A Note on ISO AMB Stability Margin

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***Abstract*— Stability is a primary robustness requirement. The stability margin is therefore a measure of the amount of system deviation from some nominal, stable configuration that can be tolerated before the system becomes unstable. The notion of 'system deviation' can imply many forms and mechanisms: engineers must, perforce, focus on those most likely to afflict a given system in order to obtain a meaningful assessment of relative stability.**

This note discusses the stability margins of rotor-AMB systems. Particular attention is focused on the output sensitivity, whose peak amplitude is a common measure of sensitivity and is adopted by the new ISO standards for AMB systems. One objective is to illustrate that many common sources of instability in rotordynamic systems may be completely missed by this measure and, further, that controller design targeting reduction in output sensitivity may, in fact, lead to very poor robustness to these mechanisms. Several examples are developed to show that, while limiting the sensitivity peak is a necessary condition for rotor-AMB system stability, it may not be sufficient to ensure commercially viable robustness to common destabilizing mechanisms such as aerodynamic cross-coupling or hysteretic damping in the rotor.

The primary observation is that, if the AMB is the destabilizing mechanism, then the output sensitivity will reveal it, but if the destabilizing mechanism is endemic to the rotor and the relevant feedback is not collocated with the AMB connectivity, then the output sensitivity may not assess its effect.

***Index Terms*— ISO standard, AMB, stability margin, robustness.**

I. Introduction

Arguably, stability is the most important requirement for a large class of rotor-AMB systems such as turbomachines. While other performance requirements such as the unbalance response may also be important, magnetic bearings feature large clearance and effective open loop control techniques to cope with them. Stability is a primary robustness requirement. Therefore, the evaluation of stability margin is an essential element for the application of magnetic bearings.

In 1996, Osami Matsushita organized an ISO committee to establish acceptance and operating standards for commercial rotating machines with AMBs [1]. This proposed ISO standard aims to improve the commercial prospect for the technology by facilitating customer specification. One aspect of the standard is assessment of the AMB system stability margin using the peak sensitivity function. For instance, the peak sensitivity function should be limited below 3.0 or 9.5 dB for newly commissioned machines. This stability margin using sensitivity peak has many advantages. First, the stability margin assessment is based on direct experimental data. The measured sensitivity function does provide realistic stability information of the AMB system under the measurement conditions. Second, the proposed sensitivity function measurement guarantees the classical gain margin and phase margin even for MIMO systems.

In spite of these advantages of using the directly measured sensitivity function as a AMB system stability margin, the application of the ISO standard to evaluate the stability margin against destablizing mechanisms has significant limitations. This note is intended to clarify the stability margin concepts in rotor-AMB systems, and investigate the potential limitations and consequences of using the ISO stability margin standard. One goal of this study is to avoid overconfidence in this measure which might lead to abandonment of the methodology altogether. Instead, we hope to show directions in which the standard might be extended in the future as familiarity with the tools and underlying concepts becomes broader.

II. AMB System Stability Margin

The stability margin, by definition, is a measure of the amount of system deviation from some nominal, stable configuration that can be tolerated before the system begins to go unstable. Classical stability margins such as gain and phase margins specify a stability safety factor describing tolerable SISO feedback loop variation. The gain margin is the smallest real gain variation that makes the system unstable while the phase margin is the smallest amount of phase variation that produces unstability. The peak sensitivity function measures the smallest complex gain variation leading to instability, thereby covering both gain and phase margins simultaneously.

A. Destablizing Mechanisms

To obtain a truly robust system, the system designer must focus on those mechanisms most likely to produce deviation in system dynamic behavior. The destabilizing mechanisms in rotor-AMB systems in general may include: (1) cross coupled stiffness; (2) AMB gain and phase; (3) parameter and dynamic uncertainties in the system (4) other destablizing sources such as the transient or steady state perturbation and disturbance as in impact or blade loss cases. Note that, for other destabilizing sources such as the disturbance by impact, blade loss, the nonlinear domain of attraction is a good stability margin criterion. In this note, we only focus on the discussion of linear system stability margin.

