Unfalsified PID Controller Design using Support Vector Machine with Gap Metric and its Application

Michihiro Kawanishi
Dept. of Mechanical Engineering
Kobe University
Rokkodai-cho 1-1, Nada ward, Kobe city, Japan
kawa@mech.kobe-u.ac.jp

Masanori Ukibune
Dept. of Mechanical Engineering
Kobe University
Rokkodai-cho 1-1, Nada ward, Kobe city, Japan
ukibune@ma-2.mech.kobe-u.ac.jp

***Abstract*— In this paper, we propose a data-based PID controller design method using unfalsified control technique with Support Vector Machine (SVM) and gap metric. SVM and gap metric are utilized for adjusting the weighting function of extended L_2 gain criterion. The effectiveness of the proposed method is evaluated by experiments on a magnetic levitation system and an active magnetic bearing system.**

I. Introduction

In this paper, we propose a new direct PID controller design method using Support Vector Machine (SVM)[1] and gap metric technique[2]. The proposed method applies the notion of "Unfalsification"[3] and extended L_2 gain criterion. So far, the unfalsified control technique is applied to several applications, e.g. [4]-[7]. The open-loop characteristics of these conventional applications are stable or marginally stable. Here, we firstly apply the method to unstable plants, i.e. a magnetic levitation system and an active magnetic bearing system. In order to manage unstable phenomena, we introduce SVM and gap metric techniques to unfalsified control method.

The notion and the technique of "Unfalsification" enable us to determine whether a candidate PID controller achieves the desired performance specification or not without performing additional experiments. Our proposed method is one of the off-line iterative controller design method. The effectiveness of the proposed method is confirmed by experiments on a magnetic levitation system and an active magnetic bearing system.

II. PID Controller Design Based on Unfalsified Control with SVM and Gap Metric

In this section, a PID controller design method with input-output data is proposed.

A. Fictitious input and performance specification

We consider one degree of freedom PID controller $\mathcal{K}$ as follows;

$$\begin{aligned}\mathcal{L}(u) &= \left(K_P + \frac{K_I}{s} + K_D s\right)\mathcal{L}(r-y) \qquad (1)\\ &=: \mathcal{K}(K_P, K_I, K_D)\mathcal{L}(r-y), \qquad (2)\end{aligned}$$

where K_P, K_I and K_D are positive real scalars, s indicates derivative operator, $u(t)$, $r(t)$ and $y(t)$ denote control input, dereference signal, and sensor output respectively, and $\mathcal{L}(f)$ indicates the Laplace transformation of $f(t)$. Suppose an initial candidate controller set $\hat{K}$ as

$$\hat{K} := \{\mathcal{K}(K_{Pi}, K_{Ii}, K_{Di}), \quad i = 1, \cdots, m\}. \qquad (3)$$

All candidate controllers $\mathcal{K}(K_{Pi}, K_{Ii}, K_{Di})$ in $\hat{K}$ are causal left-invertible. Accordingly, given input-output data $u(t)$ and $y(t)$, we can uniquely calculate the following fictitious reference $\tilde{r}_i(t)$ which makes each controller $\mathcal{K}(K_{Pi}, K_{Ii}, K_{Di})$ admissible,

$$\tilde{r}_i = \mathcal{L}^{-1}\left\{\frac{s}{K_{Di}s^2 + K_{Pi}s + K_{Ii}}\mathcal{L}(u) + \mathcal{L}(y)\right\}, \qquad (4)$$

where let $\mathcal{L}^{-1}(F)$ be the inverse Laplace transformation of $F(s)$. The signal $\tilde{r}$ is different from the actual reference signal r. Using unfalsified control theorem[3], the fictitious reference signal $\tilde{r}$ is utilized for evaluating control performance.

Let L_{2e} be the set of functions which have bounded values of extended L_2 norm defined as follows;

$$||f(t)||_{L_2[0,\tau]} := \sqrt{\int_0^\tau f(t)^2 dt}. \qquad (5)$$

For $(\tilde{r}_i, y, u) \in L_{2e} \times L_{2e} \times L_{2e}$, our performance specification is defined as

$$||w_1 * (y - \tilde{r}_i)||^2_{L_2[0,\tau]} + ||w_2 * u||^2_{L_2[0,\tau]} \le ||\tilde{r}_i||^2_{L_2[0,\tau]}, \ \forall\tau, \qquad (6)$$

where w_1 and w_2 are weighting function for which represent control specification, and $*$ denotes convolution, i.e.

