Analysis and Control of a Three Pole Radial Magnetic Bearing

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***Abstract*— This work presents a new method of formulating the current to force relationship for a three-pole radial magnetic bearing. This relationship is readily inverted to yield a general mapping from force to current that can be realized by a three-phase motor drive. Position dependence of the bearing forces and force slew rate limiting implications are also explored.**

I. Introduction

One impediment to the broader use of magnetic bearings is their expense. If magnetic bearings could be designed to use the three-phase power modules and controllers commonly used in AC servo and vector drives, the cost and complexity of magnetic bearings might be significantly reduced.

Some progress has already been made with respect to powering magnetic bearings with three-phase drives. An elegant permanent magnet biased homopolar bearing design is described in both [1] and [2]. Although the PM-biased topology has intrinsically low rotating and coil ohmic losses and is directly amenable to utilization of a standard three-phase drive, it would be useful to have an alternative heteropolar bearing design. Heteropolar bearings typically require less axial space along the shaft, and they are much simpler in construction as compared to homopolar bearings. The heteropolar design also eliminates the need for permanent magnets, which may increase cost and place restrictions on the bearings' operating environment.

Schöb also describes a six-pole heteropolar bearing driven by a three-phase supply in [2]. Although this design allows the use of a three-phase drive, an additional DC bias winding is required by this topology. A similar bearing with twelve poles is presented in [3].

Three-pole heteropolar bearings have also been investigated in the literature. Chen [4], [5] considered a bearing with two poles wound in series driven by a single-phase amplifier and the third coil driven by second single-phase amplifier. However, the economy of the three-phase drive is not realized by the two-amplifier design. In [6], a three-pole bearing driven by a three-phase drive is considered. However, the bearing is biased about the operating point dictated by the support of the rotor's weight in order to invert the relationship between currents and forces. A similar control approach is considered in [2], wherein the bias is obtained by a separate DC winding in addition to a three-phase drive. In [5], this biasing method was found to yield a system with poor robustness, due to the three-pole bearing's intrinsic nonlinearity.

This work considers the three-pole bearing topology driven by a three-phase drive, as in [6]. However, the present approach does not bias the bearing about a nominal operating point dictated by the rotor's weight. Instead, a new expression relating current and force is developed. This expression is simple to invert, yielding a power-minimal set of currents that realizes any desired force without linearizing current about a nominal operating point and in fashion that is amenable to the use of a standard three-phase drive.

The position dependence of the force is then considered. An expression for force as a general function of current and position is derived. Linearizing this expression for small rotor displacements yields a simple expression for the position dependence of the bearing near the centered position. Various control approaches are discussed for accommodating the nonlinear position dependence of the forces.

Lastly, slew rate limiting issues are considered. For the typical radial magnetic bearing geometry consisting of two more or less independent control axes, there is a desire to operate at low or zero bias current level to minimize resistive losses. However, low bias current levels can lead to excessive voltage demand at low forces due to the current-squared dependence of the magnetic force. This phenomenon is known as *slew rate limiting*[7]. Although the three-pole bearing considered in this work is run without bias currents, it is shown that the bearing is not subject to slew rate limiting problems of the severity of encountered during unbiased operation of the common eight pole bearing configuration.

II. Derivation of the Current-Force Relationship for a Centered Rotor

The relationship between current and force is first considered for the case of a centered rotor. This configuration yields a simple relationship between current and force that can readily be inverted. In later sections, the more general case of an arbitrary rotor position will be considered to investigate the variation of force with rotor position.

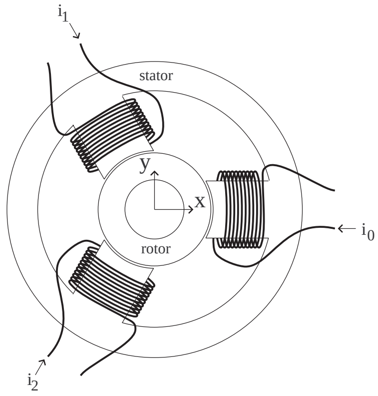

Fig. 1. Three-pole bearing geometry.

