Model and Control of Energy Storage Flywheel System Used in an Electric Vehicle*

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Abstract— **This paper is a report of the research of an energy storage flywheel system with a flexible rotor used for an electric vehicle. After introduce the energy storage flywheel system, the parameters of the rotor and the flywheel are decided by the vibration analysis of ANSYS. Based on the vibration analysis results, the model of the rotor and the flywheel is designed. In order to reduce the noise from ground, a moveable foundation is designed. Then robust controllers of the rotor system and the foundation are constructed respectively to decrease the noise from the ground. Form the simulation results, we verified that the proposed model and the controller are effective for the energy storage flywheel system used in an electric vehicle.**

Index Terms— **Energy storage, Flywheel system, Electric vehicle, Sliding mode control.**

I. INTRODUCTION

Energy storage flywheel systems, which store rotating kinetic energy by heavy spinning disks, are more promising as energy storage devices than lead batteries, because of their longer machine life and higher performance. In order to reduce the loss and achieve a un-contact flywheel system, the active magnetic bearing(AMB) and the superconducting magnetic bearing(SMB) are employed in the energy storage flywheel system. The great number of applications of AMB and SMB systems is due to their ultra-high rotational speed without contact and lubrication. And the number has been increasing at a remarkable rate.

Various studies have incorporated energy storage flywheel system. The gimbal structure design was reported by Andrei and his coworkers [1]. Cenk and his coworkers reported a gimal used for vehicle [2]. Hawkins and his coworkers reported vibration of a flywheel system with gimbal structure [3]. The energy storage flywheel with a flexible rotor used for an electric vehicle is not reported until now.

This paper reports an energy storage flywheel system with a flexible rotor used for an electric vehicle. After introduce the flywheel system with a moveable foundation, the vibration analysis of the rotational part is done to confirm the value of the natural frequency. The rotor and the flywheel are included in the rotational part. The vibration analysis is done by ANSYS which is a kind of CAE software. The parameters of the rotational part are decided by the rule that there is a flexible mode in the range of control. Then, a finite element method (FEM) model is designed for the rotational part. The reduced-order model conduced from the FEM model with two rigid modes and a flexible mode is designed to construct the controller. The sliding mode control is used for the control of the rotational part. For the control of foundation, the LQR control is used. At last, the validations of the proposed model and the controller are verified by the simulation.

II. ENERGY STORAGE FLYWHEEL SYSTEM USED IN AN ELECTRIC VEHICLE

The concept diagram of flywheel system with foundation is shown in Fig.1. The positions of the upper AMB, the below AMB and the flywheel are shown in Fig.1. The AMBs are applied to the axial and the radial directions to suspend and control the positions of the flywheel and the rotor. The axial direction of the rotor is controlled by a PID(Proportional-Integral-Derivative) controller. The control of the radial direction by a nonlinear robust control method, named sliding mode control is discussed in this paper.

A moveable foundation is designed to reduce the noise from ground. The stiffness and damping coefficient of the designed foundation are shown in Fig.1.

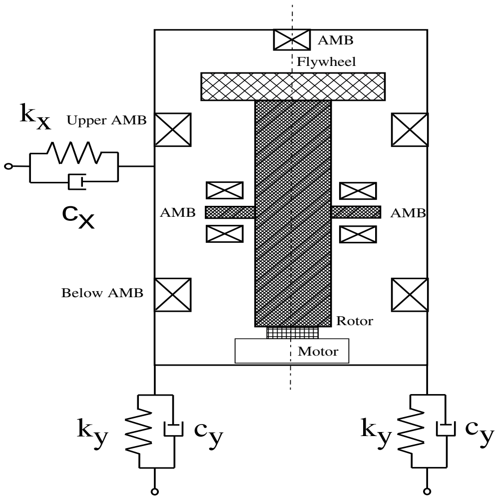

Fig. 1. Flywheel system with foundation

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III. VIBRATION ANALYSIS

The vibration analysis is done by using the damping method in ANSYS. ANSYS is a finite element method analysis program which is developed by Dr.John Swanson in 1970's to analyze various problems of the engineering field with a computer. And the development and maintenance of ANSYS are being done still now by ANSYS, inc. The ANSYS is widely used in the fields such as the structure, the heat transmission and the fluid analysis.