$$\begin{aligned}||w_1 * (y - \tilde{r}_i)||^2_{L_2[0,\tau]} &= \int_0^\tau [w_1(t-\tau)\cdot\{y(\tau) - \tilde{r}_i(\tau)\}]^2 dt,\\ ||w_2 * u||^2_{L_2[0,\tau]}|| &= \int_0^\tau \{w_2(t-\tau)\cdot u(\tau)\}^2 dt.\end{aligned}$$

In case the plant is time invariant and modeled by $P(s)$, the performance specification (6) becomes equivalent to

$$\left|\left| \begin{array}{c} W_1 S \\ W_2\mathcal{K}S \end{array} \right|\right| \le 1 \qquad (7)$$

where $W_i = \mathcal{L}(w_i)$ for $i = \{1, 2\}$ and $S := 1/(1 + P\mathcal{K})$. Notice that the performance specification (6) is plant model free representation which differs from the expression of (7). Now, we can define performance index function J as

$$J(\tilde{r}_i, y, u) \ := \ ||\tilde{r}_i||^2_{L_2[0,\tau]} - ||w_1 * (y - \tilde{r}_i)||^2_{L_2[0,\tau]} - ||w_2 * u||^2_{L_2[0,\tau]}. \quad (8)$$

B. Support vector machine and gap metric

In order to design a high performance controller, w_1 and w_2 in (6) must be properly specified. However, the design of these weights is time consuming and needs large experience. Especially, in the case of active magnetic actuators control, these weights design becomes much more difficult due to the unstable property of the plant. Here, we propose an adjustment method for these weights w_1 and w_2 by means of support vector machine and gap metric.

Briefly, we summarize support vector machine technique in this subsection. A Hyper plane classifying data $\boldsymbol{x} \in \mathcal{R}^m$ which takes $y = 1$ or $y = -1$ is defined by

$$D(\boldsymbol{x}) = (\boldsymbol{w} \cdot \boldsymbol{x}) + w_0, \quad (9)$$

where $\boldsymbol{w}$ is normal vector and w_0 is a constant. In order to define unique hyperplane, we introduce the following additional constraints.

$$(\boldsymbol{w} \cdot \boldsymbol{x}) + w_0 \geq +1 \quad \text{if} \quad y_i = +1 \quad (10)$$

$$(\boldsymbol{w} \cdot \boldsymbol{x}) + w_0 \leq -1 \quad \text{if} \quad y_i = -1 \quad (11)$$

Then, the hyperplane is denoted by

$$y_i[(\boldsymbol{w} \cdot \boldsymbol{w}_i) + w_0] \geq 1 \quad i = 1, \cdots, n. \quad (12)$$

Since minimal margin between hyper plane and the data $\boldsymbol{x}$ is $\frac{1}{\|w\|}$, we can see that the maximization of the minimal margin is carried out by solving the following optimization problem;

$$\min_{\boldsymbol{w}, w_0, \alpha} Q(\boldsymbol{w}, w_0, \alpha) \quad (13)$$

$$Q(\boldsymbol{w}, w_0, \alpha) = \frac{1}{2}||w||^2 - \sum_{i=1}^{n} \alpha_i \{y_i[(\boldsymbol{w} \cdot \boldsymbol{x}_i) + w_0] - 1\} \quad (14)$$

where $\alpha_i \geq 0$ are Lagrange variables. For optimal variables $\boldsymbol{w}^*, w_0^*$ and α^*, the following conditions hold.

$$\frac{\partial Q(\boldsymbol{w}^*, w_0^*, \alpha^*)}{\partial w_0} = 0, \quad (15)$$

$$\frac{\partial Q(\boldsymbol{w}^*, w_0^*, \alpha^*)}{\partial \boldsymbol{w}} = 0. \quad (16)$$

By these conditions, the following relations hold.

$$\sum_{i=1}^{n} \alpha_i^* y_i = 0, \quad \alpha_i^* \geq 0, \quad i = 1, \cdots, n \quad (17)$$

$$\boldsymbol{w}^* = \sum_{i=1}^{n} \alpha_i^* y_i \boldsymbol{x}_i, \quad \alpha_i^* \geq 0, \quad i = 1, \cdots, n \quad (18)$$

Kuhn-Tucker condition is denoted by

$$\alpha_i^*[y_i(\boldsymbol{w}^* \cdot \boldsymbol{x}_i + w_0^*) - 1] = 0. \quad (19)$$

Substituting (17) and (18) into (13), the following optimization problem is obtained on α.