The three-pole bearing configuration considered here is indicated in Figure 1. Since magnetic bearings do not typically run in saturation, the reluctance of the core material are neglected. Flux leakage, fringing, eddy current effects, and hysteresis effects are neglected. Each bearing pole has a cross-section area of a, a nominal air gap of g separating the rotor and stator, and is wound by n turns of wire. The magnetic permeability of free space is represented by μ_o.

For the purposes of this analysis, bearing force on the rotor is represented as a complex number where the real part of the number represents force in the x-direction and the imaginary part of the force represents force in the y-direction, $i.e.$:

$$f = f_x + jf_y \tag{1}$$

Applying Maxwell's Stress Tensor, the force contribution of the n^{th} pole, $|f_n|$, has the amplitude:

$$|f_n| = \left(\frac{a}{2\mu_o}\right) B_n^2 \tag{2}$$

where B_n is the flux density at the n^{th} pole. The force on the rotor is directed along an outward normal to the rotor through the center of the pole of interest. To combine the contributions of all poles, define matrix Λ where the diagonal entries are complex numbers representing the line of action of each force:

$$\Lambda = \text{diag}\left[\begin{array}{ccc} 1 & e^{j2\pi/3} & e^{-j2\pi/3} \end{array}\right] \tag{3}$$

The total force can then be written as:

$$f = \left(\frac{a}{2\mu_o}\right) B^T \Lambda B \tag{4}$$

where B is a vector of the fluxes going through each pole of the bearing.

Further simplification eventually results if a transformation is defined which converts bearing quantities back and forth from 3-vectors to complex-valued phasors. Defining the vector k as:

$$k = \sqrt{\tfrac{2}{3}}\left[\begin{array}{ccc} 1 & e^{-j2\pi/3} & e^{j2\pi/3} \end{array}\right] \tag{5}$$

the transformation of flux from vector B to phasor b is:

$$b = kB \tag{6}$$

The common-mode component of B is neglected in this transformation. However, since this component of B must be zero due to flux conservation on the rotor, nothing is lost in the transformation. The inverse of the transformation is:

$$B = Re\left(k^* b\right) = \tfrac{1}{2}\left(k^* b + k^T \bar{b}\right) \tag{7}$$

The over-bar denotes complex conjugate and the asterisk denotes conjugate transpose. Substituting (7) into (4) yields:

$$f = \left(\frac{a}{8\mu_o}\right)\left(k^* b + k^T \bar{b}\right)^T \Lambda \left(k^* b + k^T \bar{b}\right) \tag{8}$$

This equation can be simplified by noting some properties of Λ and k which can be verified by direct computation:

$$\begin{array}{ll} kk^T = 0 & \Lambda k^T = \sqrt{\tfrac{2}{3}}\,\{1,1,1\}^T \\ kk^* = 2 & \Lambda k^* = k^T \\ k\{1,1,1\}^T = 0 & \end{array} \tag{9}$$

When the identities of (9) are applied to (8), the result is:

$$f = \left(\frac{a}{4\mu_o}\right) b^2 \tag{10}$$

Note that up to this point, no assumption has been made about the centering of the rotor. Eq. (10) applies at an arbitrary rotor position.

When the rotor is centered, there is a simple relationship between current and flux. If the applied currents are balanced 3-phase currents, the magnetic potential of the rotor is at the same level as the stator so that each pole is acting only against the reluctance of its own air gap. In this case, the relationship between flux density and current for the nth gap is:

$$B_n = \left(\frac{\mu_o}{g}\right) n i_n \tag{11}$$

Applying the phasor transformation to both sides of this equation yields:

$$b = \left(\frac{\mu_o n}{g}\right) i \tag{12}$$

where i is the phasor representation of phase current. Substituting for flux in (10) yields a simple relationship between current and force:

$$f = \left(\frac{\mu_o n^2 a}{4g^2}\right) i^2 \tag{13}$$

III. Inversion of Current-Force Relationship

The current-force equation is reminiscent of that of a single horseshoe magnet: force is proportional to the square of the current. However, unlike the single horseshoe, the force and current are complex-valued. The current required to obtain any desired force can be derived by solving (13) for i:

$$i_{des} = \pm\sqrt{\left(\frac{4g^2}{\mu_o n^2 a}\right) f_{des}} \tag{14}$$

The desired currents can then be transformed back into real-valued phase currents via:

$$i_{phase} = Re(k^* i_{des}) \tag{15}$$

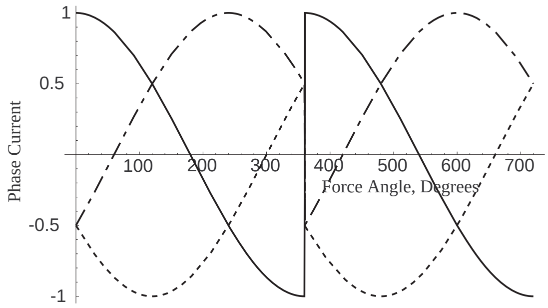

Fig. 2. Current versus force direction for positive solution branch.

It should also be noted that since k is orthogonal to the vector $\{1, 1, 1\}^T$, there is no common mode component of the desired phase currents. Since there is no common mode component, the desired phase currents are realizable with a three-wire, three-phase power supply.

Although the complex-valued inverse current-force relationship is relatively straightforward, practical usefulness and additional insight can be gleaned by converting the complex-valued results into equivalent real-valued expressions.

Let force f be broken down into real and imaginary parts as:

$$f = F\left(\cos\theta + j\sin\theta\right) \tag{16}$$

This definition of force implies that the desired force has a magnitude F that is directed at an angle of θ with respect to the $x-$axis. The square-root of f is then:

$$\sqrt{f} = \pm\sqrt{F}\left(\cos\tfrac{\theta}{2} + j\sin\tfrac{\theta}{2}\right) \tag{17}$$

The square-root operator takes the square root of the magnitude and modifies the direction to one-half of the angle of the desired force. The transformation from phasor current back into desired phase currents can be performed by unravelling (15) as:

$$i_{phase} = \pm\sqrt{\frac{8g^2F}{3\mu_o n^2 a}}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{bmatrix} \tag{18}$$

An interesting consequence of the half-angle property occurs in the case in which the bearing is creating a constant force that varies in angle as the rotor spins, as is the case when the bearing is supporting a synchronous imbalance. For every revolution of the rotor, the bearing currents vary sinusoidally but only traverse a half-cycle for every revolution. If one branch of (18) were used exclusively ($e.g.$ if the $\pm$ sign was always taken to be positive) to yield a unique mapping between force and current, there would be a large jump in commanded currents at *θ=0*. The resulting plot of current versus force angle for a constant desired force amplitude is shown in Figure 2. The ambiguity of which branch to select could be solved in practice in the context of a digital controller by picking the branch that results in the smallest change from the currents commanded during the previous time step. For the constant magnitude rotating force case, the solution branch would change with each revolution to avoid the jump at *θ=0*. The discontinuity is avoided, yielding the smooth relationship between force orientation shown in Figure 3.

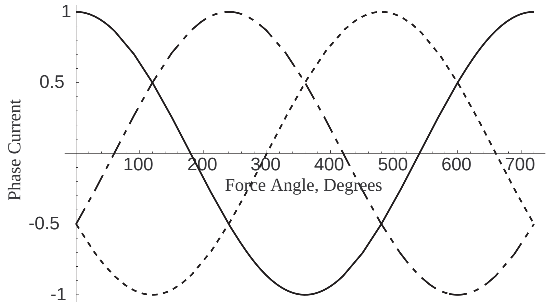

Fig. 3. Current versus force direction with adaptively selected sign.

