Smooth Stabilizing Controllers for a 3-Pole Active Magnetic Bearing System

Shyh-Leh Chen

Department of Mechanical Engineering
National Chung-Cheng University
Chia-Yi 621, TAIWAN
imeslc@ccu.edu.tw

***Abstract* – In this study, a class of smooth controllers is proposed for the stabilization of a 3-pole active magnetic bearing (AMB) system. The 3-pole AMB system is strongly nonlinear and non-affine. Conventional controllers are nonsmooth and complicated, which is difficult for implementation and further study. The proposed smooth controllers are designed utilizing a linear transformation on the system inputs (coil currents). As a result, simple smooth controllers, such as quadratic state feedback controller, can be obtained. The domain of attraction of the origin under the smooth controller is estimated. The proposed smooth controller is verified through both numerical and experimental results.**

***Key Word* – Smooth control, 3-pole, active magnetic bearing, non-affine, quadratic state feedback.**

I. Introduction

Stabilizing controller design is an important task for an active magnetic bearing (AMB) system since it is inherently unstable. For the popular 8-pole AMB system, the dynamics can be well approximated by a linear model if large bias currents are taken [1]. As a result, many linear control methods can be employed to design a suitable stabilizing controller, e.g., PID [2], Q-parameterization theory [3], μ-synthesis [4], and H_∞ control [5]. However, large bias currents imply large power loss. To reduce the heat dissipation, zero or low bias currents are preferred. For such systems, the dynamics are strongly nonlinear. In particular, the system input currents appear in a quadratic way. Thus, controller design is a nontrivial task. In general, nonsmooth (or discontinuous) nonlinear controllers are required for stabilization [6-8].

Recently, the 3-pole AMB was proposed in view of the expensive cost and large energy loss of the conventional 8-pole system [9]. It has been shown that the 3-pole system can effectively reduce the overall cost and save much energy compared to the conventional 8-pole AMB. Similar to the 8-pole AMB system, however, the 3-pole AMB system also suffers difficulties from the controller design. The situation for the 3-pole AMB is even worse. The magnetic fluxes in the 3-pole AMB are strongly coupled. Hence, the 3-pole system is a nonaffine and strongly nonlinear system. Although this nonaffine nonlinear system has been shown to be feedback linearizable, the resulting stabilizing controllers are nonsmooth and complicated [10, 11].

Nonsmooth and complicated controller can cause problems in practical applications. It can induce high frequency disturbance and cause problems in implementation. Moreover, the closed loop system with such controller is complicated, which is extremely difficult for further study. For example, observer design for such a complicated system is almost impossible. The objective of this study is to propose a class of smooth controllers for the 3-pole AMB system to overcome the problems caused by the nonsmooth and complicated controllers. The smooth controllers are designed using the special structure of the 3-pole system. More specifically, there exists a linear transformation on the input currents that greatly simplifies the system model [9-10]. Smooth controllers for the new inputs can be easily constructed.

This paper is organized as follows. After the introduction, a non-dimensional model for the 3-pole AMB system is described in Section 2. The proposed smooth controllers are presented in Section 3. In Section 4, the domain of attraction of the origin under the smooth controller is discussed. The smooth controllers are verified through numerical simulations and experimental results in Section 5. Finally, conclusions are drawn in Section 6.

II. A Nondimensional Model

Fig.1 is the 3-pole AMB system considered in this study. It is a 2-DOF system with a disk-like rigid rotor. The axial motion is constrained with thrust bearings. The AMB is Y-shaped with differential windings on the upper two poles to yield the optimal design from the viewpoint of energy and cost [9]. It is assumed that the magnetic field is linear, flux leakage and fringing effects are negligible, and gravitational field g is in the negative y direction. The system dynamics can be obtained with these assumptions, as [10-11]

$$\dot{x} = \begin{bmatrix} x_2 \\ c_0\Phi_1\Phi_2 \\ x_4 \\ \frac{c_0}{2}[\Phi_2^2 - \Phi_1^2] - g \end{bmatrix} \tag{1}$$