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} Q(\boldsymbol{\alpha}) \quad (20)$$

$$Q(\boldsymbol{\alpha}) = \sum_{i=1}^{n} \alpha_i - \gamma \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j (\boldsymbol{x}_i, \boldsymbol{x}_j) \quad (21)$$

$$\sum_{i=1}^{n} y_i \alpha_i = 0, \quad \alpha_i \geq 0, \quad i = 1, \cdots n \quad (22)$$

By using the optimal α_i^* of the optimization problem (20), optimal hyper plane is derived by

$$D(\boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i^* y_i (\boldsymbol{x} \cdot \boldsymbol{x}_i) + w_0^*. \quad (23)$$

Introducing nonlinear functions $\varphi_j(\boldsymbol{x})$, $j = 1, \cdots, m$ which map data $\boldsymbol{x}$ into m dimensional space, we can denote the hyperplane by

$$D(\boldsymbol{x}) = \sum_{j=1}^{m} w_i \varphi_j(\boldsymbol{x}). \quad (24)$$

Even for this nonlinear mapping case, we can similarly obtain the optimal hyperplane as follows;

$$D(\boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i^* y_i (\varphi(\boldsymbol{x}) \cdot \varphi(\boldsymbol{x}_i)) + w_0 \quad (25)$$

If there exists the following kernel function H, the calculation of $\varphi(\boldsymbol{x})$ can be avoided, and we can reduce the computational burden, so-called kernel trick.

$$\varphi(\boldsymbol{x}) \cdot \varphi(\boldsymbol{x}_i) = H(\boldsymbol{x}, \boldsymbol{x}_i) \quad (26)$$

These kernel functions must satisfy the Mercer condition. Here, we utilize the following Gaussian kernel.

$$H(\boldsymbol{x}, \boldsymbol{y}) = \exp\left(-\frac{||\boldsymbol{x} - \boldsymbol{y}||^2}{\sigma^2}\right) \quad (27)$$

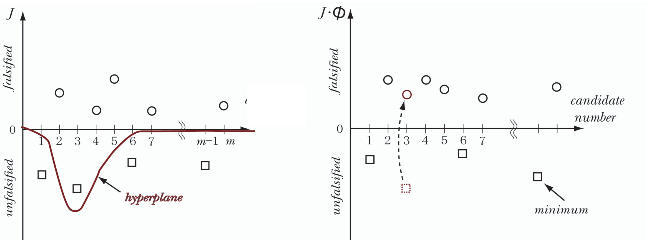
Now, the optimal hyperplane is described as follows;

$$D(\boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i^* y_i H(\boldsymbol{x}, \boldsymbol{x}_i) + w_0. \quad (28)$$

If $J(\tilde{r}_i, y, u) < 0$ holds, the controller $\mathcal{K}(K_{Pi}, K_{Ii}, K_{Di})$ satisfies the performance specification. At kth iteration of unfalsified controller design procedure, a controller $\mathcal{K}_k$ to be implemented to the plant is defined by

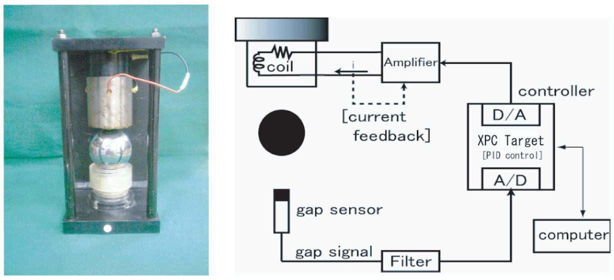
$$\mathcal{K}_k(K_{Pk^*}, K_{Ik^*}, K_{Dk^*}) := \arg\min_i J(\tilde{r}_i, y, u). \quad (29)$$

Using this $\mathcal{K}_k$, data u and y are acquired for the next design procedure. However, in case the controller $\mathcal{K}_k$ cannot stabilize the plant, we should suppose the performance index J is not adequate. In order to carry out the adjustment of the performance index function, we utilize SVM technique to derive a hyperplane classifying $\mathcal{K}_k$ in falsified set. The hyperplane by SVM theory is reformulated to the curved line $J = f(i)$ as shown in Fig. 1 (a).