IV. Generalized Current-Force Relationship

Although the bearing's current-force relationship has been derived for the rotor in the centered position, a more generalized expression needs to be obtained explore the position dependence of the bearing's force. To derive the generalized expression, flux density in the bearing as a function of coil current and position must be derived. This flux density can then be substituted into (10) to obtain force as a function of current and position. The derivation begins by considering the magnetic circuit representation of the bearing, as pictured in Figure 4.

Following the approach presented in [8], two loop equations and one flux conservation equation can be written as:

$$\frac{1}{\mu_o}\begin{bmatrix} g_0 & -g_1 & 0 \\ 0 & g_1 & -g_2 \\ g & g & g \end{bmatrix}\begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix} = n\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix} \tag{19}$$

By applying row operations, (19) can be re-arranged to:

$$\frac{\sqrt{\frac{2}{3}}}{\mu_o}\begin{bmatrix} g_0 & -\frac{1}{2}g_1 & -\frac{1}{2}g_2 \\ 0 & -\frac{\sqrt{3}}{2}g_1 & \frac{\sqrt{3}}{2}g_2 \\ \frac{1}{\sqrt{2}}g & \frac{1}{\sqrt{2}}g & \frac{1}{\sqrt{2}}g \end{bmatrix}\begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix} =$$

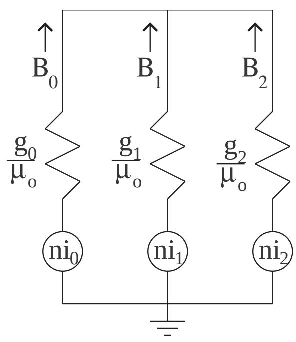

Fig. 4. Equivalent magnetic circuit representation.

$$\sqrt{\tfrac{2}{3}}n\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}i_0\\ i_1\\ i_2\end{bmatrix} \tag{20}$$

If the analysis is continued in the phasor-transformed variables, the third line of (20) is satisfied automatically. Both sides of (20) can be pre-multiplied by the vector [1,j,0] to discount the third equation and combine the two remaining equations into one complex-valued equation

$$\begin{aligned}&\frac{\sqrt{\frac{2}{3}}}{\mu_o}\begin{bmatrix} g_0 & \left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)g_1 & \left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)g_2\end{bmatrix}\begin{bmatrix}B_0\\ B_1\\ B_2\end{bmatrix}\\ &= \sqrt{\tfrac{2}{3}}n\begin{bmatrix}1 & \left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right) & \left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\end{bmatrix}\begin{bmatrix}i_0\\ i_1\\ i_2\end{bmatrix}\end{aligned} \tag{21}$$

If the matrix G is defined as:

$$G=\begin{bmatrix}g_0 & & \\ & g_1 & \\ & & g_2\end{bmatrix} \tag{22}$$

and referring to the definition of k in (5), (21) can be re-written more succinctly as:

$$\tfrac{1}{\mu_o}kGB = nki_{phase} \tag{23}$$

Substituting for flux density and current in terms of their phasor representations yields:

$$\tfrac{1}{\mu_o}kG\left(\tfrac{k^*b+k^T\bar{b}}{2}\right)=nk\left(\tfrac{k^*i+k^T\bar{i}}{2}\right) \tag{24}$$

The G matrix can be re-written to expose the dependence of gap on rotor position:

$$G = gI - xRe\left(\Lambda\right) - yIm\left(\Lambda\right) \tag{25}$$

Further simplification can be obtained if the complex-valued rotor position, d, is defined as:

$$d = x + jy \tag{26}$$

so that G can be re-arranged as:

$$G = g\mathrm{I} - \tfrac{1}{2}\left(\bar{d}\Lambda + d\bar{\Lambda}\right) \tag{27}$$

where I represents the 3×3 identity matrix. Substituting the definition of G from (27) into (24) and applying the identities of (9) allows (24) to be simplified to:

$$\frac{g}{\mu_o}\left(b-\left(\frac{d}{2g}\right)\bar{b}\right)=ni \tag{28}$$

Equation (28) contains both b and its complex conjugate. To solve for b, a second equation can be obtained by taking the complex conjugate of the entire eq. (28), implying the system of equations:

$$\frac{g}{\mu_o}\begin{bmatrix}1 & -\frac{d}{2g}\\ -\frac{\bar{d}}{2g} & 1\end{bmatrix}\begin{Bmatrix}b\\ \bar{b}\end{Bmatrix}=n\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{Bmatrix}i\\ \bar{i}\end{Bmatrix} \tag{29}$$