where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [x_r \quad \dot{x}_r \quad y_r \quad \dot{y}_r]^T$ is the state vector containing the rotor displacements x_r, y_r and their velocities. The coefficient c_0 is defined by

$$c_0 = \frac{4\mu A N^2}{3m}$$

where μ is the magnetic permeability of the air, A is pole face area, and N is the number of coil turns for each pole and m is the rotor mass. The functions Φ_1 and Φ_2 are quantities related to magnetic flux. They depend on the rotor displacements and coil currents in the following way

$$\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \frac{-1}{L}\begin{bmatrix}2l_0 - y_r & \sqrt{3}x_r\\ x_r & \sqrt{3}(2l_0 + y_r)\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} \quad (2)$$

where l_0 is the nominal air gap and $L = 4l_0^2 - (x_r^2 + y_r^2)$ is always positive in the operation range because that the rotor displacement is always smaller than the nominal air gap, i.e. $(x_r^2 + y_r^2) \le l_0^2$. The determinant of the matrix in (2) is $\sqrt{3}L$ which is always nonzero. Thus the coil currents i_1, i_2 can be expressed in terms of Φ_1, Φ_2 as

$$\begin{bmatrix}i_1\\ i_2\end{bmatrix} = \frac{-1}{\sqrt{3}}\begin{bmatrix}\sqrt{3}(2l_0 + y_r) & -\sqrt{3}x_r\\ -x_r & (2l_0 - y_r)\end{bmatrix}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} \quad (3)$$

Equations (2) and (3) will play an important role in the following controller design.

Next, a nondimensional model will be derived. It can help us choose proper feedback gains and reduce the possible numerical errors. To this aim a set of physical quantities should be taken as the bases for nondimensionalization. The nominal air gap l_0 is a natural base for the nondimensional rotor displacement. The bias current (used to support the rotor weight at the steady state) defined by

$$I_0 = \sqrt{\frac{8gl_0^2}{3c_0}}$$

is the base for the nondimensional coil current.
The desired steady state is

$$x^* = [0 \quad 0 \quad 0 \quad 0]^T, \; i^* = [0 \quad I_0]^T$$

and the linearized system matrix with respect to the steady state is

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{g}{l_0} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \frac{g}{l_0} & 0\end{bmatrix}$$

Thus, the open-loop system possesses poles of $\pm\sqrt{g/l_0}$. The time constant $\sqrt{l_0/g}$ is used here as a base for the nondimensional time. The maximum rotor displacement is l_0 and hence the nondimensional velocity is based on $l_0/\sqrt{l_0/g} = \sqrt{gl_0}$. Therefore, the nondimensional quantities are defined by

$$\bar{x}_1 = \frac{x_1}{l_0},\; \bar{x}_2 = \frac{x_2}{\sqrt{gl_0}},\; \bar{x}_3 = \frac{x_3}{l_0},\; \bar{x}_4 = \frac{x_4}{\sqrt{gl_0}}$$

$$\bar{i}_1 = \frac{i_1}{I_0},\; \bar{i}_2 = \frac{i_2}{I_0},\; \tau = \sqrt{\frac{g}{l_0}}t$$

With this set of nondimensional quantities, the state equation (1) can be expressed as

$$\frac{d\bar{x}}{d\tau} = \begin{bmatrix}\bar{x}_2\\ \frac{8}{3}\bar{\Phi}_1\bar{\Phi}_2\\ \bar{x}_4\\ \frac{4}{3}[\bar{\Phi}_2^2 - \bar{\Phi}_1^2] - 1\end{bmatrix} \quad (4)$$