In this paper, the element $BEAM4$ is used with the effect of the gyroscopic effect. $BEAM4$ is a uniaxial element with tension, compression, torsion, and bending capabilities. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. Stress stiffening and large deflection capabilities are included.

The analysis of ANSYS is done by the following steps. Firstly, a straight line represented a component is inputted into ANSYS. Secondly, the characteristic features such as the area, moment of inertia of the component are inputted. Then the three dimensional model of the component is generated by ANSYS automatically. The flywheel and rotor are represented by a line in ANSYS, respectively. Thirdly, the analysis method and the numbers of necessity mode are inputted. At last, ANSYS will given the analysis results according to the inputted parameters.

The operational speed of the system is $3000rpm$. The dimension of the rotor system is designed to include a flexible mode in the range of the operational speed.

The radius and the height of the flywheel and the rotor are $200mm$, $10mm$, $10mm$ and $700mm$, respectively. The mass of the flywheel and rotor is $11.2Kg$. From virbation analysis, the flexible natural frequency of the system is $76.5Hz$.

An example of the mode shape of ANSYS analysis is given in Fig.2.

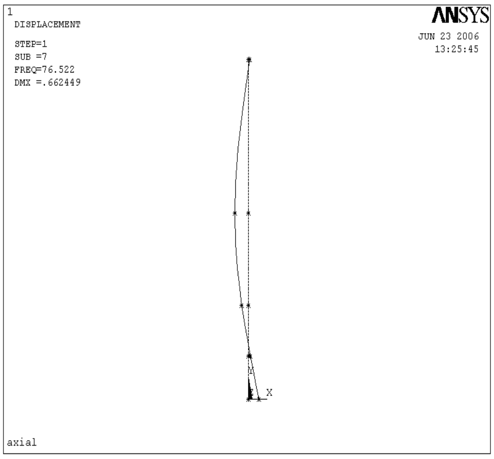

Fig. 2. An example of ANSYS anlysis

IV. CONTROL MODEL

A. Model of rotational part

The model of the rotational part used for controller design is constructed from the model of ANSYS. While matching the parameters such as the total mass, the total height, the natural frequencies and the natural modes, the control model is designed based on the three dimensional ANSYS model. The rotor is divided into three parts, and a one dimensional finite element method (FEM) model is designed. The designed one dimensional finite element method model is given in Fig.3.

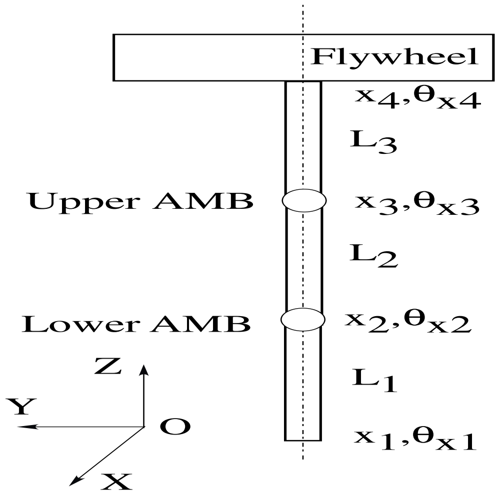

Fig. 3. One dimensional finite element method (FEM) model

The positions of the upper AMB, the below AMB, the motor and the flywheel are shown in Fig.3. $x_1, x_2, x_3, ...$ are the displacements of the position, respectively. The lengths of L_1, L_2 and L_3 are $100mm$, $500mm$ and $100mm$, respectively. Based on Fig.3, the state and the output equations are derived. The motion of equation of the free-free flexible rotor system is given in (1). Suppose the motion of the Y direction is same to that of the X direction, only the X direction is considered in the control model. The damping effect is neglected.

