Magnetic Force of Radial Magnetic Bearing Considering Eddy Currents Effect

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***Abstract*— The magnetic force of a radial magnetic bearing including the eddy current effects due to the rotation of shaft and the variation of the dynamic control current is investigated in this paper. With assumptions of moderate eddy currents developed in the rotor and stator, an approximate general form of the magnetic force is derived. The results show that the eddy current due to rotation of the rotor affects the magnitude of the static force and amplitude of the dynamic force while that induced by the dynamic control current will reduce the amplitude of the dynamic force and cause phase lag. Moreover, eddy current due to rotation results in equal decrease in the magnitude or amplitude of the static and dynamic forces. The current stiffness then derived with the dynamical force shows that the varieties of the stiffness with speed has a similar trend to that of the static one. A case study was investigated by FEM and the results agree well with the analysis. Furthermore, since the flux saturation in the eddy current regions is moderate, the nonlinear model considering flux saturation gives nearly the same results as the linear one. The result is applicable to not only solid rotor and stator but also to laminated structures.**

***Index Terms*— eddy current, magnetic bearing, stiffness, stator, rotor.**

I. Introduction

Eddy currents cannot be avoided in the applications of magnetic bearing(MB). It will cause power loss and degradation of dynamic performance of the system. Eddy currents are produced in the active magnetic bearing(AMB) system as a result of: dynamic control current, alternating of magnetic field in the rotor due to rotation and magnetic reluctance change due to variation of air gap. Laminated structures are usually used to restrict eddy currents developing in the rotor and stator. However, in the cases of high rotational speeds and high current frequencies, the effects of eddy current will still be considerable.

The effects of eddy current on MB have been studied for many years. Most of these studies were focused on the power loss due to the drag force produced by the eddy currents developed in the rotor when rotating in a radial active magnetic bearing(RAMB). The power losses due to eddy currents had been investigated experimentally [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. The magnetic forces of the RAMB and power losses had been calculated using 2D FEM model [11], [12], [13], and also obtained analytically using magnetic field analysis [14], [15], [16], [17], [18], [19]. Most of these studies assumed that the rotor was rotating concentrically in the RAMB supplied only with constant DC bias currents to produce the bias flux density in the air gap.

Eddy currents in the RAMB also cause phase lag and magnitude decrease in the force/current relationship, Therefore the dynamic performance of the system will be affected. Kim [20] incorporated eddy current model into the rotor dynamics analysis to examine its effects on the whole system closed loop stability. In his study, the FEM model was used to calculate the fields and eddy currents, and the measured frequency response of the dynamic flux density of a laminated RAMB showed obvious phase lag and magnitude decrease in the frequency range of $300Hz$. Some analytic models of RAMBs were also studied to include eddy current effects into the magnetic circuit model [21], [22]. A 1-dimensional model was used to calculate the eddy current developed in the laminated bearing and rotor. But the effects of eddy currents produced by rotation were not considered in these investigations.

This paper presented a study of the relationship of the lift force to the coil current of the RAMB considering the eddy current effects due to both rotor rotation and variation of the dynamic control current. By making some assumptions on the fields distributions in the air gap when the eddy currents produced in the stator and rotor were moderate, an approximate general form of the bearing force could be derived. Then the dynamic current stiffness at different rotation speeds could be calculated. A case study using FEM will be presented to verify the analysis.

II. Magnetic field and force

If the rotor and stator of RAMB are made of solid materials, a 2-D model can be used in the analysis of eddy currents when neglecting the end effects. In reality, nearly all RAMBs are laminated in order to reduce eddy currents. Since the eddy currents are now much smaller and the relative permeability of the conductor is much great than that of air, the variation of flux distribution in the axial direction in the air gap would be small. Therefore, it is still reasonable to assume an uniform distribution of flux in the axial direction in the air gap and a 2-D model could still be used in the following analysis.

