Nonlinear Compensation of Zero Power Magnetic Suspension for High-Performance Vibration Isolation Systems*

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***Abstract* – The treated vibration isolation system uses zero-power magnetic suspension for realizing negative stiffness and combines it with a spring that has positive stiffness (positive spring). Infinite stiffness against direct disturbance is achieved by setting the amplitude of negative stiffness to equal the positive stiffness. One of the problems of this system is that the amplitude of negative stiffness varies from the nominal value when disturabance force is applied to the isolation table. This paper presents two approaches to overcoming this problem. One is to apply a nonlinear feedback control to zero-power magnetic suspension. The other is to use a magnetic spring as a positive spring. The carried out experiment demonstrated the efficacy of both the methods.**

***Index Terms* – Magnetic suspension, Zero-power control, Nonlinear control, Magnetic Spring, Vibration Isolation.**

I. Introduction

For microvibration isolation, high performance both in isolation from ground vibration and in the suppression of direct disturbance is necessary. However, a trade-off between them is inevitable in conventional passive-type vibration isolation systems because lower stiffness of the suspension spring is better for the former while higher stiffness is better for the latter.

Mizuno has proposed a new approach to breaking through the trade-off [1]. The proposed vibration isolation system uses zero-power magnetic suspension. Since a zero-power magnetic suspension system behaves as if it has negative stiffness, infinite stiffness to disturbances on the isolation table can be achieved by combining it with a normal spring. The efficacy of this new mechanism has been confirmed experimentally.

In zero-power magnetic suspension systems, however, the magnitude of negative stiffness is a function of the gap between the electromagnet and the suspended object. When the mass of the isolation table changes, therefore, the negative stiffness varies from the nominal value so that the stiffness against direct disturbances acting on the table becomes lower [1, 2]. For solving such a problem, two approaches will be presented in this paper.

II. Vibration Isolation System

First, it will be shown that infinite stiffness can be generated by connecting a normal spring with a spring with negative stiffness (negative spring). When two springs with spring constants of k_1 and k_2 are connected in series, the total stiffness k_c is given by

$$k_c = \frac{k_1 k_2}{k_1 + k_2} \tag{1}$$

This equation shows that the total stiffness becomes lower than that of each spring when normal springs are connected. However, if one of the springs has negative stiffness that satisfies

$$k_1 = -k_2 , \tag{2}$$

the resultant stiffness becomes infinite, that is

$$|k_c| = \infty . \tag{3}$$

The proposed vibration isolation system uses this principle for generating high stiffness against direct disturbance, and zero-power magnetic suspension as negative spring [1].

Figure 1 shows the configuration of vibration isolation system [1]. A middle mass m_1 is connected to the base through a spring k_1 and a damper c_1 that work as a conventional vibration isolator. An electromagnet for zero-power magnetic suspension is fixed to the middle mass. The part of an isolation table m_2 facing the electromagnet is made of soft iron material confining magnetic fields produced by permanent magnets for zero-power control. The absolute value of negative stiffness is equal to the force-displacement coefficient of the electromagnet that is denoted by k_s.

When the mass on the isolation table increases, the gap decreases in the steady states A problem is the parameter k_s is a function of the gap between the electromagnet and

* This work is financially supported in part by a Grant-in-Aid for the Creation of Innovations through Business-Academic-Public Sector Cooperation and a Grant-in-Aid for Scientific Research (B) from the Ministry of Education, Culture, Sports, Science and Technology of Japan to T. Mizuno.