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} = 0 \tag{1}$$

Here, $\mathbf{Q} = [x_1 \quad \theta_{x1} \quad \cdots \quad \theta_{x4}]^T$, $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix. $\mathbf{Q} \in R^{8\times1}$, $\mathbf{M}, \mathbf{K} \in R^{8\times8}$.

Consider the control input and the noise, (2) is gotten.

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} = \mathbf{F}\mathbf{U} + \mathbf{E}\mathbf{W} \tag{2}$$

Here, $\mathbf{F}$ expresses the positions of the AMBs. The control inputs are inputted from the places of AMBs. $\mathbf{E}$ is used for disturbance input. $\mathbf{W}$ denotes the periodic and other disturbances. Gap-sensors are set beside AMBs to measure the positions of the rotor. The state and the output equation are shown in (3).

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U} \\ \mathbf{Y} = \mathbf{C}\mathbf{X} \end{cases} \tag{3}$$

Here, $\mathbf{A} \in R^{16\times16}$, $\mathbf{B} \in R^{16\times2}$, $\mathbf{C} \in R^{2\times16}$, $\mathbf{X} \in R^{16\times1}$, $\mathbf{U} \in R^{2\times1}$.

B. Model of the foundation

The model of the foundation is shown in Fig.4 [4]. The coordinate of the case is different from that of the rotor system shown in Fig.3.

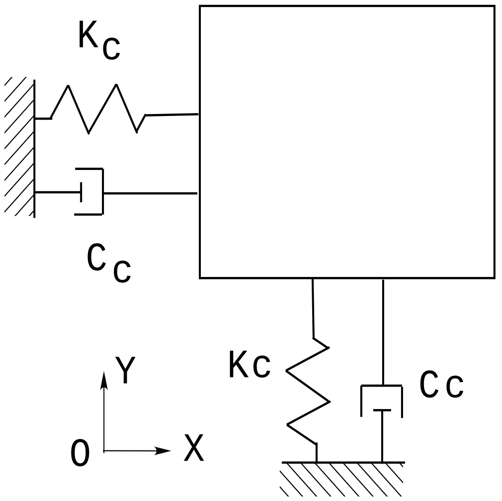

Fig. 4. Model of foundation

The stiffness and damping are shown in the vertical and the horizontal direction in Fig.4. The model of the foundation is designed when the coupled motion of the vertical direction and the horizontal direction is neglected.

$$M_c\ddot{x}_c + C_c\dot{x}_c + K_c x_c = 0 \tag{4}$$

$$M_c\ddot{y}_c + C_c\dot{y}_c + K_c y_c = 0 \tag{5}$$

Here, M_c is the mass of case, C_c is the damping coefficient, K_c is the stiffness coefficient. x_c, y_c are the displacements of the X and Y direction respectively. F_x and F_y are the control inputs of the X and the Y direction respectively. The state equation is gotten form (4) and (5), as shown in (6).

$$\dot{\mathbf{X}}_c = \mathbf{A}_c\mathbf{X}_c + \mathbf{B}_c\mathbf{U}_c \tag{6}$$

Here, $\mathbf{A}_c \in R^{4\times4}$, $\mathbf{B}_c \in R^{4\times2}$, $\mathbf{X}_c \in R^{4\times1}$, $\mathbf{U}_c \in R^{2\times1}$,

$$\mathbf{X}_c = \begin{bmatrix} \dot{x}_c & \dot{y}_c & x_c & y_c \end{bmatrix}^T, \tag{7}$$

$$\mathbf{A}_c = \begin{bmatrix} -\frac{C_c}{M_c} & 0 & -\frac{K_c}{M_c} & 0 \\ 0 & -\frac{C_c}{M_c} & 0 & -\frac{K_c}{M_c} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{8}$$

$$\mathbf{B}_c = \begin{bmatrix} -\frac{1}{M_c} & 0 \\ 0 & -\frac{1}{M_c} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{9}$$

$$\mathbf{U}_c = \begin{bmatrix} F_x & F_y \end{bmatrix}^T. \tag{10}$$

C. Reduced-order model of the rotational part

The reduced-order model of the rotational part is designed with two rigid modes and a flexible mode in the range of control. The high order vibration modes are aborted.