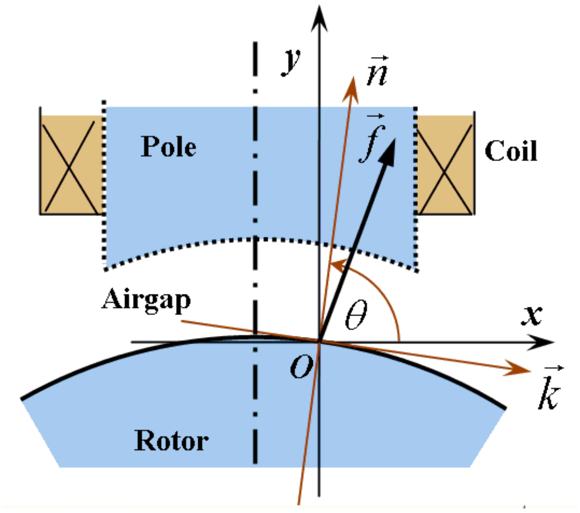

Fig. 1. Pole-rotor configuration of RAMB.

Fig.1 shows a typical pole-rotor configuration of a heteropolar RAMB. It is assumed that the rotor is rotating at steady speed Ω and with no whirling. The current of the coil i_c has two components:

$$i_c = I_0 + i \tag{1}$$

where I_0 is the bias current which produce the bias flux density in the air gap and i is the dynamic control current. Eddy currents will be induced in both the rotor and stator due to variation of the dynamic control current i. Moreover, eddy currents will also be induced in the rotor because the rotor is subjected to alternately changing polarity of the poles while rotating.

Let the bias field intensity be $\vec{H}_0$ when the rotor is not rotating and the dynamic control current $i = 0$; the additional field produced because of eddy currents developed in the rotor when it is rotating be $\vec{H}_1$; and the additional field produced due to eddy currents caused by the dynamic control current i be $\vec{H}_2$. There also exists a field $\vec{H}_3$ resulting from the coupling effect of rotor rotating and dynamic control current. Thus the magnetic field produced in the system in general can be decomposed into four components, and can be written as:

$$\vec{H}(\Omega, i_c) = \vec{H}_0 + \vec{H}_1(\Omega) + \vec{H}_2(i) + \vec{H}_3(\Omega, i) \tag{2}$$

Furthermore,

$$\vec{H}_1(0) = \vec{H}_3(0, i) = 0 \tag{3}$$

$$\vec{H}_2(0) = \vec{H}_3(\Omega, 0) = 0 \tag{4}$$

Given the magnetic field intensity $\vec{H}$, the bearing force could then be derived. Equation (2) allowed the effect of rotational speed and dynamic control current i on the eddy current and on the bearing force be studied.

Maxwell's stress tensor can be used to calculate the bearing force. The force acting on the surface of the rotor is given by:

$$\vec{f} = f_n\vec{n} + f_t\vec{k} = \frac{\mu_0}{2}(H_n^2 - H_t^2)\vec{n} + \mu_0 H_n H_t \vec{k} \tag{5}$$

where H_n is the normal component and H_t is the tangential component of the magnetic field intensity. The normal component f_n produces the lift force and f_t produces the drag force of the bearing. The total force acting on the rotor could be found by integrating $\vec{f}$ over the entire surface of the rotor in the air gap. Thus the magnetic forces per unit width of the bearing are

$$F_x = \int_L f_n cos\theta dl + \int_L f_t sin\theta dl \tag{6}$$

$$F_y = \int_L f_n sin\theta dl - \int_L f_t cos\theta dl \tag{7}$$

where the integral path L is the circumference of the rotor. The transverse force F_x is very small compared to F_y and thus could be neglected. Therefore, only the lift force F_y is considered in the following analysis.

In equation (7), the second term is very small compared with the first one because of symmetry. Thus the lift force can be approximated by

$$F_y \approx \frac{\mu_0}{2}\int_L (H_n^2 - H_t^2) sin\theta dl \tag{8}$$

Substituting (2) into (8),

$$\begin{aligned} F_y &= \frac{\mu_0}{2}\int_L [(H_{0n} + H_{1n} + H_{2n} + H_{3n})^2 \\ &\quad -(H_{0t} + H_{1t} + H_{2t} + H_{3t})^2] sin\theta dl \\ &= F_{y0} + F_{y1} + F_{y2} \end{aligned} \tag{9}$$

where

$$F_{y0} = \frac{\mu_0}{2}\int_L [(H_{0n} + H_{1n})^2 - (H_{0t} + H_{1t})^2] sin\theta dl \tag{10}$$

$$\begin{aligned} F_{y1} &= \mu_0 \int_L [(H_{0n} + H_{1n})(H_{2n} + H_{3n}) \\ &\quad -(H_{0t} + H_{1t})(H_{2t} + H_{3t})] sin\theta dl \end{aligned} \tag{11}$$

and

$$F_{y2} = \frac{\mu_0}{2}\int_L [(H_{2n} + H_{3n})^2 - (H_{2t} + H_{3t})^2] sin\theta dl \tag{12}$$