For traditional rotating machinery supported on mechanical bearings, generation of the tangential force by the cross coupled stiffness is often the major cause of lateral instability. The cross coupled stiffness represents a large class of destabilizing mechanisms including the cross coupling generated by seals, hydrodynamic bearings, turbine and pump impellers as well as the rotor internal damping. These mechanisms produce instabililty by coupling drive torque into lateral motion: they pump energy from spin into whirl. A stability margin corresponding to the cross coupled stiffness threshold level is often adopted in turbomachinery applications. The API 617 standard specifies an analysis to include the dominant destabilizing cross coupled stiffness.

Without the external cross coupled stiffness, the rotor system with fluid film bearings or ball bearings is passive. In contrast, magnetic bearing systems are not passive, and instability occurs in different patterns. Apart from the destabilizing mechanisms in the form of cross coupled stiffness, the AMB itself can act as a destabilizing source. Consequently, the stability of AMB system is not only related to the cross coupled stiffness but also depends on other parameters in the plant, operating conditions and bearing stiffness. Obviously, the API standard which targets mechanical bearings cannot adequately address the instability problems in AMB machines.

B. Sensitivity Function

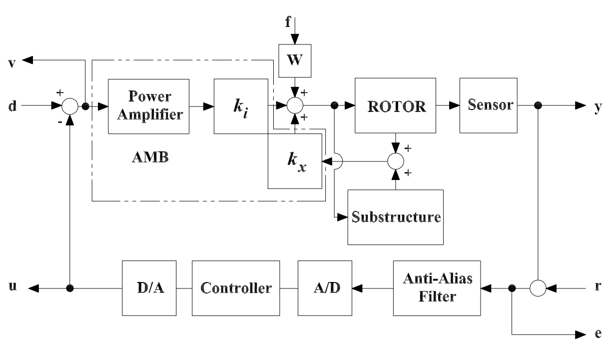

Fig. 1. AMB system block diagram.

The output sensitivity function and complementary sensitivity functions used in the ISO standard are defined as the transfer function matrix from r to e and r and y as shown in Fig. 1. Similarly, the input sensitivity and complementary sensitivity are transfer function matrix from d to v and d to u respectively.

$$\begin{aligned} S_o(s) &\triangleq (I + G(s)C(s))^{-1}, \qquad (1)\\ T_o(s) &\triangleq G(s)C(s)(I + G(s)C(s))^{-1}, \\ S_i(s) &\triangleq (I + C(s)G(s))^{-1}, \\ T_i(s) &\triangleq C(s)G(s)(I + C(s)G(s))^{-1}, \end{aligned}$$

where G represents the open loop plant transfer function and C is the controller transfer function. For multivariable systems the output sensitivity is, in general, not equal to the input sensitivity.

For the SISO case, either sensitivity function (they are equal in the SISO case) represents the inverse of the distance from $G(j\omega)C(j\omega)$ to the critical point of $(-1, 0)$. Therefore, the minimal distance which is the peak sensitivity function can be applied as a stability margin measure. In fact, the classical gain margin GM and the phase margin PM in each feedback loop of multivariable systems are bounded by the $\mathcal{H}_\infty$ norm of the sensitivity function.

For MIMO systems in general the $\mathcal{H}_\infty$ norm of the weighted output sensitivity function ensures the robustness to the inverse output multiplicative uncertainty, i.e., $(I + W_2\Delta W_1)^{-1}G$, where $W_1, W_2 \in \mathcal{RH}_\infty$ and $\Delta \in \mathcal{RH}_\infty$ with $\| \Delta \|_\infty < 1$. Similarly, the robustness to the inverse input multiplicative uncertainty $G(I + W_2\Delta W_1)^{-1}$ is guaranteed by the corresponding input sensitivity function S_i. Since the difference between the maximum singular values of S and T is at most one, the output sensitivity function also ensures the output multiplicative uncertainty $(I + W_2\Delta W_1)G$ but not the input uncertainty.

C. Multivariable Nature

Although under certain circumstances the cross talk between the bearings can be eliminated or reduced by the center of mass (*tilt–and–translate*) or other coordinate transformation, rotor-AMB systems in general are inherently multivariable. Obvious coupling mechanisms leading to this multivariable nature include modal coupling between the bearings, plant cross coupled stiffness and gyroscopic effects, and the effect of foundation dynamics. But more importantly, the structure of plant uncertainies may require a MIMO representation and add system level cross coupling beyond what is mechanically obvious. Even an apparently SISO system like the thrust axis in an AMB system becomes MIMO when structured internal uncertainties are modeled.