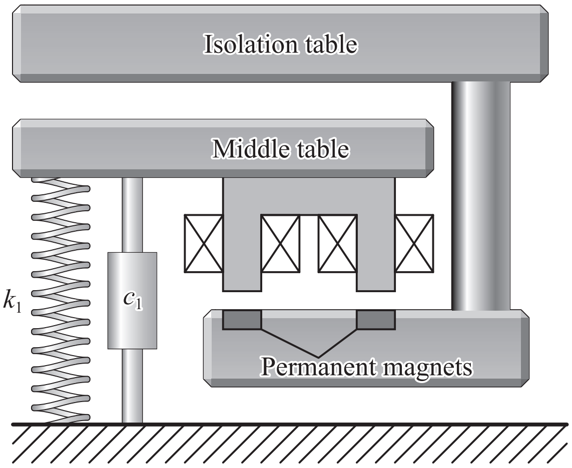


Fig.1 Vibration isolation system using zero-power magnetic suspension

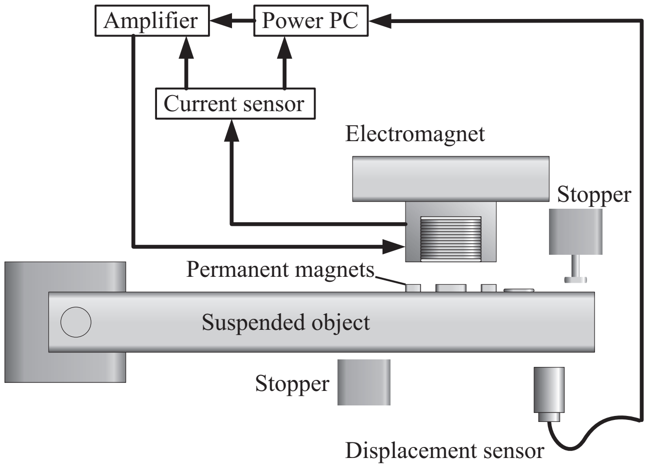


Fig.2 Experimental apparatus for zero-power control

the isolation table; roughly speaking it is inversely proportional to the cube of the gap. It indicates that it is difficult to maintain the condition (2) under variations in the mass on the isolation table when a normal mechanical spring is used as a positive spring k_1. The experiment carried out with the manufactured apparatus has supported this prediction [1, 2]. It causes the stiffness against direct disturbances to be lower. This paper will present two approaches to this problem in the following.

III. Nonlinear Compensation

A. Principles

The attractive force of a PM-biased electromagnet can approximately be presented by

$$F_e = K \frac{(I_0 + i)^2}{(D_0 - x)^2}\,, \tag{4}$$

where K is the attractive force coefficient of the electromagnet, I_0 is the equivalent current provided by the permanent magnet, and D_0 is the nominal air gap in the steady state including the height of the permanent magnet. Using Taylor principle, (4) can be expanded as

$$F_e \cong K \frac{I_0^2}{D_0^2}(1 + 2\frac{x}{D_0} + \cdots + (n+1)(\frac{x}{D_0})^n + \cdots) \\ \times (1 + 2\frac{i}{I_0} + \frac{i^2}{I_0^2})\,. \tag{5}$$

In the zero-power control system, the control current is small especially near the steady states so that

$$F_e \cong F_0 + k_i i + k_s \sum_{n=1}^{\infty} c_n x^n\,. \tag{6}$$

where

$$F_0 = K \frac{I_0^2}{D_0^2}\,, \tag{7}$$

$$k_i = 2K \frac{I_0}{D_0^2}\,, \tag{8}$$

$$k_s = 2K \frac{I_0^2}{D_0^3}\,, \tag{9}$$

$$c_n = \frac{n+1}{2D_0^{n-1}} \quad (n = 1, 2 \cdots)\,. \tag{10}$$

It is found from (6) that nonlinearity comes from the third term in the right-hand side. Therefore, the control current including the compensation for the nonlinearity should be given by

$$i = i_{zp} - \frac{k_s}{k_i} \sum_{n=2}^{\infty} c_n x^n\,. \tag{11}$$

where i_{zp} is the input for zero-power control.

B. Experimental Setup

Figure 2 shows a schematic drawing of a single-degree-of-freedom experimental apparatus for basic study on magnetic suspension [3]. It has an arm as a suspended object and an electromagnet for control. Permanent magnets providing bias flux are made of NdFeB material. They are built in the target iron of the suspended object instead of the electromagnet. Such configuration has an effect of widening the operation range because repulsive force can be generated between the suspended object and the electromagnet. The motion of the arm is detected by an eddy-current displacement sensor. The moving part of a voice coil motor is attached to the arm to produce disturbances. Designed control algorithms are implemented with a digital controller DS1102 manufactured by dSPACE*TM*. The control period is 100μs.

C. Experimental Results

To estimate the *negative* stiffness of the zero-power magnetic suspension, its gap-force characteristics are measured; downward force is produced by placing weights on the arm. The measurement results are shown by the graph for $d_2 = 0$ in Fig.3. The gap decreases as the downward force increases so that the stiffness is negative. The graph for $d_2 = 0$ in Fig.4 shows the magnitude of negative stiffness versus the applied force, which is calculated based on the measurement results shown in Fig.3. As the downward force increases, the gap decreases so that the force-displacement coefficient k_s becomes larger. As a result, the amplitude of negative stiffness also becomes larger.