Solving (29) for b yields:

$$b=\frac{4g\mu_o ni}{4g^2-|d|^2}+\frac{2d\mu_o n\bar{i}}{4g^2-|d|^2} \tag{30}$$

which can be substituted into (10) to yield a general expression for force as a function of position:

$$f=\left(\frac{a}{4\mu_o}\right)\left(\frac{4g\mu_o ni}{4g^2-|d|^2}+\frac{2d\mu_o n\bar{i}}{4g^2-|d|^2}\right)^2 \tag{31}$$

A much simpler form can be obtained by linearizing for small d:

$$f=\left(\frac{\mu_o n^2 a}{4g^2}\right)\left(i^2+\frac{d}{g}i\bar{i}\right) \tag{32}$$

If the current is specified as a function of desired force by (14), force on the rotor simplifies to:

$$f = f_{des} + |f_{des}|\,\frac{d}{g} \tag{33}$$

V. Control Approaches

Three possible methods of addressing the position dependence of the force in a current-controlled bearing are:

- Position Feedback Linearization
- Flux Feedback
- Robust Controller Design

A brief description of each approach follows.

A. Position Feedback Linearization

It is common to control magnetic bearings by first closing a stiff, high bandwidth control loop on the desired bearing currents. If hysteresis and eddy current effects can be neglected, this control loop effectively eliminates internal dynamics of the bearing itself, leaving the bearing as a more or less static mapping from current and position to force.

It can be noted from (10) that, if the flux in the bearing is specified, the position dependence of the force is removed. Using (28), which maps a known bearing flux density onto a set of coil currents, a correction on the centered rotor force-to-current mapping can be obtained which exactly inverts the position dependence of the bearing.

Let i_{cmd} denote the current that is actually commanded to the current control. As in (14), i_{des} is the desired current that produced the desired rotor force in the centered rotor position. From (12), it can be inferred that the desired flux density is:

$$b_{des}=\left(\frac{\mu_o n}{g}\right)i_{des} \tag{34}$$

Substituting b_{des} for b in (28) and simplifying yields:

$$i_{cmd}=i_{des}-\left(\frac{d}{2g}\right)\bar{i}_{des} \tag{35}$$

This simple correction to the commanded current inverts the position dependence of the bearing, assuming that the model accurately represents the current-force relationship of the bearing. A controller for the rotor system as a whole would then be designed assuming that the bearing acts like a source of prescribed force. The current command that produces this force is then obtained by (14) and (35).

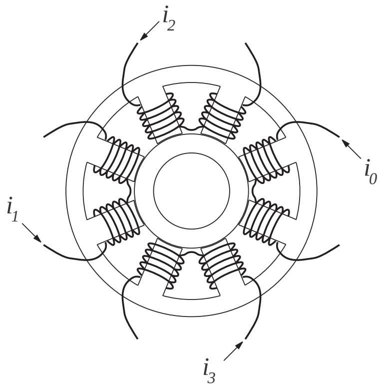

Fig. 5. Eight-pole bearing wound in a four-quadrant configuration.

B. Flux Feedback

Alternatively if flux (or flux density) can be measured, the high bandwidth loop can be closed on flux. The bearing then becomes a flux-controlled bearing, rather than a current-controlled bearing, and (10) can be inverted directly to compute the required flux density corresponding to any desired force. Although it may be possible to mount Hall Effect sensors directly in the bearing's air gaps to measure flux density, a more robust method of implementing flux feedback is through the use of small search coils wound from fine gauge wire located around the bearing's poles close to the air gap. By integrating the search coil voltage (and possibly fusing this estimate with a DC flux estimate based on (30)), a measurement of flux in the bearing's air gaps can be obtained. A more detailed description of search coil-based flux feedback is contained in [9].