(a) (b)

Fig. 1. Performance index adjustment using SVM with gap metric



(a) (b)

Fig. 2. Experimental setup of magnetic levitation system

The horizontal axes in Fig. 1 are the controllers index which is sorted by the gap metric value. That is , the following condition holds;

$$||\mathcal{K}(K_{Pi}, K_{Ii}, K_{Di}) - \mathcal{K}_{k-1}||_G < ||\mathcal{K}(K_{Pj}, K_{Ij}, K_{Dj}) - \mathcal{K}_{k-1}||_G, \quad \forall i < j, \tag{30}$$

where $|| \cdot ||_G$ denotes gap metric[2]. The calculation of the gap metric is summarized as follows;

$$||P_1 - P_2||_G := \inf_{Q \in H_\infty} \left\| \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} Q \right\|_\infty \tag{31}$$

where

$$P_i = C_i(sI - A_i)^{-1}B_i + D_i \tag{32}$$

$$\begin{bmatrix} M_i \\ N_i \end{bmatrix} = \left[\begin{array}{c|c} A_i + B_iF_i & B_iR_i^{-\frac{1}{2}} \\ \hline F_i & R_i^{-\frac{1}{2}} \\ C_i + D_iF_i & D_iR_i^{-\frac{1}{2}} \end{array}\right] \tag{33}$$

$$F_i = -R_i^{-\frac{1}{2}}(B_i^* X_i + D_i^* C_i) \tag{34}$$

$$X_i = \mathrm{Ric}\begin{bmatrix} A_i - B_iR_i^{-\frac{1}{2}}D_i^*C_i & -B_iR_i^{-\frac{1}{2}}B_i^* \\ -C_i^*\tilde{R}_i^{-\frac{1}{2}}C_i & -(A_i - B_iR_i^{-\frac{1}{2}}D_i^*C_i)^* \end{bmatrix}$$

$$R_i = I + D_i^* D_i \tag{35}$$

$$\tilde{R}_i = I + D_i D_i^* \tag{36}$$

where "Ric" means Algebraic Riccati Equation solution.

This SVM classification enables us to avoid unnecessary experiments by unpromising controllers whose closed loop characteristic is similar to the unstable controller.

Now, renewing the performance index function J as $J \cdot \Phi$ where

$$J \cdot \Phi(i) := J(\tilde{r}_i, y, u) - |f(i)|, \tag{37}$$

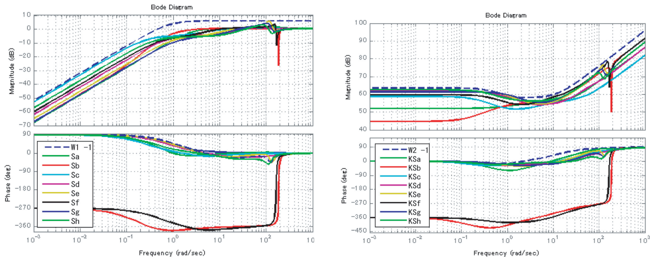
we can obtain an adjusted controller, i.e. $\arg\min J \cdot \Phi$, for the implementation to the plant as shown in Fig. 1 (b).

C. Unfalsified PID controller design method with SVM

In this subsection, we briefly summarize our proposed design procedure.

[Unfalsified PID Design Procedure with SVM]

1 Define $\hat{K}$, $\mathcal{I} := \{1, 2, \cdots, m\}$ and $\mathcal{K}_{\text{prev}} := \emptyset$.
2 Select an implement controller $\mathcal{K}_1 \in \hat{K}$, $k = 1$.
3 while $\mathcal{I} \neq \emptyset$ and $K_k \notin \mathcal{K}_{\text{prev}}$
4 Acquire data $u(t)$ and $y(t)$ with controller $\mathcal{K}_k$.
5 for $i \in \mathcal{I}$
6 Solve $\tilde{r}_i := \mathcal{L}^{-1}\left\{\frac{s}{K_{Di}s^2 + K_{Pi}s + K_{Ii}}\mathcal{L}(u) + \mathcal{L}(y)\right\}$.
7 Calculate $f(i)$ with SVM and gap metric
8 (In case $y(t)$ is stable, $f(i) = 0$.)
9 if $J \cdot \Phi(i) > 0$
10 $\mathcal{I} := \mathcal{I} \backslash \{i\}$.
11 endif
12 endfor
13 $i_{\min} := \arg\min_{\mathcal{I}} J \cdot \Phi(i)$.
14 $\mathcal{K}_{k+1} := \mathcal{K}(K_{Pi_{\min}}, K_{Ii_{\min}}, K_{Di_{\min}})$
15 $\mathcal{K}_{\text{prev}} := \mathcal{K}_{\text{prev}} \cup \{\mathcal{K}_k\}$.
16 $k := k + 1$
17 endwhile



(a) (b)

Fig. 3. Frequency responses of weighting functions

Due to the stopping criterion in Step3, the procedure is terminated when the all of the candidate controllers are falsified or the same controller is selected again. In case that the all controllers are falsified, i.e.$\mathcal{I} = \emptyset$, it is necessary to restart the procedure after enlarging the candidate controller set $\hat{K}$ or changing the performance specification.