When the cross coupling cannot be neglected, using the output sensitivity peak as a stability margin for multivariable system can be a problem. Unlike SISO systems where uncertainties are completely specified by the loop gain or equivalently the sensitivity function bound, some uncertainties in MIMO systems cannot be characterized by the loop gain without introducing substantial conservatism. In addition, uncertainty occurs in the input may not be revealed by the output sensitivity function. Consequently, the sensitivity function is only a necessary condition for stability but not sufficient to guarantee the stability margin to specific uncertainties in general rotor-AMB systems.

D. MIMO Stability Margin μ

For the multivariable systems, it is well known that the classical loop-at-a-time analysis[1] may not be sufficient to

[1] *Loop-at-a-time* means that the sensitivity of the closed loop system to individual uncertainties is examined one at a time, holding all other uncertainties to zero. The measure adopted in the ISO standard: maximum diagonal element of the MIMO sensitivity function is a loop-at-a-time measure.

characterize the stability under the simultaneous perturbation within different loops [10]. The structured singular value μ is the stability margin for multivariable systems.

E. Uncertainty Assumption

Uncertainty may arise from a variety of sources. The most obvious is model error, but perhaps more important are actual plant perturbations due to the changes in operating condition, parameter varying dynamics, process dynamics, aging, etc. Accordingly, the stability margin should be assessed relative to deviations which may not be captured in testing under specific operating conditions.

III. Potential Problems and Limitations

If a system has a very high sensitivity function peak, then it is certain to have robustness problems and is likely to become unstable when operating conditions or equipment condition changes slightly. However, a system with relatively low sensitivity function peaks may also have serious robustness problems and this is the central issue we explore below.

This in general may occur in rotor-AMB systems including but not limited to the following cases.

- Plant uncertainty mechanisms couple to the nominal plant at different locations than AMB input and output locations. An example is a seal. Here, the uncertainty mechanism is deviation of the seal cross–coupled stiffness from nominal value and this mechanism is coupled to the rotor at a point other than where the AMB acts or senses.
- While the sensitivity function is measured at the output, the uncertainty occurs at the bearing location.
- There are parametric uncertainties, such as elastic modulus.

In these cases, the sensitivity function peak is not a reliable stability margin measurement. In other words, even with a low peak sensitivity function, the stability margin to the uncertainty above can be very small. This is illustrated through the following cases.

A. Cross Coupled Stiffness

Rotor lateral instability due to cross–coupled stiffness is one of the most prevalent problems in industry, and instability problems are extensively investigated in traditional rotordynamic applications [5]. However, the instability pattern for AMB supported rotors can be different.

Instability Patterns: To illustrate the cross coupled stiffness destabilizing mechanism, consider a modal second order system with cross coupled stiffness:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{q}_x \\ \ddot{q}_y \end{bmatrix} + \begin{bmatrix} c_x & g \\ -g & c_y \end{bmatrix} \begin{bmatrix} \dot{q}_x \\ \dot{q}_y \end{bmatrix} + \begin{bmatrix} k_x & -\kappa \\ \kappa & k_y \end{bmatrix} \begin{bmatrix} q_x \\ q_y \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}, \tag{2}$$

where κ represents the modal equivalent magnitude of the cross coupled stiffness, and m,c,k and g are the modal mass, damping, stiffness and gyroscopic terms respectively.

Without loss of generality we can assume that $m = 1$. Also, we assume that the gyroscopic effects are weak and can be neglected, and c and k are positive. Ruth-Hurwitz stability criterion yields the threshold value of κ.

$$\kappa^2 \leq \frac{\left[(c_x k_y + c_y k_x)(c_x + c_y) + (k_x - k_y)^2\right] c_x c_y}{(c_x + c_y)^2}. \tag{3}$$

The result reveals a well known fact that support stiffness orthotropy ($k_x \neq k_y$) can increase the stability threshold for cross coupled stiffness [4].

Equation (3) is nonlinear in the stiffness and damping parameters but can be simplified by assuming symmetry: $k_x = k_y = k$, $c_x = c_y = c$. This yields the modal threshold value of $\kappa = c\sqrt{k} = 2\zeta\omega_n^2$, where $\omega_n = \sqrt{k/m}$, and $\zeta = c\sqrt{m/4k}$. Thus, the stability margin to κ is linearly related to the damping ratio and square of the natural frequencies.