C. Robust Controller Design

A third alternative, applicable to a current-controlled bearing, would be to specify current as a function of force using the centered rotor inversion in (10). Many magnetic bearing applications ($e.g.$ flywheels) support mainly a synchronous imbalance load. If a synchronous imbalance is being accommodated, the problem of interest would an assessment of the stability of the bearing's orbit. In this case, a controller could be designed using the position-linearized description of the bearing force from (33). The nominal force magnitude required to counteract the mass imbalance would replace the $|f_{des}|$ in (33), resulting in a system with a constant linear stiffness. However, since mass imbalance is most likely *a priori* unknown, the value of this constant stiffness would represent a structured uncertainty in the system. Various robust control methods could then be employed to design a linear controller that is robust to the range of bearing stiffness implied by the expected range of steady-state force.

VI. Slew Rate Limiting/No Bias Operation

Consider the four-quadrant eight-pole bearing pictured in Figure 5. If each horseshoe has a total of $2n$ turns and

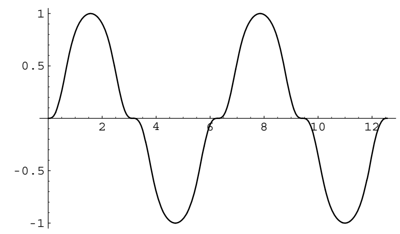

Fig. 6. Effects of slew-rate limiting on a sinusoidal waveform.

each pole has an area of a, the total force along an axis is:

$$f_x = \left(\frac{\mu_o n^2 a}{g^2}\right)\left(i_0^2 - i_1^2\right) \tag{36}$$

where i_0 represents the current in the horseshoe that pulls in the positive direction and i_1 is the current in the horseshoe that pulls in the negative direction. The force slew rate (the rate at which force changes with respect to time) is:

$$\frac{df_x}{dt} = \left(\frac{2\mu_o n^2 a}{g^2}\right)\left(\frac{di_0}{dt}i_0 - \frac{di_1}{dt}i_1\right) \tag{37}$$

The power-optimal method of selecting currents to realize a desired force uses only one horseshoe at a time: the current in the active horseshoe varies proportional to the square root of the desired current, and the other horseshoe carries no current. When the force passes through zero, both currents are zero, implying that the obtainable force slew rate is zero. In a typical bearing operation, each axis produces a sinusoidally varying force to counteract the imbalance of the shaft. With a sine function, the moment at which the force passes through zero (where *df/dt* = 0) coincides with the *peak* required force slew rate. The result of the slew rate limiting is cross-over distortion around zero force, as pictured in Figure 6. If the synchronous imbalance load dominates the bearing force, the unbiased horseshoe bearing system would traverse four slew rate limited zero crossings per revolution.

For the three-pole bearing, the slew rate limiting condition can be explored by differentiating the centered force equation, eq. (13), with respect to time:

$$\frac{df}{dt} = \left(\frac{\mu_o n^2 a}{2g^2}\right) i\frac{di}{dt} \tag{38}$$

The slew rate limiting condition occurs when i= 0. However, since (13) addresses both force axes simultaneously, the i= 0 condition only occurs when force is zero *on both axes simultaneously* (*i.e.* $f = 0$). During normal operations, when the bearings support the rotor's gravitational load plus a synchronous imbalance force, forces along both axes are typically never zero simultaneously.

For example, consider a symmetric rigid rotor supported by two radial magnetic bearings with symmetric control. Each bearing must support half of the gravitational load and half of the imbalance load:

$$f = \frac{mg}{2} + \frac{m\omega^2}{2}e^{j\omega t}\left(\epsilon - x\right) \tag{39}$$

Solving (39) for $f = 0$ shows that the slew rate limiting condition can occur when

$$\omega = \sqrt{\frac{g}{|\epsilon - x|}} \tag{40}$$

where m is the mass of the rotor, g is acceleration due to gravity, ϵ is the rotor unbalance eccentricity, and x is the rotor motion phasor. The slew rate limiting condition only occurs when the rotor's speed is such that the force due to the synchronous imbalance is exactly the same as the gravitational load supported by the bearing, creating a once-per-revolution slew rate limiting condition.