F_{y0} is a static force at constant speed Ω, F_{y1} and F_{y2} corresponding to the linear and nonlinear part of the dynamic force related to the fields $\vec{H}_1$, $\vec{H}_2$ and $\vec{H}_3$. Although the dynamic force F_{y1} is linear with fields $\vec{H}_2$ and $\vec{H}_3$, it is nonlinear with the current i because the fields are not linear with the dynamic current due to hysteresis and flux saturation.

Usually the dynamic magnetic fields $\vec{H}_2$ and $\vec{H}_3$ are small compared to $\vec{H}_0$ and $\vec{H}_1$ because the dynamic control current i is much smaller than I_0. So F_{y1} will be much greater than F_{y2} and is dominant in the dynamic fore. At lower rotational speed, the eddy currents produced in the rotor by rotation are not so large and $\vec{H}_1$ and $\vec{H}_3$ will be small compared to $\vec{H}_0$ and $\vec{H}_2$. These are the assumptions used in the following analysis to get a simplified form of the magnetic force by ignoring the higher order terms of $\vec{H}_1$ and $\vec{H}_3$.

A. Static force F_{y0}

The static component F_{y0} of the force is given by (10), and can be written as:

$$\begin{aligned} F_{y0} &= \frac{\mu_0}{2}\int_L (H_{0n}^2 - H_{0t}^2) sin\theta dl \\ &+\mu_0 \int_L [H_{0n}H_{1n} - H_{0t}H_{1t}] sin\theta dl \\ &+\frac{\mu_0}{2}\int_L [H_{1n}^2 - H_{1t}^2] sin\theta dl \end{aligned} \quad (13)$$

It is obvious that the first term is constant and only depends on the bias current I_0, The second term is linear with respect to the field H_1. Since $H_{1n}(0) = H_{1t}(0) = 0$, the magnetic force when the rotor is stationary is

$$F_{y0}(\Omega)|_{\Omega=0} = F_{y0,0} = \frac{\mu_0}{2}\int_L (H_{0n}^2 - H_{0t}^2) sin\theta dl \quad (14)$$

F_{y0} can be normalized by dividing by $F_{y0,0}$. Thus

$$\bar{F}_{y0} = \frac{F_{y0}}{F_{y0,0}} = p_1(\Omega) \quad (15)$$

and

$$p_1(\Omega)|_{\Omega=0} = 1 \quad (16)$$

B. Linear part of the dynamic force F_{y1}

Assume that the dynamic control current varies sinusoidally, that is $i = i_0 e^{j\omega t}$. Usually, the magnetic field component $\vec{H}_2$ and $\vec{H}_3$ at steady rotation speed Ω would not be sinusoidal at the given frequency of the dynamic control current because of hysteresis and flux saturation. However, if the nonlinearity is weak, the fundamental components will be dominant in the fields and can be used as approximations of the fields, and the dynamic force as well. Therefore, an approximate linear relationship between the dynamic force and the dynamic control current could be obtained with the fundamental component of the dynamic force. Since F_{y1} is linear with $\vec{H}_2$ and $\vec{H}_3$, its fundamental component can be calculated directly from $\vec{H}_2$ and $\vec{H}_3$ with (11). For simplicity, $\vec{H}_2$, $\vec{H}_3$ and $\vec{F}_{y1}$ are to denote the respective fundamental components in the following analysis.

The dynamic fields could be written as:

$$\vec{H}_2 = [\hat{H}_{2n}\vec{n} + \hat{H}_{2t}\vec{k}] e^{j\phi(\omega)} e^{j\omega t} \quad (17)$$

$$\vec{H}_3 = [\hat{H}_{3n}\vec{n} + \hat{H}_{3t}\vec{k}] e^{j\phi(\omega)} e^{j\omega t} \quad (18)$$

where $\phi(\omega)$ is the phase lag of the magnetic field with respect to the current i, $\hat{H}_{2n,t}$ and $\hat{H}_{3n,t}$ are both dependent on frequency ω and amplitude i_0.