Since the modal damping ratio ζ is closely related to the threshold cross coupled stiffness, it may be used as an approximate measure of the corresponding stability margin in some cases. For instance, the API 617 standard specifies a log decrement greater than 0.1 as the final stability acceptance criteria [2].

To better understand the stability margin problem, we first explore instability patterns with the cross coupled stiffness. For rotors supported on fluid film bearings or rolling element bearings with squeeze-film dampers, the instability pattern is relatively straightforward. With the exception of tilting pad bearings, the stiffness and damping of mechanical bearings are usually adequately modeled as frequency independent for the first few modes below a certain frequency. Assume that the modal damping mainly comes from the bearings, and assume that the modal equivalent cross coupled stiffness is roughly the same for certain low frequency modes and high frequency modes. It is clear that the lowest mode is almost always destabilized first. In addition, the cross coupled stiffness is generally related to the speed, and most stability problems are subsynchronous in nature.

In contrast, the AMB parameters especially the damping can be different for each mode. Consequently, the instability patterns are much more complex. The instability can occur in different modes either subsynchronously or supersynchronously. The relatively low stiffness of AMB systems means that the first two modes in AMB machines are almost always rigid body cylindrical and conical motions. In addition, the rotor rigid body modes are typically well separated from the bending modes in frequency. As a result, the rigid body modes benefit from the low support stiffness and high effective damping while the high frequency bending modes are provided only limited damping by the AMB. The active damping on bending modes is restricted in part by the power bandwidth (slew rate limits.) It is further constrained by component dynamics as well as the control strategy.

The result is that the bending modes may tolerate less κ than do the rigid modes, and thus can be destabilzed first. Which mode is the limiting mode (first destabilized by κ), affects the implication of using the ISO sensitivity peak as a stability margin to cross coupled stiffness.

Rigid Body Mode: We first consider that a rigid body mode is destabilized by the cross coupled stiffness. To simplify the analysis, we assume the rotor is rigid. The two rigid body cylindrical and conical modes can be decoupled by the center of mass transformation. The resulting open loop transfer function $G(s) = K_i/(ms^2 - K_x)$, where K_i and K_x are the actuator gain and open loop stiffness of AMBs. Since most AMB controllers in the rigid bode frequency range are essentially PD controllers, we adopt an ideal PD controller, i.e., $C(s) = k_d s + k_p$. Note that in practice ideal lead compensator cannot be implemented, and PD controllers are also equipped with roll off filters. There are always phase lags introduced by the filters, amplifiers, sensors and time delays. Next we show that even with the ideal PD controller, the stability margin defined by the sensitivity peak and the stability margin to the cross coupled stiffness is not equivalent.

With the ideal PD controller, the closed-loop system is also a second order system $ms^2 + K_i k_d s + K_i k_p - K_x = 0$. The damping and natural frequency can be defined as

$$\omega_n = \sqrt{\frac{K_i k_p - K_x}{m}}, \quad \zeta = \frac{K_i k_d}{2\sqrt{(K_i k_p - K_x)m}}. \tag{4}$$

To simplify the notation, replace $K_i k_p$ with K_P and $K_i k_d$ with K_D. The resulting output sensitivity function is

$$S(s) = \frac{ms^2 - K_x}{ms^2 + K_D s + K_P - K_x}. \tag{5}$$

For the underdamped case, $\zeta \leq 1 \Leftrightarrow K_D^2 \leq 4(K_P - K_x)$, the peak sensitivity of the function M_s is

$$\begin{cases} \frac{2}{K_D}\sqrt{\frac{m^2 K_P^2 - mK_x K_D^2}{4m(K_P - K_x) - K_D^2}} & \text{if} \quad \frac{2mK_P^2 - 2mK_x K_P - K_x K_D^2}{2mK_P - K_D^2} > 0 \\ \max\left\{\frac{K_x}{K_P - K_x}, \quad 1\right\} & \text{Otherwise, } \omega = 0 \text{ or } \omega = \infty \end{cases} \tag{6}$$