It should be stressed that this slew rate limiting condition only occurs near a single speed. In contrast, the unbiased eight-pole bearing is subject to slew rate limiting at all speeds.

A. *Simulations*

To illustrate this, consider a simple mass of 10 kg supported in a pair of three pole AMBs with pole area = 6.5 cm^2, 328 coil turns on each leg, a nominal radial air gap of 1.0 mm, and a coil resistance of 0.5 Ω. For this example, a simple PD control is implemented for position with a current minor loop. The current loop gain on current error is 400 V/amp while the target PD gains are 3600000 N/m and 8400 N-sec/m. In addition, (35) is implemented to compensate for postion dependence of gap. The controller is presumed digital with a sampling rate of 10 kSa/sec. The mass is subject to a gravity load equal to its own weight plus an unbalance due to mass eccentricity of 0.001 kg-m. Each actuator is assumed connected in a "Wye" configuration to a 300 volt link through a conventional three phase drive. Selection of the phase voltages uses a space vector modulation approach[10] which means that the phase voltages themselves tend not to be very sinusoidal and are sometimes non-smooth.

Figure 7 illustrates the system behavior at a typical rotor running speed. Although both force components pass through zero, the voltage is well behaved and acceptably small: for this system, phase voltage is limited to $\pm$ 173 volts. The coil currents are obviously not perfectly sinusoidal, but the bearing forces are nearly exactly so.

Figure 8 shows the system behavior at the pathological rotor speed, $\omega = \sqrt{g/|e-x|} = 288.01$ rad/sec. Clearly, the coil voltages spike once in each revolution as predicted. However, because the controller samples at a finite rate, the voltage actually never exceeds 154.61 volts in the simulation. With a much higher sampling rate, the controller might attempt to request a much higher voltage. In any case, limiting this peak voltage to 173 volts would clearly not cause the system any problems. Indeed, the force time history is nearly perfectly sinusoidal.

Figure 9 shows the response of the system to a step vertical load applied to the mass at 0.05 seconds. The initial lift–off event requires 173 volts while the step load requires a peak of 146.0 volts. In any case, the system is well behaved.

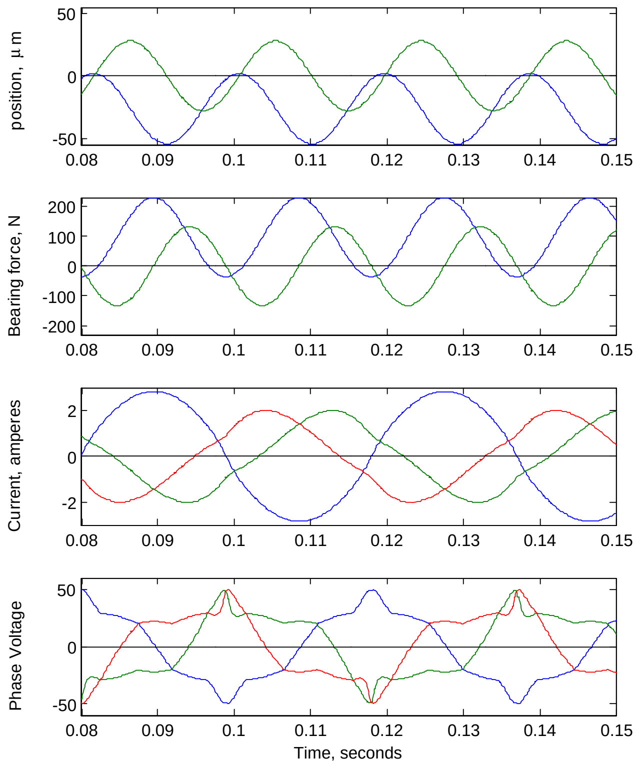

Fig. 7. Position, bearing force, current, and phase voltage at 3150 RPM. Maximum voltage is 51.4 volts