The relation of $\bar{\Phi}_1$ and $\bar{\Phi}_2$ to $\bar{i}_1$ and $\bar{i}_2$ now becomes

$$\begin{bmatrix}\bar{\Phi}_1\\ \bar{\Phi}_2\end{bmatrix} = \frac{-1}{\bar{L}}\begin{bmatrix}2 - \bar{x}_3 & \sqrt{3}\bar{x}_1\\ \bar{x}_1 & \sqrt{3}(2 + \bar{x}_3)\end{bmatrix}\begin{bmatrix}\bar{i}_1\\ \bar{i}_2\end{bmatrix} \quad (5)$$

and

$$\begin{bmatrix}\bar{i}_1\\ \bar{i}_2\end{bmatrix} = \frac{-1}{\sqrt{3}}\begin{bmatrix}\sqrt{3}(2 + \bar{x}_3) & -\sqrt{3}\bar{x}_1\\ -\bar{x}_1 & 2 - \bar{x}_3\end{bmatrix}\begin{bmatrix}\bar{\Phi}_1\\ \bar{\Phi}_2\end{bmatrix} \quad (6)$$

where $\bar{L} = 4 - (\bar{x}_1^2 + \bar{x}_3^2)$.

III. Design of Smooth Controllers

Based on the nondimensional model, a class of smooth stabilization controllers for the 3-pole AMB system is proposed in this section. For conciseness, we shall abuse the notation by dropping the over bar on the nondimensional quantities in this and next sections.

First, it is observed that Φ_1 and Φ_2 are equivalent to the control inputs i_1 and i_2 by equation (5) and (6). In other words, one can first design a smooth feedback law for Φ_1 and Φ_2. The actual control law for i_1 and i_2 can be obtained easily by (6). Let

$$\Phi_1 = \hat{\psi}_1(x) \text{ and } \Phi_2 = \hat{\psi}_2(x) \quad (7)$$

be the feedback law. From the state equation (4), it is easy to see that an equilibrium must satisfy

$$x_2 = x_4 = 0,\; \hat{\psi}_1(x) = 0, \text{ and } \hat{\psi}_2(x) = \pm\frac{\sqrt{3}}{2} \quad (8)$$

Recall that the desired steady state is the origin. Hence, that feedback law in (7) must satisfy

$$\hat{\psi}_1(0) = 0, \text{ and } \hat{\psi}_2(0) = \pm\frac{\sqrt{3}}{2} \quad (9)$$

Here, the positive and negative ones in (9) are equivalent since they only represent opposite coil current directions at the steady state. We shall take the positive one without loss of generality. That is, the feedback law will take the form

$$\Phi_1 = \psi_1(x) \text{ and } \Phi_2 = \frac{\sqrt{3}}{2} + \psi_2(x) \quad (10)$$

where $\psi_1(0) = \psi_2(0) = 0$. For the controller to be smooth, we have to require that $\psi_1(x)$ and $\psi_2(x)$ be smooth.

With equation (10), the closed-loop system becomes

$$\dot{x} = \begin{bmatrix} x_2 \\ \frac{4}{\sqrt{3}}\psi_1(x) + \frac{8}{3}\psi_1(x)\psi_2(x) \\ x_4 \\ \frac{4}{\sqrt{3}}\psi_2(x) + \frac{4}{3}[\psi_2^2(x) - \psi_1^2(x)] \end{bmatrix} \triangleq f(x) \quad (11)$$

It is clear that the quadratic terms in ψ_1 and ψ_2 do not contribute to the local stability since they contain only higher order terms. Let

$$\left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \end{bmatrix} \triangleq k_1^T \quad (12)$$

$$\left.\frac{\partial \psi_2}{\partial x}\right|_{x=0} = \begin{bmatrix} k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \triangleq k_2^T \quad (13)$$

Then the local stability of the origin is determined by the eigenvalues of

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=0} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{4}{\sqrt{3}}k_{11} & \frac{4}{\sqrt{3}}k_{12} & \frac{4}{\sqrt{3}}k_{13} & \frac{4}{\sqrt{3}}k_{14} \\ 0 & 0 & 0 & 1 \\ \frac{4}{\sqrt{3}}k_{21} & \frac{4}{\sqrt{3}}k_{22} & \frac{4}{\sqrt{3}}k_{23} & \frac{4}{\sqrt{3}}k_{24} \end{bmatrix} \quad (14)$$

The feedback laws $\psi_1(x)$ and $\psi_2(x)$ have to be chosen such that the matrix A in (14) is Hurwitz.