The corresponding threshold value of cross coupled stiffness is $\kappa = K_D\sqrt{(K_P - K_x)/m}$. Comparing the peak sensitivity M_s and the threshold κ, notice that a small sensitivity function peak does not necessarily translate to large threshold value of cross coupled stiffness. The reason is that κ depends on both the stiffness $K_P - K_x$ and damping K_D while the sensitivity peak M_s is mainly determined by the effective damping. For instance, the minimal peak sensitivity is 1. This can be achieved by selecting $K_D = 2mK_P$. In other words, to reach the minimal sensitivity peak, it is not necessary to use high stiffness. However, in order to tolerate a large κ, it *is* necessary to use high stiffness.

Rotor Bending Mode: If the flexible modes are destabilized first, whether the sensitivity peak represents the stability margin to the cross coupled stiffness depends on the flexible mode stabilization strategies.

Unlike the unstable rigid body modes on AMB support which must be stabilized by active stiffness and damping, the flexible mode stabilization strategies can be different. Although flexible modes are often lightly damped, some small structural damping guarantees that these modes are stable without controller intervention. With a stable mode, either phase stabilization or gain stabilization can be applied. To phase stabilize a mode, phase lead is applied at the mode crossover region by phase shifting or phase bump. This results in active damping being applied to the mode. Alternatively, the gain stabilizing applies notch filters or roll off filters to make the open loop gain $L = GK$ small, and thus guarantees stability regardless of the phase variation. However, the gain stabilization approach typically does not result in active damping of the mode.

Both gain and phase stabilization can result in low peak sensitivity, but the implication to the stability margin to the cross coupled stiffness is completely different. With phase stabilization adding active damping into the modes, robustness to cross coupled stiffness can be enhanced. In contrast, gain stabilization provides almost no damping improvement and therefore less stability margin to cross coupled stiffness. If the controller gain at the mode is zero, or if the mode is uncontrollable or unobservable, then the sensitivity function is 1.0 near this mode and the mode will never be destabilized by the AMB. However, the amount of cross coupled stiffness the mode can cope with depends on the structural damping, the modal frequency, and the mode shape at the cross coupled stiffness location. Therefore, the sensitivity function in this case yields no direct information about the stability margin to the cross coupled stiffness.

A Case Study: This is illustrated by the test rig control design and testing [6]. An μ controller was designed assuming no cross coupled stiffness. The measured diagonal sensitivity function peaks were all below 2.5. Fig. 2 compares the model prediction and the output sensitivity function measurement in terms of singular values. According to

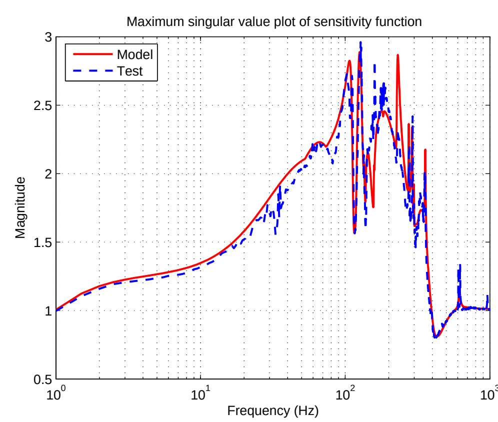

Fig. 2. Maximum singular value plot of S with μ controller.

the ISO stability margin criterion, this machine can be classified in Zone A/B for unrestricted long term operation.

However, the closed loop system is very sensitive to cross–coupled stiffness. A small cross coupled stiffness introduced at the middle disk and the closed-loop system became unstable. The instability was associated with the first free–free bending mode. This is mainly because the rigid body modes were well damped while the damping at the rotor bending mode was quite small. The first rigid bode mode was at 27 Hz with a damping ratio of 0.8. According to (3), the resulting cross coupled stiffness threshold value is 46,000. In contrast, the rotor bending mode occurs at 153 Hz with a structural damping of 1.7%. If there is no external active damping added, a cross coupled stiffness exceeding 31,390 destabilizes the rotor bending mode. Clearly, the required destablizing cross coupled stiffness to the rotor bending mode is lower than that of the rigid rotor mode. Interestingly, depending on the location of the cross coupled stiffness, other modes such as the substructure modes can be destabilized first. The sensitivity function provides little information on how much cross coupled stiffness each mode can cope with or which mode can be destablized first.