III. Experiments

The effectiveness of our proposed method is evaluated by the experiments of a magnetic levitation system and an active magnetic bearing system.

A. Magnetic Levitation System

Fig. 2 shows the experimental setup of our magnetic levitation system. The steel ball is levitated by the electro-magnetic force. The mass of the steel ball is 4.2[kg], The

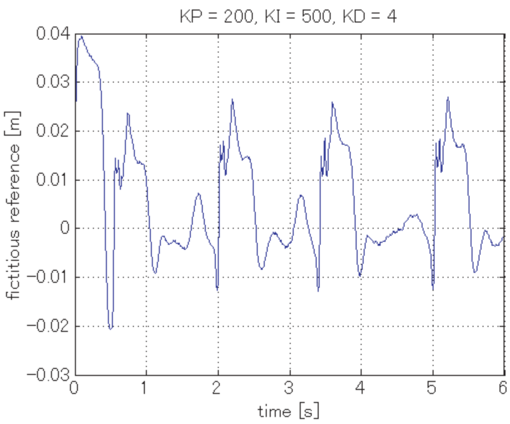
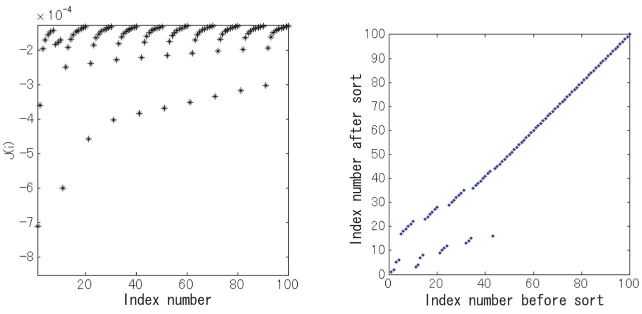


Fig. 4. Fictitious reference signal



(a) (b)

Fig. 5. (a) Performance index function and (b) Gap metric sorting

diameter of the steel ball is 100[mm]. The gap between coil and steel ball is measured by Eddy current sensor and the maximal gap is 10[mm]. We utilize MATLAB xPC Target System for controlling the electro-magnetic force.

The frequency domain representations of the weighting functions w_1 and w_2 are determined by frequency responses of stabilizing controllers. The weighting functions are derived as follows;

$$W_1(s) = \frac{0.5s + 0.4}{s}, \tag{38}$$

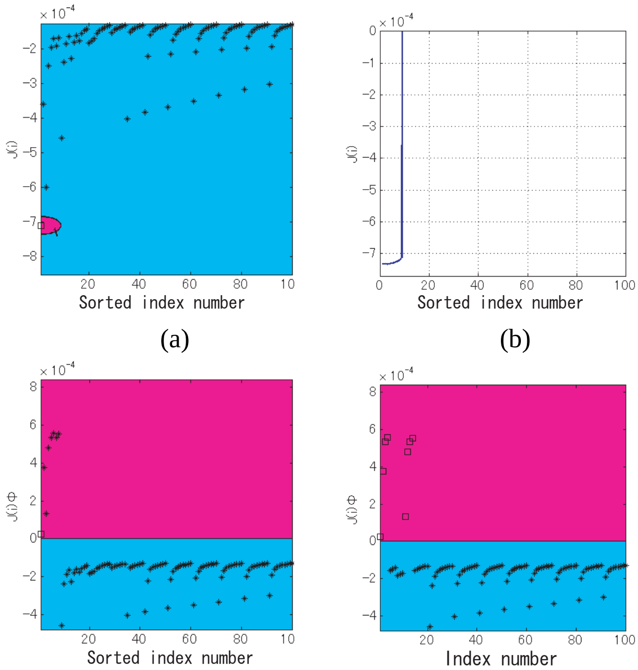
$$W_2(s) = \frac{2s + 1}{125s^2 + 1625s + 1500}. \tag{39}$$

The frequency responses concerning $W_1(s)$ and $W_2(s)$ are shown in Fig. 3 (a) and (b) respectively.

