## Investigations of an AMBs – Flexible Rotor for Pass through Two Bending Mode Frequencies

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### **1** Introduction

In the project of the 10MW high temperature gascooling reactor in Tsinghua university, the helium turbine rotor and the motor rotor in the reactor were designed to be supported by AMBs in order to meet the requirements of the clean helium environment of high temperature, contactless and no lubrication, whose working speed is above the first two flexible critical speeds. The rotor is 3.5 meters long and 540 kg weight. To reach the work speed, it is necessary to cross 2 critical speeds. According to the demarcation of the rotor dynamics characteristic and its rotate speed, an AMB-flexible rotor experiment named AMB-P is built up as a model of the helium turbine rotor supported by AMBs, and some research about how the flexible rotor passes through the flexible critical speed smoothly supported by AMBs, is carried out.

In 1984, Doctor Larsonneur started to research on flexible rotor suspended by AMBs[1], designed a kind of controller of SPOC-D. The rotor can operate at supercritical speeds. Prof. Schweitzer give a conclusion in [2] about how to distinguish high speed and low speed rotor which supported by AMBs, describe the different between the designing of machinery and control system for rigid rotor and flexible rotor. A method of analyzing the observability and controllability of the rotor modes is also put forward. Conrad Gähler design a controller based on phase shaping and analyzing of flexible rotor dynamics, make the rotor operate supercritical speed [3]. Florian Lösch gives a arithmetic for automatic controller, which is a kind of robust controller on u synthesize[5]. coefficient building based on identification of AMBs-flexible rotor.

Therefore, under the control of AMBs, it can make the flexible rotor of the thin-long type to acquire the electromagnetism damping, compensating and reducing the vibration, passing through smoothly the bending critical speeds. But there are still many works to be carried on. To accumulate the experimental data, an experiment device is designed and establishment for pass through two bending critical speeds, according to the helium gas turbine characteristics. In order to avoid the risk and difficulty, a small and similar system need to be researched firstly.

### 2 Design of Experiment Setup 2.1 Specification for Design

According to turbine rotor characteristics, and the corresponding electromagnetism force and the control bandwidth effect, some specification have been put forward from the optimal of both the rotor dynamics and the machine structure similarity as follows,

First bending mode frequency is 300Hz, second bending mode frequency is 700Hz, the rotor length is nearly 500 ~ 600mm, and its operate speed is 800 Hz.



Fig.1 A photo of the rotor

#### 2.2 Configuration of the Experiment Setup

According to the rotor machinery requirements, the radial, axial AMBs and auxiliary bearings are designed. And a high speed drive motor is been chooses to form a long electro spindle. The total configuration is shown at Fig.2.



Fig.2 Configuration of the setup

# **3** Observability and Controllability of the Rotor and Choice of Stiffness

## **3.1** Observability and Controllability with the Rotor Mode

The result of mode shape corresponding to the rotor structure is shown at Fig.2, this result come from the optimality analysis between mode shape and observability and controllability of the rotor, by adjusting the position of the three disks and the AMBs with the sensors. of the stiffness of AMBs had it special for the flexible rotor. Too low stiffness can not meet the requirement of the machine, at the other hand too high may cause unstable or uncontrollable. The Fig.5 describes the relationship between mode parameter and the stiffness. According to the stiffness magnitude, the diagram can be divided into three sections, which shown at the Fig5 with mark 1,2,3. At first section, the rotor nearly comes to be a free-free rotor. At third section, the rotor is supported by hinged bearing, and the nodes are very near to the supporter. The second section is a kind of transition area, the bearing stiffness approach to the



Fig.3 Configuration of the rotor with its mode shape

The Gram matrix of observability and controllability of the rotor can be obtained from the state equation (5) of the flexible rotor-AMBs, they are:

$$\boldsymbol{W}_{c} = \int_{0}^{\infty} e^{-At} \boldsymbol{B} \boldsymbol{B}^{T} e^{-A^{T} t} dt \quad ,$$
$$\boldsymbol{W}_{o} = \int_{0}^{\infty} e^{A^{T} t} \boldsymbol{C}^{T} \boldsymbol{C} e^{At} dt \qquad (1)$$

Fig. 4 show the observability and controllability of the rotor modes, it shown that the first two rigid modes are good for observability and controllability, and the two flexible modes are little, but are enough for the requirement of super the two bending mode critical speeds.



Fig.4 Observability and Controllability of the rotor modes

3.2 Selection of the Stiffness of AMBs

Usually, the requirement to the stiffness should come from the machine application. But the selection rigidity of the rotor, and mode parameters change violence when the stiffness has small change. So these curves describe the nature of critical speeds and node position change with the stiffness, the former concern with the design of controller, and the latter concern with the performance of controllability.