B. *Modal Frequency Variation*

Flexible rotors and substructures exhibit multiple lightly damped modes and multiple gain crossover frequencies. Thus, robustness to modal parameter variation at the gain crossover frequencies is an important requirement. This is not only because modeling and testing errors are inevitable, but also because the rotor or substructure modal parameters can vary with the operating conditions. Change in modal frequencies can arise for many reasons including (1) the gyroscopic effect splits the modes; (2) shrink fits on the rotor vary with speed and temperature; (3) substructure modes are sensitive to connecting part contact stiffness. Consequently, stability robustness to modal parameter uncertainty must be evaluated. Compared to modal damping, modal frequency uncertainty is more critical to the closed-loop stability [7]. Unfortunately, the sensitivity function is not a good measure of the stability margin to the modal frequency variation. The problem is that modal frequency

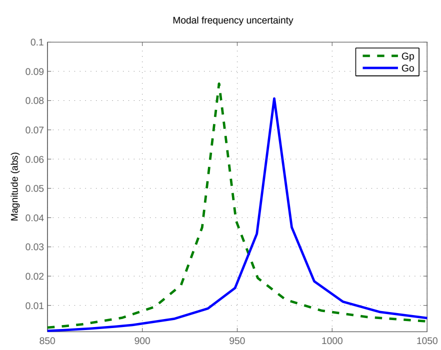

Fig. 3. Modal frequency perturbation.

uncertainty is not readily covered by multiplicative uncertainty which is directly related to the sensitivity function. Intuitively, for a lightly damped mode, a slight shifting in modal frequency can cause large multiplicative errors in the gain. Consider the test rig rotor for example: a 3% modal frequency perturbation can result in 800% uncertainty in terms of the multiplicative uncertainty model as shown in Fig. 3.

As a result, the design based on nominal modal frequency can have a low sensitivity peak but be very sensitive to perturbation of the modal frequency. This is illustrated by an test rig instability incidence. Several $\mathcal{H}_\infty$ and μ controllers were implemented and tested a few years ago with acceptable levels of sensitivity peak specified in ISO standard. After a few years of operation, a μ-controller became unstable during testing. This controller was de-

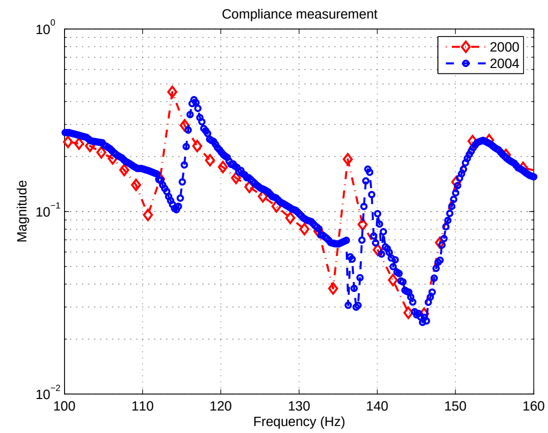

Fig. 4. Compliance measurement comparison.

signed based on 2% modal frequency uncertainty for the substructure mode. A careful examination of substructure compliance measurements taken in 2000 and again in 2004 revealed substantial shifting of the substructure modes. Fig. 4 shows that modes at 113 Hz and 134 Hz shift about 2%. This mode variation was in part due to reassembly of the rig.

Figure 5 shows the sensitivity function of the closed loop system computed using the original substructure model (red curve) and with the re–identified model (blue curve). Although the shift in modal frequency is small, the change in peak sensitivity is dramatic (the modified model is not stable). However, both the sensitivity prediction and measurement with the original structure modes do not reveal this vulnerability. Experience suggests that, for systems with flexible rotors and substructures, the controller should tolerate as much as $\pm5\%$ modal uncertainty for modes within the bandwidth of the controller. Unfortunately, this margin is not guaranteed by the sensitivity measurement specified in the ISO standard.

C. *Gyroscopic Coupling*

Gyrosocpic effects couple the rotor motion in two radial directions and make the system multivariable in nature. The gyroscopic moment itself is conservative and is not a destabilizing factor. However, the gyroscopic system presents large condition numbers. Certain control solutions which produce decent sensitivity functions can be dubious.