We set the sampling time 0.002[s] and the number of data 3000. The pseudo derivative $\frac{s}{s/5000+1}$ is utilized. We consider the following initial candidate controller set;

$$\begin{aligned} K_P &= \{200 + 200i \mid i = 0, \cdots, 9\} \\ K_I &= \{500i \mid i = 0, \cdots, 9\} \\ K_D &= \{4 + 8i \mid i = 0, \cdots, 9\} \end{aligned}$$

The number of candidate controllers is 1000. We select an initial implemented controller as $\{K_P, K_I, K_D\} = \{400, 500, 4\}$. Based on the experimental data by the implemented controller, we calculate the fictitious reference



(a) (b) (c) (d)

Fig. 6. Specification adjustment with SVM and Gap metric

signal by (4) and evaluate performance index function J by (8), (38), (39) and (40) for each candidate controllers. As the result of these evaluation, a controller $\{K_P = 200, K_I = 500, K_D = 4\}$ takes the best value of J. Fig. 4 shows the time response of the fictitious reference signal $\tilde{r}_1$ of the controller, i.e.

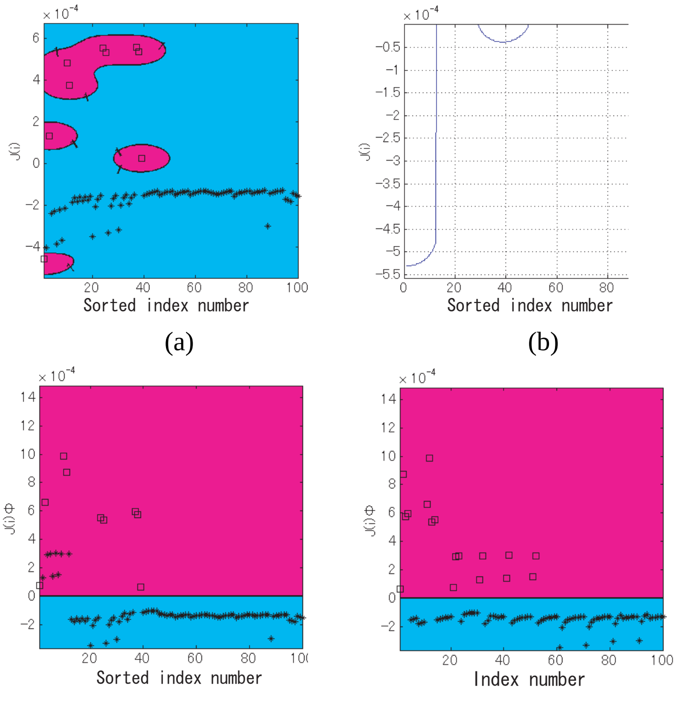
$$\tilde{r}_1 = \mathcal{L}^{-1}\left[\frac{s}{4s^2 + 200s + 500}\mathcal{L}(u) + \mathcal{L}(y)\right]. \tag{40}$$

Next, we fix integral gain K_I at the best value of J for intensive optimization on K_P and K_D gain. In this case, K_I is fixed at 500 and the number of the candidate controllers can be reduced to 100. The value of performance index function J is shown in Fig. 5 (a).

The time response of the controller $\{K_P = 200, K_I = 500, K_D = 4\}$ is unstable. Then, we adjust the performance index function J. First, the gap metric values are calculated by (31)-(36) for each candidate controllers and the controller index is sorted by the gap metric value. The sorting result is shown in Fig. 5 (b).

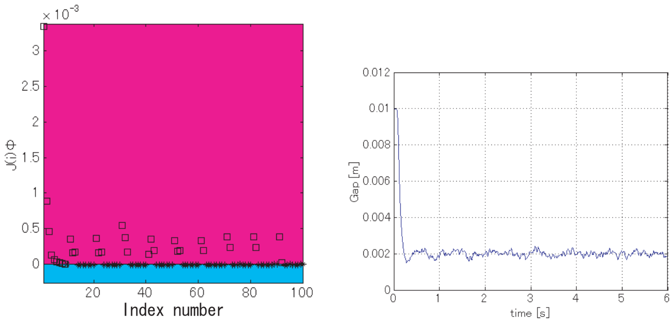
We set the parameter of Gaussian Kernel $\sigma = 0.1$ in (27). Carrying out the optimization with SVM software[1], we obtain optimal classification as shown in Fig. 6 (a), where the region colored by magenta is unfalsified. Based on the optimization, we can derive the additional term f in performance index function (37). In order to avoid unnecessary penalty, we set $f = 0$ for $J > 0$. The resultant f is shown in Fig. 6 (b). The adjusted performance index function $J \cdot \Phi$ is shown in Fig. 6 (c) and $J \cdot \Phi$ versus original controller index number is shown in 6 (d).