Next, we can simplify the control law (10) by taking $\psi_1(x)$ as a function of x_1 and x_2 only, and $\psi_2(x)$ as that of x_3 and x_4 only. In doing so, we must have

$$k_{13} = k_{14} = k_{21} = k_{22} = 0$$

in equations (12)-(14). In other words, the linearized dynamics are decoupled in the x and y motions. Physically, the x dynamics are controlled by $\psi_1(x)$, whereas the y dynamics are determined by $\psi_2(x)$. Therefore, the desired equilibrium is exponentially stable if and only if k_{11}, k_{12}, k_{23}, and k_{24} are all negative.

One can further simplify the control law by taking $\psi_i(x)$ as a linear function of the states, i.e.

$$\psi_1(x) = k_{11}x_1 + k_{12}x_2 \quad (15)$$

$$\psi_2(x) = k_{23}x_3 + k_{24}x_4 \quad (16)$$

Then, the closed-loop system (11) becomes a quadratic nonlinear system

$$\dot{x} = \begin{bmatrix} x_2 \\ \frac{4}{\sqrt{3}}(k_{11}x_1 + k_{12}x_2) + \frac{8}{3}(k_{11}x_1 + k_{12}x_2)(k_{23}x_3 + k_{24}x_4) \\ x_4 \\ \frac{4}{\sqrt{3}}(k_{23}x_3 + k_{24}x_4) + \frac{4}{3}\left[(k_{23}x_3 + k_{24}x_4)^2 - (k_{11}x_1 + k_{12}x_2)^2\right] \end{bmatrix} \quad (17)$$

where the local dynamics are dominated by the decoupled linear parts. When the feedback law is taken as the linear form (15) and (16), the control law for the actual input i can be obtained by (6) and is given by

$$i_1 = (\frac{\sqrt{3}}{2} - 2k_{11})x_1 - 2k_{12}x_2 + x_1(k_{23}x_3 + k_{24}x_4) - x_3(k_{11}x_1 + k_{12}x_2) \quad (18)$$

$$i_2 = -\sqrt{3} + (\frac{\sqrt{3}}{2} - 2k_{23})x_3 - 2k_{24}x_4 + x_1(k_{11}x_1 + k_{12}x_2) + x_3(k_{23}x_3 + k_{24}x_4) \quad (19)$$

which are polynomial functions of degree 2 in the states. One interesting fact for this smooth controller is noted. Recall that the original state equation (4) is strongly nonlinear in the states and inputs. However, if the quadratic state feedback laws (18) and (19) are applied, the closed-loop system (17) is also quadratic. In the rest of this study, we will focus on this quadratic state feedback system.

IV. Domain of Attraction

The local stability of the desired equilibrium point, i.e., the origin, has been imposed in designing the smooth controller, as demonstrated in the previous section. Here, the domain of attraction (DOA) of the origin will be discussed. One special feature of the AMB system is that no impacts between the rotor and stator are allowed during the operation. Thus, one can define the operation domain to be

$$D = \left\{ x \in \Re^4 \middle| x_1^2 + x_3^2 \le 1 \text{ and } x_2^2 + x_4^2 \le 1 \right\}$$

Therefore, it is hoped that the DOA of the origin is large enough so that it can cover the operation domain. However, the DOA will be constrained by other equilibrium points. Let us begin by investigating the existence of other equilibrium points.