To illustrate this problem, a decoupled rigid rotor model

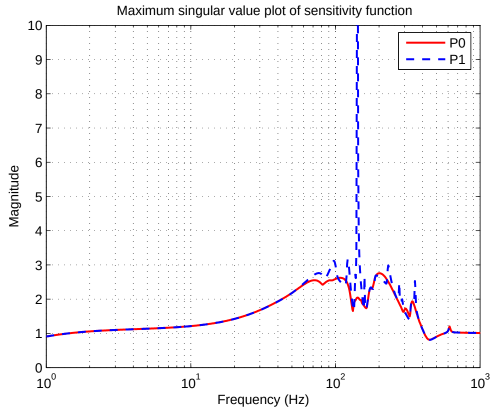

Fig. 5. Sensitivity function with original and modified structure model.

is examined.

$$\begin{bmatrix} I_t & 0 \\ 0 & I_t \end{bmatrix}\begin{bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \end{bmatrix} + \Omega \begin{bmatrix} 0 & I_p \\ -I_p & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \end{bmatrix} + \begin{bmatrix} -k_\theta & 0 \\ 0 & -k_\theta \end{bmatrix}\begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}. \quad (7)$$

The condition number is the ratio of largest to smallest singular value of the 2×2 system, i.e.,

$$\gamma(G(j\omega)) = \frac{\omega^2 + \Omega I_\gamma \omega + \bar{k}_\theta}{\omega^2 - \Omega I_\gamma \omega + \bar{k}_\theta}. \quad (8)$$

The condition number approaches infinity at the pole frequency.

$$\omega_{1,2} = \frac{\Omega I_\gamma \mp \sqrt{(\Omega I_\gamma)^2 - 4\bar{k}_\theta}}{2}. \quad (9)$$

Inverse-based control design such as internal mode control (IMC) can be a very useful approach to many control problems. Zames [9] showed that approximate invertibility of the plant is a necessary and sufficient condition for achieving good sensitivity reduction. An $\mathcal{H}_\infty$ optimization with the objective to minimize the sensitivity function results in inverse-based control. With inverse-based controller $C = l(s)G^{-1}(s)$, it is clear that closed-loop system yields good nominal sensitivity.

However, when an inverse-based controller is used, the resulting system is very sensitive to input uncertainty when the plant has a large condition number [8]. Since gyroscopic systems can have very high condition numbers, they can have very high input sensitivity over a range of operating speeds. This problem would not be revealed by an output sensitivity measurement - even if taken in the problematic speed range.

It is interesting to observe that an inverse-based controller generates cross coupled damping. Physically, cross coupled damping force is perpendicular to the whirling orbit. Thus no effective modal damping or energy dissipation is introduced through cross coupled damping feedback. Cross coupled damping has been applied to gyroscopic systems [3] and will likely yield low output sensitivity, but this approach can be very sensitive to input uncertainty. This further illustrates that output sensitivity alone does not necessarily reveal strong potential robustness problems.

IV. Conclusions

For rotating machinery applications such as turbomachinery, instability remains one of the most prevalent problems due to the presence of various uncertain destabilizing mechanisms in rotors and in the process dynamics. Use of magnetic bearings in these machines may further exacerbate the problem. This is not only because the magnetic bearings are open loop unstable, but also because uncertainties are introduced which makes the system multivariable.

Due to the multivariable nature, the sensitivity function peak specified in the current ISO draft standard may not be a reliable measure of the stability margin to the destablizing mechanisms other than the magnetic bearing itself. In particular, we point out that a good sensitivity function does not guarantee robustness to cross coupled stiffness or modal frequency uncertainty. With strong gyroscopic coupling, certain controller structures such as the inverse-based controller are fragile to the skewed uncertainty. For multivariable systems the structured singular value μ provides a reliable stability margin.

Finally, we emphasize that we are not suggesting that ISO sensitivity measurement should not be considered as a stability margin evaluation criteria for AMB systems due to the aforementioned shortcomings. On the contrary, the sensitivity function is a good screening tool and should always be applied first for the evaluation of the stability margin against the magnetic bearing loop gain variations. However, we are also noting that, as with any application standard, one should be aware of the limit of the standard and should not attempt to extrapolate the information beyond those assumptions.

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