When the dynamic control current i is much less than the bias current I_0, the force component F_{y2} will be much less than F_{y1} and thus could be neglected. Substituting $\hat{H}_{2n,t}$ and $\hat{H}_{3n,t}$ into (11), gives the following expression for F_{y1}.

$$\begin{aligned} F_{y1} &= \mu_0 \int_L \big[\big(H_{0n} + H_{1n}\big)\big(\hat{H}_{2n} + \hat{H}_{3n}\big) - \big(H_{0t} + H_{1t}\big) \\ &\quad \big(\hat{H}_{2n} + \hat{H}_{3n}\big)\big] sin\theta dl \, e^{j\phi(\omega)} e^{j\omega t} \\ &= \hat{F}_{y1} e^{j\phi(\omega)} e^{j\omega t} \end{aligned} \quad (19)$$

where

$$\begin{aligned} \hat{F}_{y1} &= \mu_0 \int_L \big[H_{0n}\hat{H}_{2n} - H_{0t}\hat{H}_{2t}\big] sin\theta dl \\ &+\mu_0 \int_L \big[H_{1n}\hat{H}_{2n} - H_{1t}\hat{H}_{2t}\big] sin\theta dl \\ &+\mu_0 \int_L \big[H_{0n}\hat{H}_{3n} - H_{0t}\hat{H}_{3t}\big] sin\theta dl \\ &+\mu_0 \int_L \big[H_{1n}\hat{H}_{3n} - H_{1t}\hat{H}_{3t}\big] sin\theta dl \end{aligned} \quad (20)$$

is the amplitude of the linear force. It can be seen that the first term is independent of the rotation speed Ω and the second term is linear in field $\vec{H}_1$ and the third term in $\vec{H}_3$. The last term is a higher order term in speed Ω and would be much smaller compared to the second and third terms.

When the rotor is not rotating, $H_{1n,t} = 0$ and $\hat{H}_{3n,t} = 0$, and the amplitude of the dynamic force at frequency ω will be

$$\begin{aligned} \hat{F}_{y1,0} &= \hat{F}_{y1}(\Omega, \omega)|_{\Omega=0} \\ &= \mu_0 \int_L \big[H_{0n}\hat{H}_{2n} - H_{0t}\hat{H}_{2t}\big] sin\theta dl \end{aligned} \quad (21)$$

The normalized force can be written as

$$\bar{F}_{y1} = \frac{\hat{F}_{y1}}{\hat{F}_{y1,0}} = p_2(\Omega, \omega) \quad (22)$$

where p_2 is a function of the rotation speed Ω and frequency ω and given by

$$p_2(\Omega, \omega)|_{\Omega=0} = 1 \quad (23)$$

C. Linear approximation of magnetic force

Although the eddy currents will cause redistribution of the flux in the pole of the bearing and the rotor due to skin effect, there will be little change of the distribution of the field in the air gap when the eddy current is small because the relative permeability of the rotor and bearing are far greater than that of air. Therefore, when the amplitude of the dynamic control current is small and the frequency is not very high, the eddy current produced is weak. Its effect on the distribution of the field $\vec{H}_2$ and $\vec{H}_3$ in the air gap will be very small and mainly cause the decrease of amplitude and phase lag of the dynamic field. So it is reasonable to assume that the amplitude of $\vec{H}_2$ has the same distribution as $\vec{H}_0$ and $\vec{H}_3$ as $\vec{H}_1$ at lower frequencies, and that the eddy currents have equal effects on them at the same frequency ω. That is:

$$\frac{\hat{H}_{2n}}{H_{0n}} = \frac{\hat{H}_{2t}}{H_{0t}} \approx q(\omega) \quad (24)$$

$$\frac{\hat{H}_{3n}}{H_{1n}} = \frac{\hat{H}_{3t}}{H_{1t}} \approx q(\omega) \tag{25}$$