From the closed-loop system (17), it is easy to see that there exists another equilibrium point given by

$$x^* = \begin{bmatrix} 0 & 0 & -\frac{\sqrt{3}}{k_{23}} & 0 \end{bmatrix}^T \quad (20)$$

Then, one can get

$$A^* = \left.\frac{\partial f}{\partial x}\right|_{x=x^*} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{4}{\sqrt{3}}k_{11} & -\frac{4}{\sqrt{3}}k_{12} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{4}{\sqrt{3}}k_{23} & -\frac{4}{\sqrt{3}}k_{24} \end{bmatrix} \quad (21)$$

Since all k_{ij} 's are all negative (so that the matrix A in (14) is Hurwitz), the eigenvalues of A^* must be positive or negative real numbers. We conclude that x^* must be a saddle point. The DOA of the origin must be constrained by x^*. The parameter k_{23} should be taken as

$$-\sqrt{3} < k_{23} < 0$$

so that x^* is located out of the operation domain. To enlarge the DOA, the parameter k_{23} should be taken as close to zero as possible.

Next, we shall estimate the DOA utilizing the quadratic structure of the system. The closed-loop system (17) can be rewritten as

$$\dot{x} = Ax + g(x)$$

where

$$g(x) = \begin{bmatrix} 0 \\ \frac{8}{3}\psi_1(x)\psi_2(x) \\ 0 \\ \frac{4}{3}\left[\psi_2^2(x) - \psi_1^2(x)\right] \end{bmatrix}$$

Now, consider the Lyapunov function

$$V(x) = x^T P x$$

where $P = P^T > 0$ is the positive definite matrix satisfying the Lyapunov's equation

$$PA + A^T P = -I$$

Then, one can obtain

$$\dot{V}(x) = -x^T x + 2x^T P g(x) \le -\|x\|^2 + 2\|x\|\|P\|\|g(x)\| \quad (22)$$

where $\|\bullet\|$ denotes the Euclidean 2-norm. The norm $\|g(x)\|$ can be obtained as

$$\|g(x)\|^2 = \left(\frac{4}{3}\right)^2 \left[4\psi_1^2(x)\psi_2^2(x) + \left(\psi_2^2(x) - \psi_1^2(x)\right)^2\right]$$

resulting in

$$\|g(x)\| = \frac{4}{3}\left[\psi_1^2(x) + \psi_2^2(x)\right] = \frac{4}{3} x^T Q x \le \frac{4}{3}\|Q\|\|x\|^2$$

where

$$Q = k_1 k_1^T + k_2 k_2^T = \begin{bmatrix} k_{11}^2 & k_{11}k_{12} & 0 & 0 \\ k_{11}k_{12} & k_{12}^2 & 0 & 0 \\ 0 & 0 & k_{23}^2 & k_{23}k_{24} \\ 0 & 0 & k_{23}k_{24} & k_{24}^2 \end{bmatrix}$$

Hence, equation (22) becomes

$$\dot{V}(x) \le -\|x\|^2 \left(1 - \frac{8}{3}\|P\|\|Q\|\|x\|\right)$$

which is negative definite if

$$\|x\| < \frac{3}{8\|P\|\|Q\|}$$

Therefore, the DOA can be estimated by [12]

$$\left\{ V(x) \le \frac{9\lambda_{\min}(P)}{64\|P\|^2\|Q\|^2} \right\}$$

where $\lambda_{\min}(P)$ denotes the minimum eigenvalue of the matrix P. Obviously, this estimated DOA depends on the control gains k_{ij} 's, which can be maximized using optimization technique. We will not pursue it here, but leave it as a future work.

V. Numerical and Experimental Results

Both numerical and experimental studies will be performed to verify the analysis.

The system parameters are: m=0.634 kg, g=9.81 m/s^2, l_0 =1×10^{-3} m, μ =4π×10^{-7} H/m, A=4×10^{-4} m^2, N=300. The control gains are chosen to be: $k_{11} = 300$, $k_{12} = 350$, $k_{23} = 300$, and $k_{24} = 350$, corresponding to nondimensional control gains of $k_{11} = 6.14 \times 10^{-4}$, $k_{12} = 0.071$, $k_{23} = 6.14 \times 10^{-4}$, and $k_{24} = 0.071$. The initial condition is:

$$x(0) = \begin{bmatrix} 0 & 0 & -5 \times 10^{-4} m & 0 \end{bmatrix}$$

Note that there is a back-up bearing placed on half way between the stator and the rotor. In other words, the practical allowable operation range for the rotor is a circle with radius of $5 \times 10^{-4} m$, which is marked by dashed lines in the following figures. Hence, the initial condition represents the situation that the rotor is initially at rest on the back-up bearing. This is the case that coincides with the experiment presented below. The simulation results are shown in Figs. 2 to 3. As one can see clearly, the rotor can be brought to the origin with zero steady state error.