The designed reduced-order model is given in (11).

$$\begin{cases} \dot{\mathbf{X}}_r = \mathbf{A}_r\mathbf{X}_r + \mathbf{B}_r\mathbf{U}_r \\ \mathbf{Y}_r = \mathbf{C}_r\mathbf{X}_r \end{cases} \tag{11}$$

Here, $\mathbf{A}_r \in R^{6\times6}$, $\mathbf{B}_r \in R^{6\times2}$, $\mathbf{C}_r \in R^{2\times6}$, $\mathbf{X}_r \in R^{6\times1}$, $\mathbf{U}_r \in R^{2\times1}$.

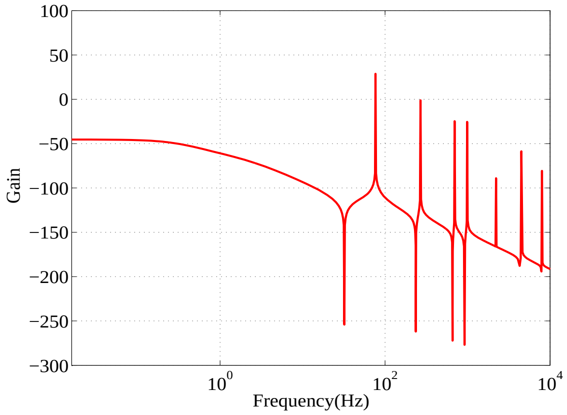

Fig. 5. Bode plot of the actural system

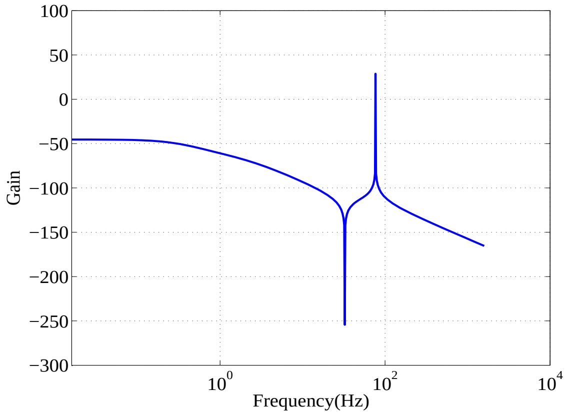

Fig. 6. Bode plot of the reduced-order system

The bode plot of the full order model shwon in (3) is given in Fig.5. And the bode plot of the reduced order model shown in (11) is given in Fig.6. Form Fig.5 and Fig.6, we see the modes in the range of the control are separated from the high order vibration modes successfully. Equation (11) is used in the controller design later.

V. CONTROL METHOD

A. Sliding mode control

The sliding mode control is applied in this paper. In this section, the sliding mode control is applied to the energy storage flywheel used in an electric vehicle. The observer is used to estimate the states of the mode coordinate. The states of the mode coordinate are estimated by the control input and the system output. The block diagram is shown in Fig.7.

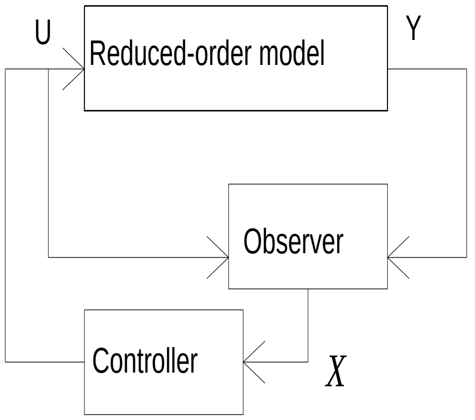

Fig. 7. Block diagramm of slidling mode control

Sliding mode control is a control scheme based on the concept of changing the structure of the controller in response to the changing state of the system in order to obtain a desired response [5], [6]. A high speed switching control action is used to switch between different structures in the state space. The behavior of the closed loop system is determined by the sliding surface. The sliding mode control is insensitivity to variation in system parameters and external disturbances.