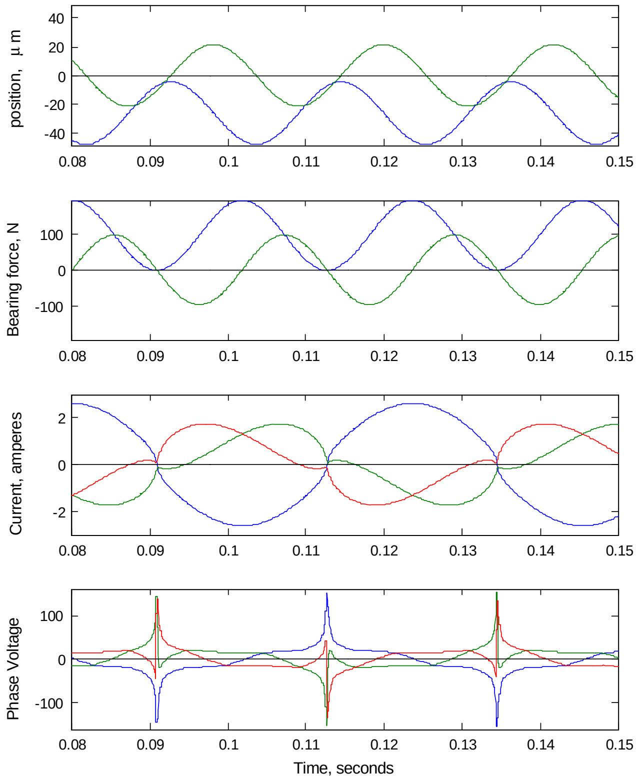

Fig. 8. Position, bearing force, current, and phase voltage at 2750.4 RPM. Maximum voltage is 154.1 volts.

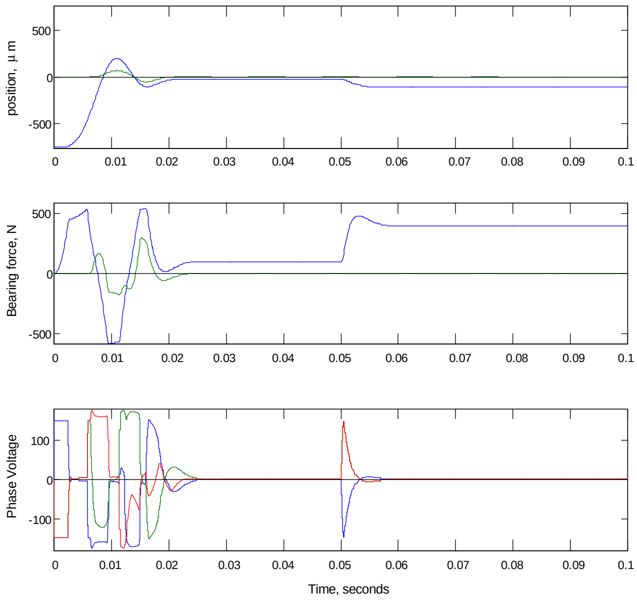

Fig. 9. Position, bearing force, and phase voltage at 2989.4 RPM: transient response to 300 N step load at 0.05 seconds with no unbalance.

VII. Conclusions

The current-to-force relationship for a three-pole AMB has been re-derived in a way that permits simple inversion. The inverse force-to-current mapping could be implemented with a three-phase motor drive, potentially reducing the complexity and cost of the magnetic bearing system.

The resulting force-to-current mapping operates the bearing without bias currents. Elimination of bias currents is beneficial for both coil losses and rotating losses; however, the lack of bias currents can also lead to force slew rate limiting problems. Although scenarios exist in which force slew rate limiting can occur for the three-pole bearing the practical consequences are negligible, in contrast to more conventional AMB control schemes.

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