The experimental set-up is shown in Fig. 4. It is a single 3-pole AMB with a disk-like rotor. The frame and base are made of aluminum alloy to shield the magnetic flux. To reduce the eddy current loss, both the stator and rotor are made of laminated sheets of silicon steels with thickness of 20 mm. The outer diameter of the stator is 150 mm, and that of the rotor is 70 mm. Each side of the rotor shaft is constrained with a thrust bearing. The coil currents are provided by PWM power amplifiers (Advanced Motion Control model 25A20). The power amplifier can provide currents between ± 12.5 A with bandwidth of 2.5 KHz. The displacements of the rotor are measured by two eddy current sensors (Applied Electronics Corporation AEC 7606-34), with resolution of 0.5μm and measurement range of 0 to 1.3 mm. The control algorithms are implemented on a dSpace's DS1102 control card for the real time control with sampling time of 10^{-4} sec. The system parameters are the same as those used in simulations. The rotor is initially at rest on the back-up bearing and is to be levitated to the origin.

The experimental results are shown in Figs. 5 to 6. Indeed, the rotor can be levitated to a neighborhood of the origin. However, there exists steady state error, as indicated in Fig. 6. The existence of the steady state error could be due to uncertainties. There exist many uncertainty sources for an AMB system, such as magnetic flux leakage, magnetic hysteresis and saturation, and manufacturing and assembly errors, etc [13]. As suggested in [11], delicate calibration and robust controller are necessary for good performance. The present smooth controller is far from robust. The steady state error can be eliminated by including an integral control. Fig. 7 shows the experimental results with integral control, where the rms steady state error is $4.28\mu m$.

VI. Conclusions

A class of smooth controllers has been proposed for the stabilization of a 3-pole AMB system. The system is strongly nonlinear and non-affine. Conventional controllers are nonsmooth and complicated, which are difficult for practical implementation and further study. The proposed smooth controllers are designed utilizing a linear transformation on the system inputs (coil currents). Based on the new inputs, a class of smooth controllers can be designed easily. In particular, a quadratic state feedback controller has been proposed. With such a smooth controller, there exists another undesirable equilibrium point that can constrain the domain of attraction of the origin. The stability of the undesirable equilibrium and the domain of attraction of the origin have been discussed. It is found that the undesirable equilibrium is a saddle point.

Moreover, the estimated domain of attraction of the origin depends on the control parameters. One can take proper feedback gains to enlarge the domain of attraction of the origin.

The analyses have been verified by numerical and experimental results. Perfect results have been obtained in simulations. In the experimental results, although the rotor can be levitated to a neighborhood of the origin, there exists steady state error, which could be due to the uncertainty. The steady state error can be eliminated using integral control. Both numerical and experimental results verify that the smooth controller is indeed feasible.

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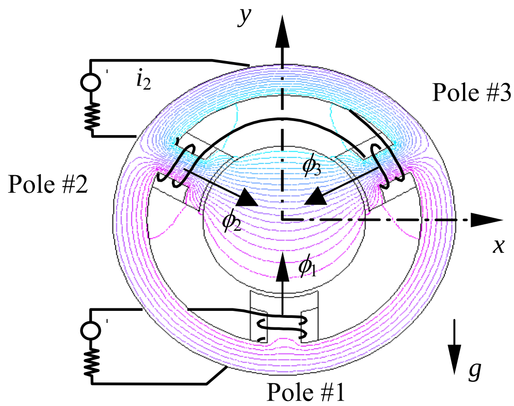