where $q(\omega)$ is only a function of ω. As $\omega \rightarrow 0$, the eddy currents will disappear. Since the field will be approximately linear to the current when there is no eddy currents, the ratio of the dynamic field to the bias field should approximately equal to that of the current if i_0 is small enough. Thus

$$q_0 = q(\omega)|_{\omega=0} \approx \frac{i_0}{I_0} \tag{26}$$

and the normalized form of $q(\omega)$ will be

$$q(\omega) = q_0\bar{q}(\omega) \tag{27}$$

Substituting (24) into (21), the amplitude of the dynamic force when the rotor does not rotate is given by

$$\hat{F}_{y1,0} = q(\omega)\mu_0 \int_L (H_{0n}^2 - H_{0t}^2) sin\theta dl = 2q(\omega)F_{y0,0} \tag{28}$$

and the normalized amplitude of the dynamic force at rotation speed Ω will be

$$p_2(\Omega, \omega) = p_1(\Omega) \tag{29}$$

The equation (29) implies that the rotation has equal effects on the static and dynamic forces. It is to be noted that p_2 is independent of frequency ω now, and only depends on the rotation speed Ω. Therefore, by substituting (15), (19), (22) and (28) into (9), the magnetic force could be written as

$$\bar{F}_y = \frac{F_y}{F_{y0,0}} = p_1(\Omega)\left[1 + 2q(\omega)e^{j\phi(\omega)}e^{j\omega t}\right] + h.o.t. \tag{30}$$

The above result is of particular interest as the effects of the two types of eddy current can be separated now. The eddy current due to rotor rotation only affects the static force F_{y0} and the amplitude of the dynamic force $\hat{F}_{y1}$, and that induced by current i will reduce the amplitude of $\hat{F}_{y1}$ and cause phase lag. Moveover, because of separation of the two effects, the magnitude plots of the frequency response of the dynamic force at different speeds are similar and only offset by $20log[p_1(\Omega)]$ from the one when the rotor is not rotating.

D. Current stiffness

The current stiffness k_i of the bearing is defined as the ratio of linear part of the dynamic force to the dynamic control current:

$$k_i = \frac{\hat{F}_{y1}e^{j\phi(\omega)}e^{j\omega t}}{i_0 e^{j\omega t}} \tag{31}$$

Substituting (30) into (31), then the normalized form of the stiffness can be written as

$$\bar{k}_i = \frac{k_i}{k_{i0}} = p_1(\Omega)\bar{q}(\omega)e^{j\phi(\omega)} \tag{32}$$

where

$$k_{i0} = \frac{2F_{y0,0}}{I_0} \tag{33}$$

is the static current stiffness. So the magnitude of the dynamic current stiffness decreases by an amount of $p_1(\Omega)$ at different rotation speeds. Hence, if the variation of the static force p_1 with rotation speed, and the dynamic current stiffness when the rotor is not rotating is known(these two situations can be more easily investigated theoretically and experimentally), then the dynamic current stiffness at any speed could be derived.

III. Case study using FEM

TABLE I
PARAMETERS OF THE MODEL

G_0	$0.25\ mm$	Air gap
r_1	$29\ mm$	Radius of the rotor
r_2	$29.25\ mm$	Radius of the pole surface
r_3	$31.25\ mm$	Radius of inner surface of the coil region
r_4	$45\ mm$	Inner radius of the back-iron of the stator
r_5	$60\ mm$	Outer radius of the stator
d	$25\ mm$	Axial length of the bearing
l	$10\ mm$	Width of the pole
I_0	$2\ A$	Bias current
N_i	48	Number of turns in the coil
u_r	2700	Relative permeability of the iron
R_1	$1.00E{-}5\ \Omega{\cdot}m$	Equivalent resistivity of the silicon steel
R_c	$3.05E{-}8\ \Omega{\cdot}m$	Resistivity of the coil