Fig.5 Diagram of mode frequencies with stiffness

Fig. 6 and Fig.7 show the two "rigid" mode shapes of the rotor corresponding to fore kinds of bearing stiffness.



Fig.6 the first mode shapes with different stiffness



Fig.7 The second mode shapes with different stiffness

The figurer show that, when the stiffness is high, the rigid mode shape becomes not rigidity, that means that at this kind of stiffness, the node move to the sensor and the bearing, may cause poor observability and controllability of the rotor modes.

Based on the analyses above, the stiffness is selected form  $1 \times 10^5$  to  $6.5 \times 10^5$  N/m.

### 4 Control System Design 4.1 Modelling and Reduced-order Model

Fig.8 describe the force diagram of the rotor, when ignoring the coupling of magnetic circuit and sensor position,

direction. The mass matrix, stiffness matrix and gyroscopic effect matrix are described as follow,

$$\begin{bmatrix} \boldsymbol{M} \end{bmatrix}_{(4N\times4N)} = \begin{bmatrix} [\boldsymbol{M}_{x}] & \boldsymbol{0} \\ \boldsymbol{0} & [\boldsymbol{M}_{y}] \end{bmatrix}$$
$$\begin{bmatrix} \boldsymbol{K} \end{bmatrix}_{(4N\times4N)} = \begin{bmatrix} [\boldsymbol{K}_{x}] & \boldsymbol{0} \\ \boldsymbol{0} & [\boldsymbol{K}_{y}] \end{bmatrix}$$
$$\begin{bmatrix} \boldsymbol{G} \end{bmatrix}_{(4N\times4N)} = \begin{bmatrix} \boldsymbol{0} & [\boldsymbol{G}_{y}] \\ -[\boldsymbol{G}_{x}] & \boldsymbol{0} \end{bmatrix}$$
(4)

Supposing N=66, where N is the segment number of the rotor as mode analyzing, the dimension is 264 for the equation of (2). Fortunately, only the first two bending critical speeds need to pass by, it is enough to deal with the first rigid and the first bending modes. Therefore, the dimension can be reduced by mode truncation. And the equation (2) become,

$$[\boldsymbol{M}_{R}]Y_{R} + \Omega[\boldsymbol{G}_{R}]Y_{R} + [\boldsymbol{K}_{R}]Y_{R} = P_{R}(t)$$
  
$$y = \boldsymbol{C}\boldsymbol{\Phi}Y_{R}$$
(5)

Where  $\emptyset$  is the mode transfer matrix. [*M*<sub>R</sub>], [*G*<sub>R</sub>],



Fig.8 Force diagram of the rotor

The equation is,

$$[\boldsymbol{M}]\ddot{\boldsymbol{Y}} + \Omega[\boldsymbol{G}]\dot{\boldsymbol{Y}} + [\boldsymbol{K}]\boldsymbol{Y} = P(t)$$
(2)

Where , the  $\Omega$  is the rotate speed, the Y is nodal point of the rotor in Cartesian coordinate, and the P is the force at each nodal point. They are,

$$Y_{(4N\times1)} = \begin{bmatrix} x_{1,}\theta_{x1,}x_{2,}\theta_{x2,}...,x_{N,}\theta_{xN}, y_{1,}\theta_{y1,}y_{2,}\theta_{y2,}...,y_{N,}\theta_{yN} \end{bmatrix}^{T}$$

$$P_{(4N\times1)} = \begin{bmatrix} P_{x}, P_{y} \end{bmatrix}^{T}$$
(3)

Where x,  $\theta_x$  and y,  $\theta_y$  are the deflection and the slope at x, y direction,  $P_x$  and  $P_y$  are the force at x, y  $[K_R]$ , and  $P_R$  (t) and  $Y_R$  are separately the mass matrix, the gyroscopic matrix, the stiffness matrix, the force vector and the modal degrees of freedom.

For designing the controller and analyzing the system capability conveniently, the equation (4) can be changed to the state space form (5) while the input signal is the coil current.

$$\begin{cases} \dot{Z} = AZ + BU \\ y = CZ \end{cases}$$
(6)

with

$$Z = \begin{bmatrix} Y_R & \dot{Y}_R \end{bmatrix}^T \quad U = \begin{bmatrix} i_1 & i_2 & i_3 & i_4 \end{bmatrix}^T$$

The dimension of the system model is 20, which meets the system modal analysis demands and the controller design. Compared with the former highorder system model, the dimension of the model with the modal decomposition is greatly reduced and the dynamic performance is also guaranteed.