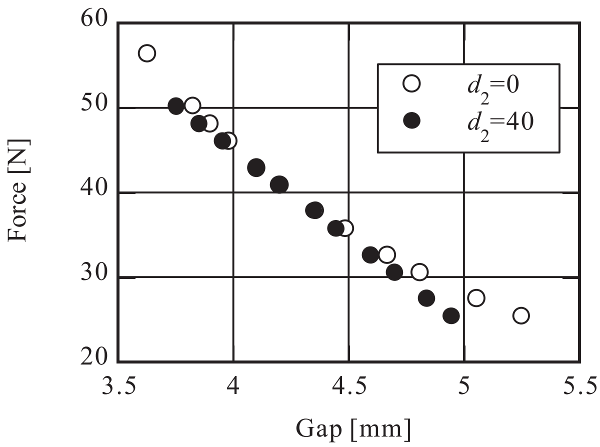


Fig.3 Load -gap characteristics of the zero-power suspension system

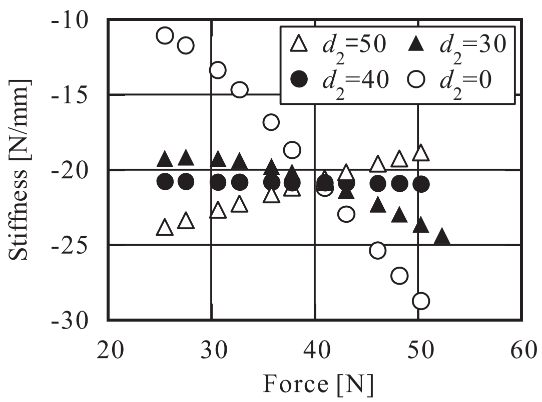


Fig.4 Load -stiffness characteristics of the zero-power suspension system using nonlinear compensation with various gains.

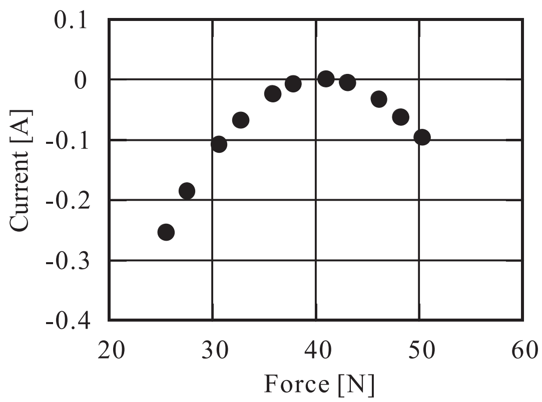


Fig.5 Control current of electromagnet for nonlinear compensation.

Next, the same characteristics were measured when the proposed nonlinear compensation was activated. In this experiment, only the square of displacement x^2 was fed back so the control input was represented as

$$i = i_{zp} - d_2 \frac{k_s}{k_i} \frac{x^2}{D_0^2}, \qquad (12)$$

where d_2 is the coefficient of nonlinear feedback, which should equal $c_2 D_0^2$. The gap-force characteristic is linearized as shown by the graph for $d_2 = 40$ in Fig.3. Figure 4 shows the magnitude of negative stiffness versus the applied force for several values of d_2. The negative stiffness is almost constant when $d_2 = 40$. This demonstrates the efficacy of the nonlinear compensation. This compensation has been applied to a three-axis vibration isolation system [4].

One of the problems of this method is that the property of zero power is lost by adding such compensation. Figure 5 shows the control current when $d_2 = 40$. Since the nominal value of negative stiffness is set to be the value when the applied force is 40N and the gap is 4.2mm, the coil current becomes zero at this force. However, some steady-state current flows in the other conditions.

IV NONLINEAR SPRING

A. Principles

The former method has a problem that the zero-power characteristic may be lost due to the compensation. Another approach, which does not have such problem, is to replace the normal spring by a spring that has an appropriate nonlinearity [5]. Figure 6 shows presents a modified configuration in which a magnetic spring using repulsive forces between permanent magnets replaces the normal spring in Fig.1. The stiffness of the magnetic spring is also a function of the gap; it increases as the gap between the magnet decreases. When an additional downward force is applied to the isolation table, the gap decreases both in the zero-power magnetic suspension system and in the magnetic spring so that both negative and positive stiffness increases. Therefore, it is expected that the modified system can maintain high stiffness against direct disturbances for a wider range of mass on the isolation table.