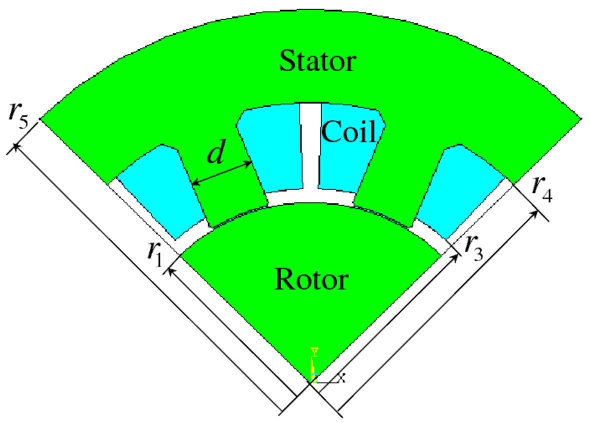

Fig. 2. FEM model

A typical 8-pole RAMB was studied using FEM to investigate the eddy current effects. It was assumed that the field distribution was uniform along the axial direction and thus a 2-D model was used in the analysis. It was also assumed that the rotor was positioned and rotating at the center of the bearing so that the whole bearing-rotor model was periodical symmetric. Thus a $1/4$ part of the model with periodical symmetric boundary conditions was used in the analysis to reduce the model size. Moreover, in order to study the eddy current effects of the laminated structure, an equivalent resistivity(much greater than that of silicon iron)was used in this model.

The FEM package ANSYS(8.1) was used for the numerical calculation. Fig.2 shows the FEM calculation regions of the model. The parameters used are listed in Table I. A total of 8582 elements and 17259 nodes were used in the model. In order to reduce the calculation error, very fine meshes were used in the eddy current region. The smallest size of the element in that region is $0.5G_0(0.125mm)$ which corresponds to the skin depth at about $60KHz$, so as to give good results in the eddy current region in the frequency range of interest. The pole configuration of the whole bearing is NSSN. The bias flux density was about 0.4T at the air gap produced by a bias current $I_0 = 2A$ of the coil which has 47 turns each.

A dynamic current of $0.5sin\omega tA$ was used in addition to the bias current. After the transient analysis of the magnetic fields, the bearing force can be obtained for each rotation speed Ω and frequency ω. The results showed that the force approached to a steady state after the initial transition time. Then the last two periods could be used as the steady state responses to calculate the amplitude and phase lag of the force.

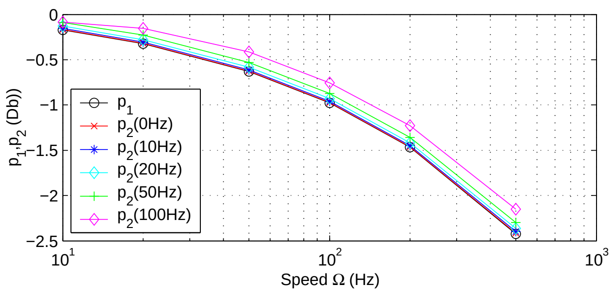

Fig. 3. Normalized force $p_1(\Omega)$,$p_2(\Omega, \omega)$

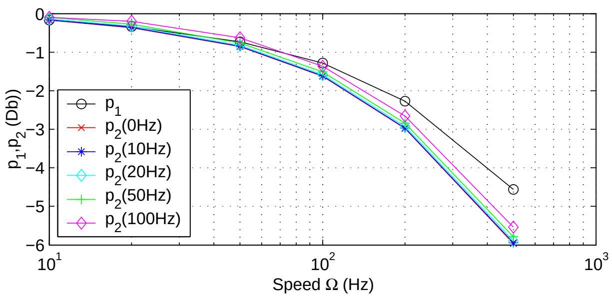

Fig. 4. Normalized force $p_1(\Omega)$,$p_2(\Omega, \omega)$ considering flux saturation

The variation of the normalized static force p_1 and amplitude of the dynamic force p_2 with rotation speed are shown in Fig.3 and Fig.4. It can be seen that p_2 is approximately constant when the frequencies is lower than $50Hz$. This is accordance with the assumption used in the analysis before. Moreover, p_2 is nearly equal to p_1 when ω is less than $50Hz$ and the rotation speed is less than $100Hz$. p_2 is also nearly identical to p_1 in the whole range of rotation speed at low frequencies when flux saturation is not considered as is in the linear analysis. The larger difference between p_1 and p_2 at high speeds in the nonlinear analysis is due to extreme saturation of flux in the eddy currents region near the rotor surface and the variation of magnitude of i_c around I_0 will affect the extent of saturation(this will cause the distortion of the waveform of the dynamic force even at lower frequency). Thus the assumption used in (24) will bring some errors in the result. However, the assumption still gives good approximations in moderate saturation case.