By these procedure, 8 controllers are falsified. The next controller which takes the best value of $J\Phi$ is decided



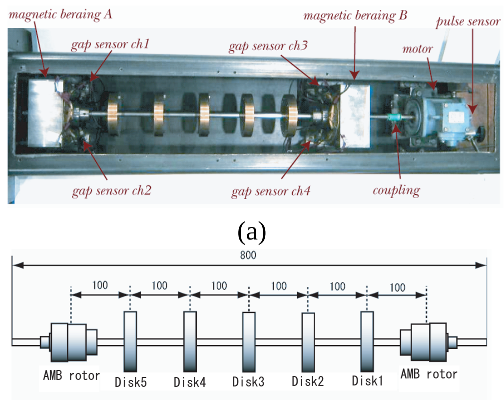
(a) (b) (c) (d)

Fig. 7. Specification adjustment with SVM and gap metric



(a) (b)

Fig. 8. (a) Performance index value and (b) Time response



(a) (b)

Fig. 9. Experimental setup of AMB rotor

TABLE I

UNFALSIFIED DESIGN PROCEDURE RESULTS

#	(K_P, K_I, K_D)	Stability	Falsified controllers #
1	$(400, 500, 4)$	Unstable	0
2	$(200, 500, 4)$	Unstable	8
3	$(200, 500, 20)$	Unstable	17
4	$(200, 500, 52)$	Unstable	23
5	$(1600, 500, 4)$	Unstable	27
6	$(800, 500, 20)$	Stable	35
7	$(200, 500, 20)$	Unstable	41
8	$(200, 2000, 20)$	Unstable	45
9	$(200, 3500, 20)$	Unstable	48
10	$(400, 2500, 12)$	Unstable	54
11	$(800, 1000, 20)$	Stable	55
12	$(400, 4500, 20)$	Unstable	56
13	$(800, 1000, 20)$	Stable	57

as $\{K_P = 200, K_I = 500, K_D = 20\}$. Experimental data is collected by this controller and similar procedure is repeated as shown in Fig. 7 (a)∼(d).

Table I shows the iterations, implemented controllers, stabilities and the number of falsified controllers. In this case, after 13 iterations, the proposed algorithm is terminated. As the result, 57 controllers are falsified and we obtain a high performance controller whose time response is shown in Fig. 8 (b). The resultant performance index is shown in Fig. 8 (a).

B. Active Magnetic Bearing

Our experimental setup is shown in Fig. 9 (a). The rotor schematic is shown in Fig. 9 (b). The mass and moment of inertia of AMB rotor are 1.56[kg] and 1.04×10^{-3}[kg·m^2]. The mass and moment of inertia of Disk 1∼5 are 1.293[kg] and 9.53×10^{-4}[kg·m^2]. Flexural rigidity EI of the shaft is 209[N·m^2].

First, in order to design weighting functions w_1 and w_2 in (8), we derive mathematical model of AMB rotor system. Figure and model of AMB is shown in Fig. 10 (a) and (b) respectively, where bias current I_1 and I_2 are 3.8[A] and 1.17[A], resistance R is 0.81[Ω], equilibrium gap W is 0.4×10^{-3}[m]. The inductance L_1 and L_2 are described as follows;

$$L_1 = \frac{Q}{X+W-x} + L_0, \tag{41}$$

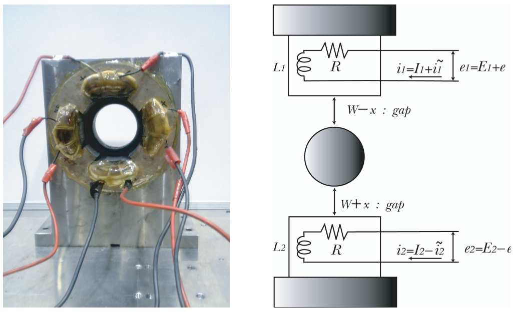
$$L_2 = \frac{Q}{X+W+x} + L_0, \tag{42}$$

where the parameters $Q = 6.34\times10^{-6}$[m·H], $X = 2.34\times10^{-4}$[m] and $L_0 = 8.55\times10^{-3}$[H] are identified by experiments. The total mass M is 10.34[kg]. The gravity acceleration g is 9.8[m/s^2]. The model of AMB rotor system is now described as follows;

$$\ddot{x} = \frac{Q(I_1^2+I_2^2)}{M(X+W)^3}x + \frac{QI_1}{M(X+W)^2}\tilde{i}_1 + \frac{QI_2}{M(X+W)^2}\tilde{i}_2, \tag{43}$$

$$\dot{\tilde{i}}_1 = -\frac{R}{L_c}\tilde{i}_1 - \frac{QI_1}{(X+W)^2L_c}\dot{x} + \frac{1}{L_c}e, \tag{44}$$

$$\dot{\tilde{i}}_2 = -\frac{R}{L_c}\tilde{i}_2 - \frac{QI_2}{(X+W)^2L_c}\dot{x} + \frac{1}{L_c}e, \tag{45}$$