Fig. 1 The 3-pole AMB system

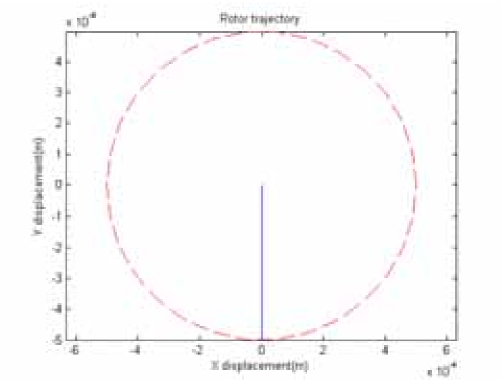

Fig. 2 Numerical results: rotor trajectory

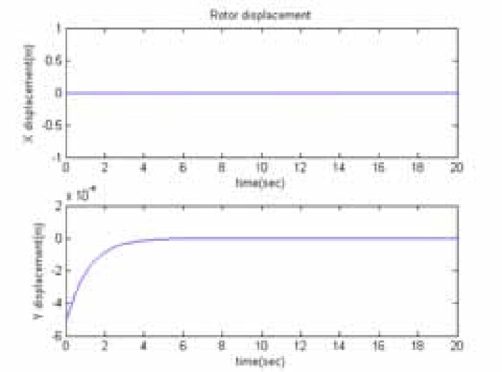

Fig. 3 Numerical results: displacement responses

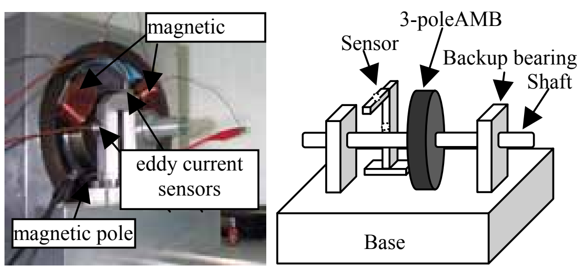

Fig. 4 The experimental set-up

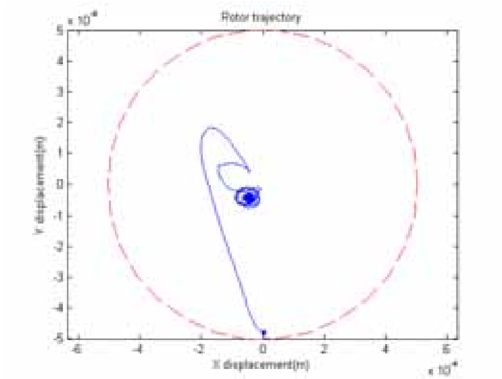

Fig. 5 Experimental results: rotor trajectory

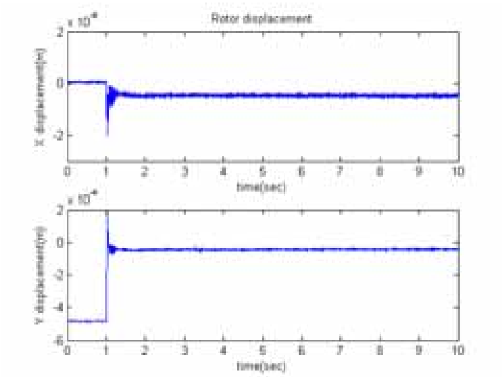

Fig. 6 Experimental results: displacement responses

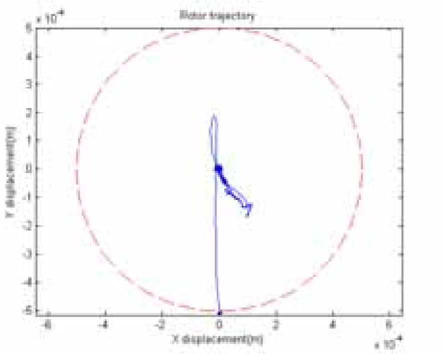

Fig. 7 Experimental results with integral control: rotor trajectory