#### 4.2 System Dynamic Analysis

The system state space model is obtained with the stiffness of  $1.5 \times 10^5$ N/m and the damping ratio of 0.003. Therefore the static system Bode diagram is shown as the Fig.8.



Fig.8 System Bode diagram in static state

And the Fig. 9 is the system Bode diagram at the speed – 800Hz. Effected by the gyroscopic effect, the eigen frequencies will be bifurcated.



Fig.9 System Bode diagram with the speed of 800Hz

To suspend the static rotor stably and pass through the first two flexible critical speeds, the controller will be designed with strict demands.

#### **4.3 System Identification**



Fig. 10 The process of the system identification

To identify the system, a rigid controller and the appropriate excitation signal need to be designed. The flexible modes and gyroscopic effect are ignored in the rigid controller. The process of the system identification is shown in Fig. 10.

If the time delay of the A/D, D/A and the calculation and the dynamic effect of the power amplifier and the sensor are ignored in the system model, the measured and fitted open-loop Bode diagram of AMB-P is shown in Fig. 11.



Fig.11 system open-loop transfer function

## 4.4 Controller Design and Test at High Speed

The controller design of AMB- flexible rotor system is an iterative process with the rotor speedup. The whole process could be divided into three steps.

A rigid controller is designed to identify the system. This controller ignores the flexible modes and the gyroscopic effect of the rotor. To reduce the gain in high frequency, the low-pass filter is used in the differentiator. But vibrations in various frequency were detected while the rotor could be suspended stably in static state. Compared to the eigen frequency of the rotor, the vibrations are high frequency modal vibrations. The frequency and time domain of the vibrations of the X direction in the side A (AX) are shown in fig.12.



Fig.12 Time and frequency domain diagram at static state

To control the modal vibrations, especially the first three flexible modes, the phase compensation in series was used. To increasing the damping of the flexible modal vibrations and suspending the rotor stably, the pole-zero and center position of the compensator were adjusted. The rotor could exceed the two rigid critical speeds and the first flexible critical speed, which is 634Hz, with the modified controller. The orbits of the rotor and the spectrum of the AX are shown in Fig. 13.



Fig.13 Frequency and time domain signal at 634Hz

A vibration at 730Hz appears at speed 634Hz, and its amplitude increases step by step, so it's impossible to increase the rotor speed continually. The controller should be improved.

The gyroscopic effect of high speed rotor makes the eigen frequency bifurcate, as figure 14. Obviously, the mode-frequency of the rotor has frequency-changing characteristic, and the way to solve it is improve the adaptability of constant parameters. Design a robust controller which considers about the perturbation of these mode parameters.



Fig.14 Characteristic frequency bifurcate

Set the static stiffness equals to  $1.5 \times 105$  N/m, openloop cut-off frequency 150 Hz, damping ratio of flexible mode more than 0.2. Based on the H<sub>∞</sub> theory, choose the appropriate weight function  $W_1$ ,  $W_2$  and  $W_3$ , then the optimized H<sub>∞</sub> controller could be solved out. Its Bode diagram is Fig.15. In the figure, the dashed line is the Bode diagram of controller which input from side A and output from side A, the continuous line is the Bode diagram of controller which input from side A and output from side B.



Fig.15 weight factor diagram of amplitude frequency

With this controller, system can restrain the system mode vibration very well, and exceeds first two flexible critical speed of the rotor. Rotor can work stably between 0-800Hz.

Fig.16 shows the 2 ends of rotor axis center orbit and the frequency domain analysis of AX direction at speed-324Hz, which is the first flexible critical speed of the rotor.



Fig.16 Time and frequency domain diagram at first flexible critical speed

From the frequency domain analysis in Fig.16, the same frequency vibration of the rotor is the main ingredient, and the frequency-doubled vibration is secondary and very small. Rotor can run stably at this speed for a long time.

Increase the speed of the rotor by the motor till 800Hz. The time and frequency domain diagram at 800Hz is shown as Fig.17. In this process, the rotor exceeded the second flexible critical speed successfully.



Fig.17 Time and frequency domain diagram at 800Hz

### **5** Conclusion

Based on the appropriate structure design and rotor dynamics, AMB control can provide damping for the flexible modes of the rotor, and the rotor can pass through the first two flexible critical speed, and can stay at the critical speed stably.

However, the supporting stiffness of flexible rotor must be chosen appropriately. The supporting stiffness should be decided appropriately to meet the actual requirement in application.

The phase compensation for the flexible modes of the rotor maybe can't adapt the parameters perturbation caused by gyroscopic effect at high speed.  $H_{\infty}$  controller has been proved effectively.

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