The transient motion of a variable structure control system consists of two independent stages, a motion bringing the state of the system to the sliding mode, and a slower motion during which the state slides toward the origin of the state space.

The sliding surface is defined as (12).

$$\sigma(\hat{\mathbf{x}}) = \mathbf{S} \times \hat{\mathbf{x}} \tag{12}$$

The elements of $\mathbf{S}$ are S_i, and S_i are designed by the zero point of the system. The principle is that the real parts of eigen-values of $\mathbf{A_r} - \mathbf{B_r} \times (\mathbf{S} \times \mathbf{B_r})^{-1} \times \mathbf{S} \times \mathbf{A_r}$ are negative numbers.

The switching function is given in (13) when the switching mode is reached.

$$\sigma(\hat{\mathbf{x}}) = 0 \tag{13}$$

The equivalent control input is given in (14).

$$\mathbf{U_{eq}} = -(\mathbf{S} \times \mathbf{B_r}) \times (\mathbf{S} \times \mathbf{A_r})\hat{\mathbf{x}} \tag{14}$$

Where $\mathbf{A_r}$ and $\mathbf{B_r}$ are shown in (11). $\hat{\mathbf{x}}$ is the estimated value of $\mathbf{x}$ in (11).

B. LQR control

In this paper, the LQR control is applied to the foundation control. The objective is to find a control function to stabilize the case. A system is stabilizeable if there exists a state feedback control such that the closed loop system is exponentially stable.

C. Controller design

The controller design in this paper is realized by $Matlab$ [7]. The commands LQR and $LQR2$ with $Potter$ and $Schue$ method are used in controller design. The controller used for rotational part is given as follows.

$$\mathbf{Q}_r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$\mathbf{R}_r = 8.1 \times \begin{bmatrix} 0.15 & 0 \\ 0 & 0.13 \end{bmatrix} \tag{16}$$

The design observer is given as follows.

$$\mathbf{Q}_{ob} = 60 \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$\mathbf{R}_{ob} = 1.1 \times \begin{bmatrix} 0.01 & 0 \\ 0 & 0.001 \end{bmatrix} \tag{18}$$

The controller used for the control of the foundation is given as follows.

$$\mathbf{Q}_c = 110 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$\mathbf{R}_c = 0.02 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{20}$$

Next, the designed controller and observer are used for the closed loop simulation.

VI. SIMULATION

A. Validity of the model

The vibration analysis is done by the one dimensional FEM model shown in (3), the first flexible natural frequency is about $76.7Hz$. The value is near to that of ANSYS analysis.

The mode shape is given in Fig.8.

B. Simulations

The designed controllers are applied to the model of the rotational part and the foundation to verify the validation of the model and the controller. Here, the initial value responses are given. The initial value response of rotor is given in Fig.9. The measured value of the upper sensor and the below sensor are given in Fig.9(a) and Fig.9(b), respectively. The outputs of rotor are converged to zero from Fig.9(a) and Fig.9(b). The case is stabilized concurrently. The responses of the case are given in Fig.10. The

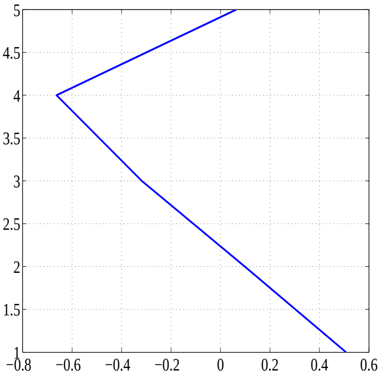

Fig. 8. Mode shape of FEM analysis

responses of the x direction and the y direction are given in Fig.10(a) and Fig.10(b), respectively.