B. Experimental Setup

Figures 7 is a schematic drawing the manufactured apparatus for basic experimental study. The middle and isolation tables are mechanically guided to be in translation in the vertical direction by the combination of vertical poles and ball bearings. The middle table is suspended by a repulsive-type magnetic spring; a ring-shape permanent magnet is attached to the lower side of the middle table and another is fixed to the base. An electromagnet is fixed to the upper side of the middle table. Permanent magnets providing bias flux for zero-power magnetic suspension are built in the target iron of the isolation table. All the permanent magnets are made of NdFeB materials.

Two auxiliary electromagnets, omitted in Fig.7, are fixed to the base. They are operated to adjust the positive stiffness and to improve the damping characteristics of the middle table. Since the positive stiffness of the magnetic spring was lower than the negative stiffness of the zero-power magnetic suspension in the initial state, the magnitude of the positive stiffness was adjusted by using the auxiliary electromagnets. It is to be noted that such adjustment could be possible with a mechanical spring or avoidable if the magnetic spring were carefully designed.

The relative displacement of the middle table to the base and that of the isolation table to the middle table are detected by eddy-current gap sensors with a resolution of m.

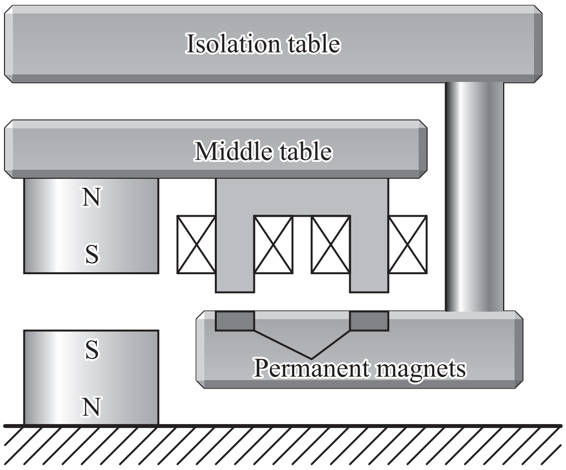


Fig.6 Modified vibration isolation system using zero-power magnetic suspension

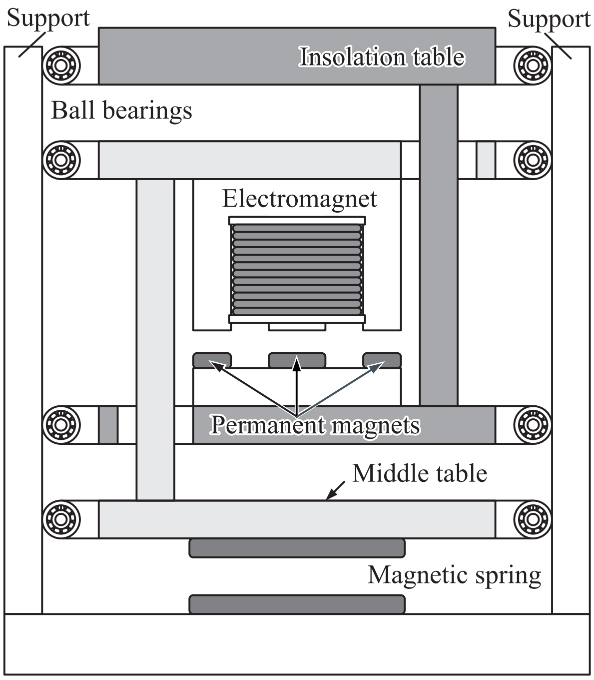
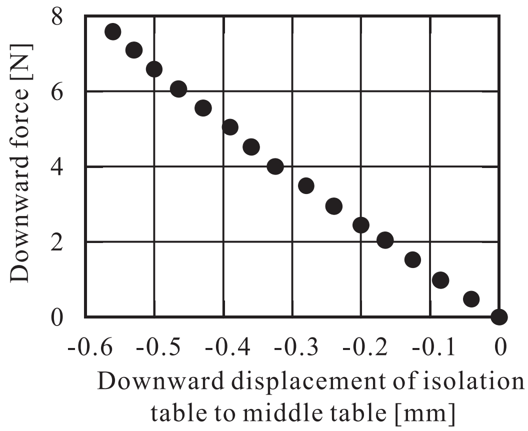


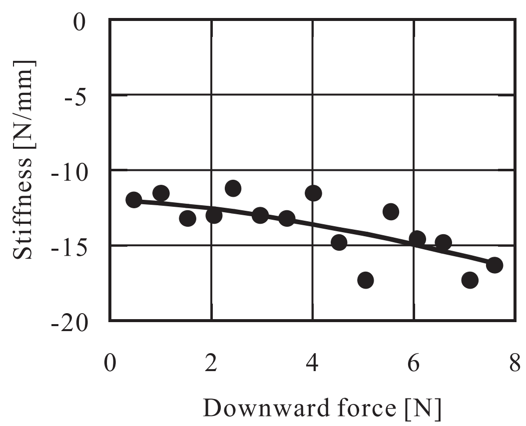
Fig.7 Schematic drawing of the apparatus with a magnetic spring