(a) (b)

Fig. 10. Active magnetic bearing

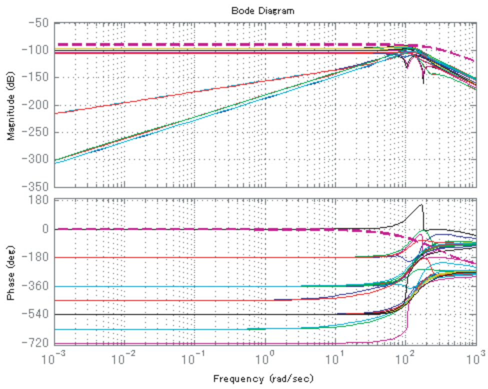


Fig. 11. Frequency response of weighting function $W_2(s)$

$$L_c = \frac{Q}{X + W} + L_0. \tag{46}$$

Considering ± 3[%] error for the parameters R, W, W, X and L_0, we can estimate the perturbation of the plant as shown in Fig. 11. Then, $W_2(s)$ depicted by dotted line in Fig. 11 is determined to cover all the perturbation as

$$W_2 = \frac{900}{s^3 + 900s^2 + 2.7 \times 10^5 s + 2.7 \times 10^7}. \tag{47}$$

On the sensitivity function, we consider the following two specifications;

$$W_{11} = \frac{0.7s + 0.005}{s}, \quad W_{12} = \frac{0.7s + 0.25}{s}. \tag{48}$$

Notice that W_{12} is tighter than W_{11}. We set the sampling time 0.002[s], the number of collected data 5000 and the candidate controllers set as follows;

$$K_P = \{2000 + 2000i \mid i = 0, \cdots, 9\}, \tag{49}$$
$$K_I = \{2000 + 2000i \mid i = 0, \cdots, 9\}, \tag{50}$$
$$K_D = \{5 + 5i \mid i = 0, \cdots, 9\}. \tag{51}$$

The number of initial controllers is 1000. Carrying out the proposed design procedure, we designed 4 controllers as summarized in Table. II, where $(1.6e4, 8e3, 45)$ indicates $(K_P, K_I, K_D) = (16000, 8000, 45)$. In the case of W_{12}, i.e. tighter specification case, we can confirm the numbers of falsified controllers increase for each initial contollers cases. All resultant controllers can stabilize the rotor system and achieve the highest operation speed. These facts indicate that the proposed method is effective for AMB controller design. The rotate operation by one of the resultant controller $(1e4, 1.8e4, 20)$ is shown in Fig. 12.

TABLE II

UNFALSIFIED DESIGN PROCEDURE RESULTS

Initial controller	Specification	Iter.	Falsified#	Final controller
$(1.6e4, 8e3, 45)$	W_{11}, W_2	12	65	$(1.4e4, 2e3, 15)$
$(6e3, 8e3, 10)$	W_{11}, W_2	11	72	$(1e4, 2e3, 15)$
$(1.6e4, 8e3, 45)$	W_{12}, W_2	15	179	$(1e4, 1.4e4, 15)$
$(6e3, 8e3, 10)$	W_{12}, W_2	16	132	$(1e4, 1.8e4, 20)$

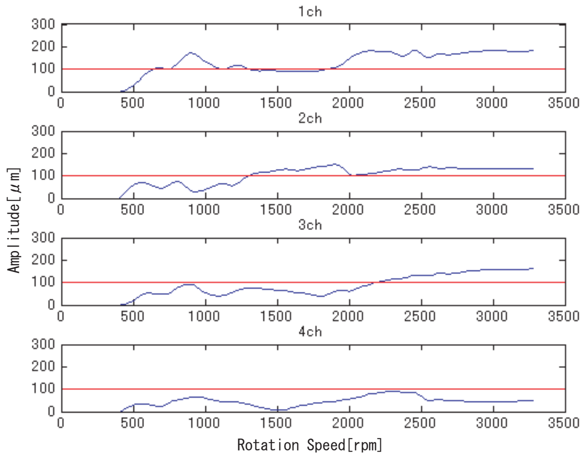


Fig. 12. Rotate operation of AMB rotor

IV. CONCLUSION

In this paper, we proposed a data-based PID controller design method using unfalsified control technique with Support Vector Machine (SVM) and gap metric. SVM and gap metric were utilized for adjusting the weighting function of extended L_2 gain criterion. The effectiveness of the proposed method was confirmed by experiments on a magnetic levitation system and an active magnetic bearing system.

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