The control inputs of the rotor are given in Fig.11. The control inputs of the upper AMB and the below AMB are given in Fig.11(a) and Fig.11(b), respectively. The switching functions of the sliding mode controller are given in Fig.12(a) and Fig.12(b). The control inputs of the case are given in Fig.13. The control inputs of the X direction and the Y direction are given in Fig.13(a) and Fig.13(b) respectively. From Fig.9 and Fig.10, the outputs of the rotor and the case are not big. From Fig.11 and Fig.13, the control inputs are smaller than the max output of the actuator. The control input can be generated from the actuator.

VII. CONCLUSIONS

This paper reported the model design and controller design of an energy storage flywheel with a flexible rotor used for electric vehicle. The vibration analysis, the design of foundation, the parameters of the flywheel and the rotor, the model of the flywheel and the rotor, the model of the foundation, the design of a reduced-order model with two rigid modes and a flexible mode, the sliding mode controller design and LQR controller design, closed loop simulation are reported.

The conclusions of this paper are given as follows.

1. The parameters of the flywheel and the rotor are designed based on the vibration analysis of ANSYS.

2. The one dimensional finite element method model of the flywheel and the rotor is designed. Based on this model, a reduced-order model with two rigid modes and a flexible mode is designed.

3. The model of the foundation is designed.

4. The controller of the flywheel and the rotor is designed by the reduced order model.

5. A LQR controller is designed for the foundation.

6. The validation of the proposed model and the controller are verified by the closed loop simulations.

Now, we are making an experiment equipment to verify the proposed model and designed controller.

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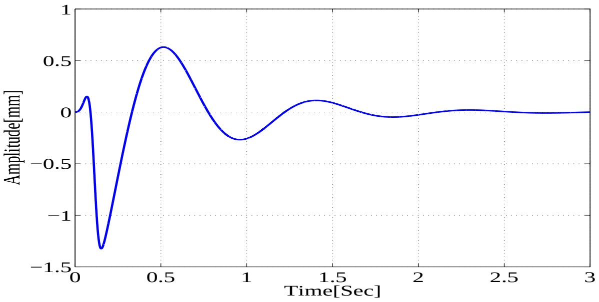
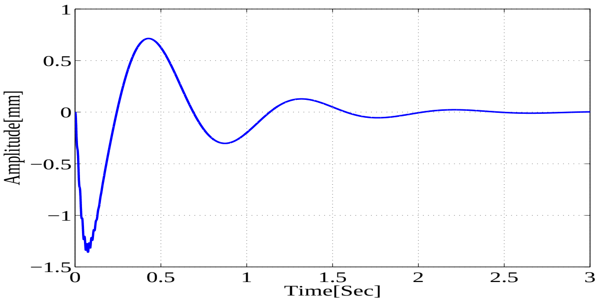
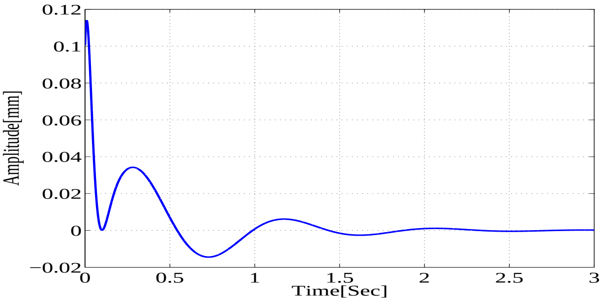