C. *Experimental Results*

First, the *negative* stiffness of the zero-power magnetic suspension was estimated. Downward direct disturbance force is produced by adding weights on the isolation table. Figure 8 shows the results. The relative downward displacement of the isolation table to the middle table, which equals the decrease in the gap, is plotted to downward force in Fig.8a. Since the additional forces produced by weights are downward, the stiffness of the magnetic suspension is clearly negative. The negative stiffness estimated from the measurement results is also shown in Fig.8b. It is found that the magnitude of *negative* stiffness becomes larger as the added downward force increases.

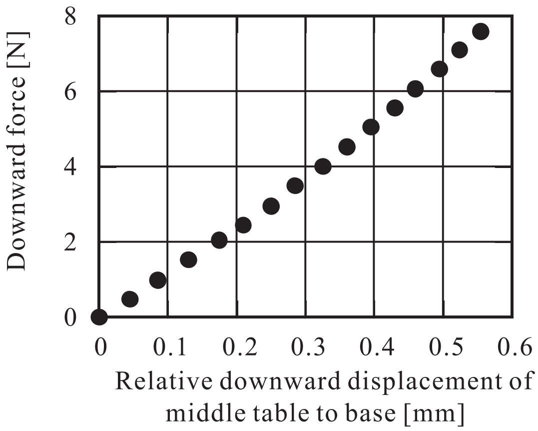


(a) Force-displacement characteristics

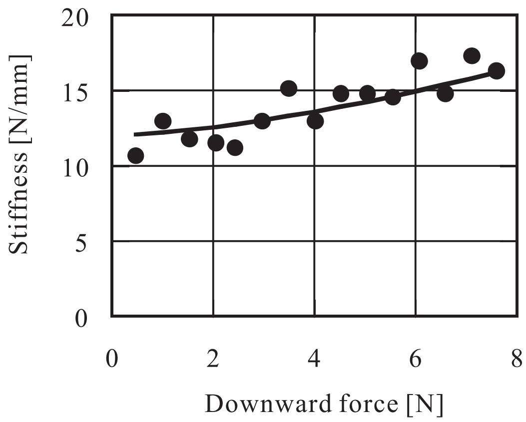


(b) Stiffness

Fig.8 Characteristics of the zero-power magnetic suspension system



(a) Force-displacement characteristics



(b) Stiffness

Fig.9 Characteristics of the magnetic spring

Second, the *positive* stiffness of the magnetic spring was estimated. The *downward* displacement of the middle table to the base is plotted to downward force in Fig.9a; the estimated positive stiffness is also shown in Fig.9b. This result demonstrates well that the magnitude of *positive* stiffness becomes larger as downward load force increases.

Figure 10 shows the displacement of the isolation table to the base in addition to the relative displacements of the middle table to the base and of the isolation table to the middle table when weights are added on the isolation table. This result demonstrates well that combining a zero-power magnetic suspension with a magnetic spring can generate high stiffness against static direct disturbance acting on the isolation table.

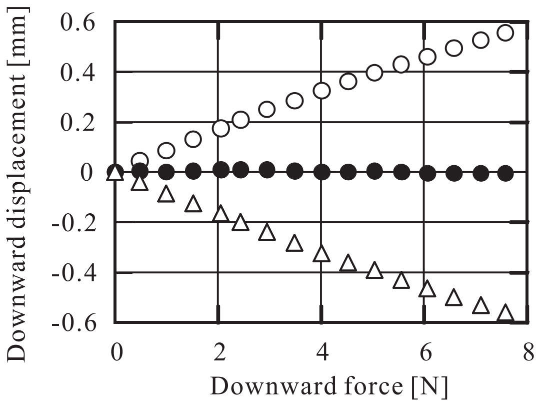


Fig.10 Displacements of the isolation table when downward force is applied to the isolation table.

V CONCLUSION

Two approaches were presented for solving a problem due to the nonlinearity of zero-power magnetic suspension. One is to compensate the nonlinearity with a nonlinear feedback. The other is to compensate the nonlinearity with nonlinearity; replacing a normal mechanical spring by a magnetic spring using repulsive forces between permanent magnets. The efficacy of each method was experimentally confirmed.

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