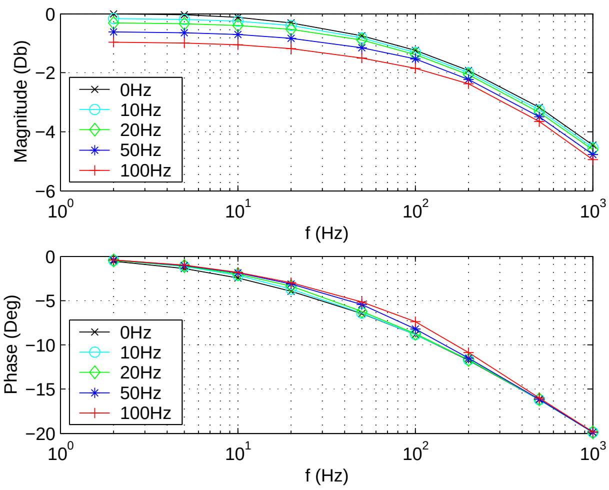

Fig. 5. Frequency response of current stiffness at different rotation speeds

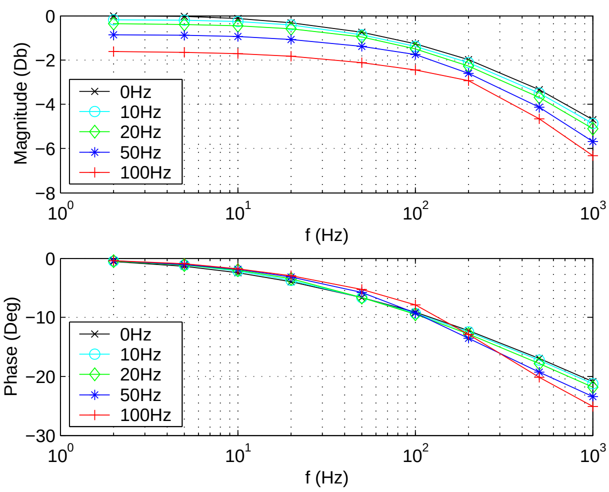

Fig. 6. Frequency response of current stiffness at different rotation speeds considering flux saturation

Fig.5 and Fig.6 show the frequency response of current stiffness at different rotation speeds. The stiffness decreases as the frequency increases. The $3dB$ bandwidth is about $500Hz$ and the phase lag is about 16 degrees at the cutoff frequency. The magnitude of the stiffness at different speeds just offset from the one for zero rotational speed by an amount of $p_1(\Omega)$. Furthermore, it could also be seen in the phase plot that the rotation does not change the phase lag much. It is mainly determined by the eddy currents induced by the dynamic current i. These agree well with the analysis in part D of Section II. Since the rotational speed is lower, the flux saturation in the rotor is moderate, and the nonlinear model gives nearly the same results as the linear one.

IV. Conclusion

The magnetic force of the RAMB considering the eddy current effects due to both the rotation and variation of the dynamic control current is investigated in this paper. The approximated general form of the force is derived with the assumption of moderate eddy currents developed in the rotor and stator. The results show that the effects of the two types of eddy currents on the bearing force could be separated. The eddy current due to rotation of the rotor only affects the magnitude of the static force and amplitude of the dynamic force and that induced by dynamic control current will reduce the amplitude of the dynamic force and cause phase lag. A case study using FEM is performed to verify the analysis. The FEM results agree well with the analytical ones.

The study presented in this paper is most interesting not only because it gives a description of how the eddy currents affect the magnetic force and hence the dynamic properties of the bearing but also it is usefulness in experiments and applications. If the variation of the static force with rotation speed and the dynamic current stiffness when the rotor is not rotating are known(both of which could be more easily analyzed and tested), the dynamic current stiffness at any speed could be derived. Hence the effect of eddy currents on the performance of AMB can be more readily estimated in design stage.

Moreover, there is no particular assumptions on the eddy current distributions but only assumptions on the fields distributions in the air gap based on the moderate eddy currents situation. This is particular important because the analysis and result is applicable to not only solid rotor and stator but also of laminated structures.

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