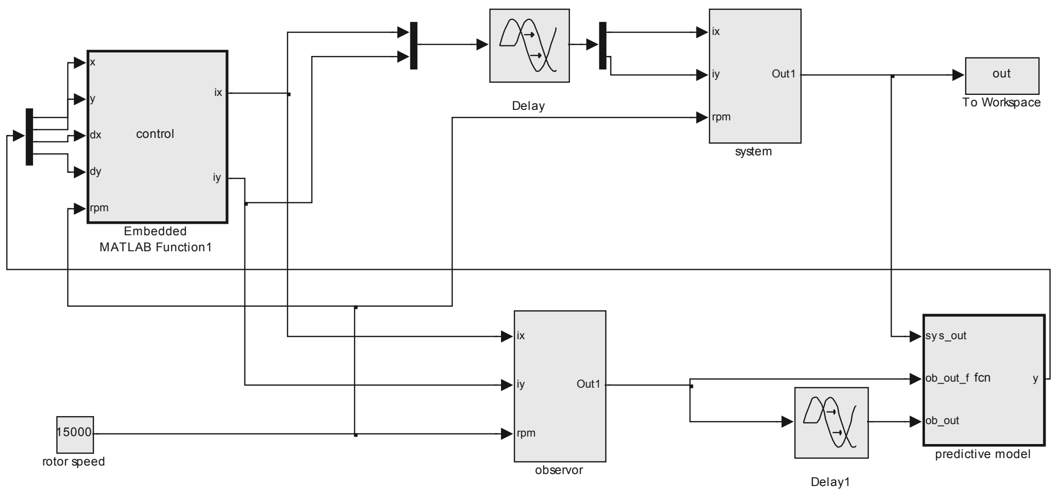


Fig.5. The time delay system with addition of compensating module in Simulink of Matlab

Then we design a predictive observer as mentioned in section 3 for this system. Fig 5 shows the structure of the time delay system with addition of a compensating module developed in Simulink of Matlab. Assume that the value of computing time delay is $32\mu s$, and a comparison is made between the system with time delay compensation, as showed in Fig 6(b), and without, as showed in Fig 6(a).

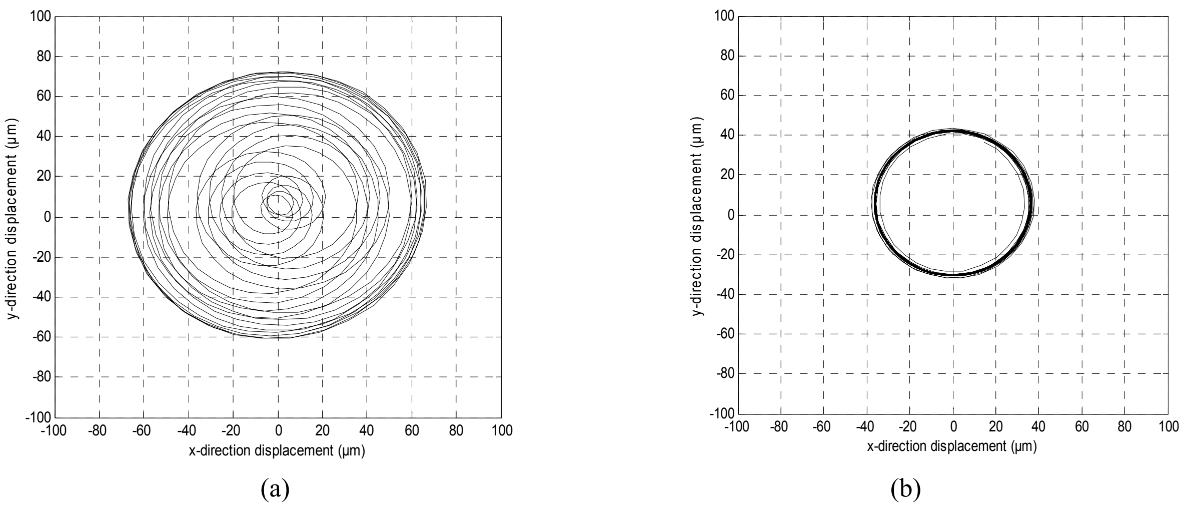


Fig.6 The transient orbits of rotor systems with time delay $\tau = 32\mu s$

(a) without compensation module (b) with compensating module

The result shows that the method presented in section 3 has been demonstrated for the purpose of compensating time delay, and a significant improvement in AMB control performance was obtained.

V. CONCLUSION

In this paper, we proposed the problem caused by computing time delay in digital control of AMB system. We discuss the negative effect of the time delay on a 2-degree rigid rotor magnetic bearing system and give the upper bounds of time delay according to system stability. Then a solution through improving control strategies is employed. We try to compensate the time delay by adding a predictive observer into closed loop. Simulation results show that the control performance of AMB system degrades owning to computing time delay, even causing instability when the time delay exceed the upper bounds given above. By the addition of compensating module, the time delay is well compensated, indicating the effectiveness of the proposed method.

Hereafter, more detailed analysis for obtaining the relations between control performances and computing time delay must be conducted. Moreover, we should find more robust solution which is insusceptible to inaccurate model parameters and insensitive to outside disturbance.

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