(a) Upper sensor

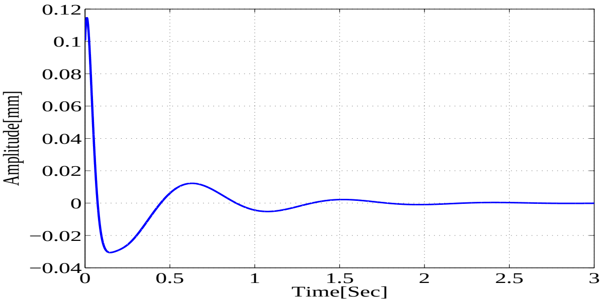


(b) Below sensor

Fig. 9. Outputs of rotor

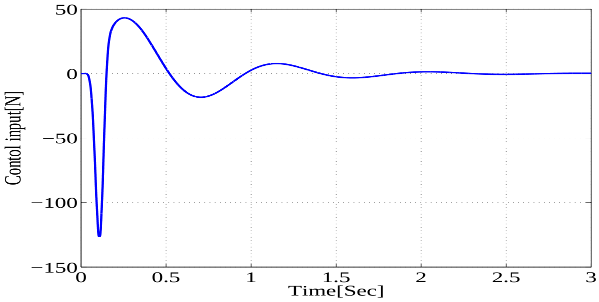


(a) x direction of case

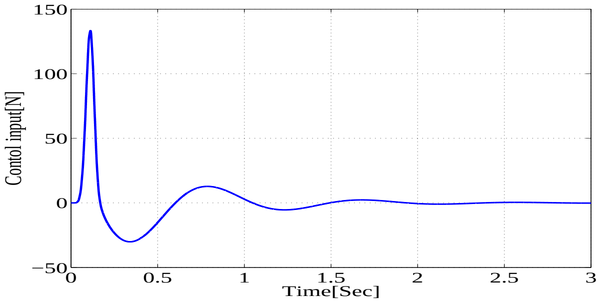


(b) y direction of case

Fig. 10. Outputs of foundation

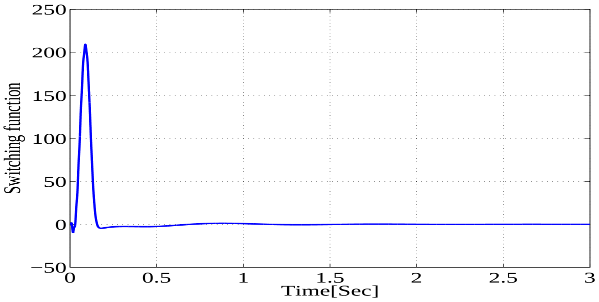


(a) Upper AMB

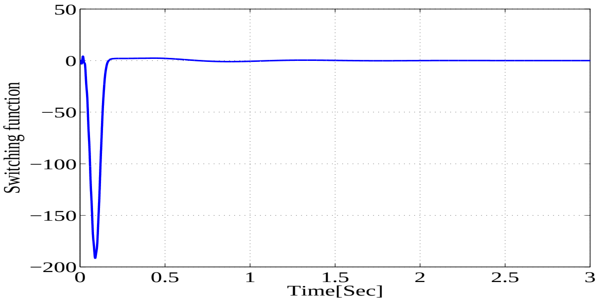


(b) Below AMB

Fig. 11. Control inputs of rotational part

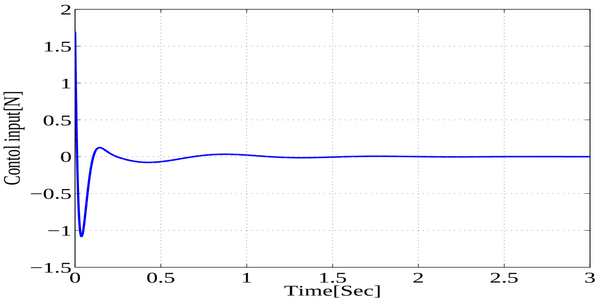


(a) Upper sensor

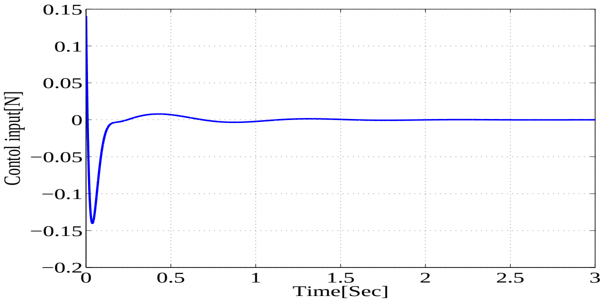


(b) Below sensor

Fig. 12. Switching function



(a) X direction



(b) Y direction

Fig. 13. Control inputs of foundation