Modal Control Method for Rotors Supported by Active Magnetic Bearings based on a Condensed Reduced Model

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***Abstract*— The work presented in this paper exposes a new approach for the control of the dynamic behavior of flexible rotors for rotating machines supported by AMB. In order to perform this control, an accurate model of the structure reduced on the modes to be controlled and an accurate model of the AMB are used. An approach involving the use of a condensed reduced model is introduced. The identification approach presents the main advantage to be easy to realize, and efficient for control. An experimental identification of an accurate inverse model of the AMB is performed, as it is necessary for modal control. The results are exposed for a flexible beam, and validates the method experimentally.**

***Index Terms*— condensed reduced model, inverse model of the AMB, modal control.**

I. Introduction

Turbomolecular pumps are high vacuum pumps used in many industrial fields. Their rotating elements are levitating on Active Magnetic Bearings (AMB) as a solution for chemical (corrosion) and mechanical (sealing, maintenance, sustainable development, ...) problems inherent to the manufacturing process [1]. The behavior of these AMB is highly non linear, they are unstable, and thus require to be continuously controlled. Industrially, AMB are mostly controlled by PID controllers that permit the control of the rigid body modes of the rotor [2]. As they are not well designed to cross several critical speeds of rotation, new methods have been developed [3]. Their principles rely on very accurate mathematical models of the coupling relations between the system and the AMB linearized around a working point. Firstly, the mathematical model is estimated, re-adjustments are then necessary to insure the accuracy of the model. Consequently, the efficiency of the control will depend on the precision of the model. Secondly, the linear controllers that are obtained do not take into account the nonlinear characteristics of the AMB. Moreover, the displacements amplitudes would be small around the working point.

The work presented in this paper exposes the control of a flexible beam supported by AMB that would be applied for the control of a flexible rotor of a rotating machine. In order to perform this control, an accurate reduced model of the structure on a Target Frequency Bandwidth (TFB) and accurate models of the AMB (direct and inverse) are needed. An approach involving the use of a Condensed Reduced Model (CRM) is introduced. It consists on the direct identification of a condensation of the structure on the available sensors. The identification approach presents the main advantage to be easy to realize, and efficient for control [4]. For the actuator concerns, the AMB is chosen to be controlled by intensity. However, the electromagnetic force is a non linear function of the air gap and the intensity. In order to apply the control forces, the implementation of an inverse model is necessary. In that case, the effort generated by the AMB will be the force computed by the controller. Furthermore, analytical models do not take into account all specifications inherent to experimental conditions. That was the reason for the development of an experimental identification of an accurate inverse model of the AMB that will eventually be introduced in the control loop.

The paper is organized as follow. In the first part, the CRM aspects are presented. It involves the theoretical aspects, the CRM-based observer and the complex modes of the CRM. In second part, the experimental method for the inverse model identification of the AMB is shown. In the third part, the CRM and the inverse model of the magnetic bearings are implemented in the feedback modal controller, that is applied to the structure. For the demonstration, flexible beam is used, because its behavior is similar to the rotor's at rest. Then, the method is assessed experimentally. The paper ends on the concluding remarks.

II. Condensed Reduced Model (CRM)

A. Theoretical aspects

Equations of motion of a n degrees of freedom system can be expressed as:

$$F_{n\times 1} = M_{n\times n}\ddot{x}_{n\times 1} + C_{n\times n}\dot{x}_{n\times 1} + K_{n\times n}x_{n\times 1} \qquad (1)$$

with $M_{n\times n}$, $C_{n\times n}$ and $K_{n\times n}$ respectively the mass, damping and stiffness matrices. $\ddot{x}_{n\times 1}$, $\dot{x}_{n\times 1}$ and $x_{n\times 1}$ are the acceleration, velocity and displacement vectors. $F_{n\times 1}$ represents the external forces vector acting on the system.

The control of the modes of the system allows a global action on the structure with a minimum number of actuators [5], as long as the controllability criterium is respected. The symetric matrices of mass, stiffness and damping are diagonalized, so that the equation of motion of the system can be decoupled. In this work, the use of a CRM of the structure is proposed. This model represents the structure condensed on the measured degrees of freedom (dof). The CRM has as much poles as the number of measurable dof. Although the method is similar to [6], the approach is different.

Equation (1) can be expressed as:

$$\left\{ \begin{array}{c} F_m \\ F_{nm} \end{array} \right\} = \left[\begin{array}{cc} M_m & M_{nm} \\ M_{nm}^T & M_f \end{array} \right] \left\{ \begin{array}{c} \ddot{x}_m \\ \ddot{x}_{nm} \end{array} \right\} + \left[\begin{array}{cc} C_m & C_{nm} \\ C_{nm}^T & C_f \end{array} \right] \left\{ \begin{array}{c} \dot{x}_m \\ \dot{x}_{nm} \end{array} \right\} + \left[\begin{array}{cc} K_m & K_{nm} \\ K_{nm}^T & K_f \end{array} \right] \left\{ \begin{array}{c} x_m \\ x_{nm} \end{array} \right\} \quad (2)$$

Sub-indices refer to the parts of the matrices that correspond to the dof that can be measured (m) or not (nm). The dimension of x_m and its time-derivations is $r \times 1$, while r is the number of measurable dof. x_{nm} and its time-derivations are the dof that cannot be measured, and the dimension is $(n - r) \times 1$. The sub-indice f refers to the parts of the matrices corresponding to the last $(n - r)$ equations for the dof that are not measurable.

In modal space, the displacement vector can be written as:

$$\left\{ \begin{array}{c} x_m \\ x_{nm} \end{array} \right\} = \underbrace{\left[\begin{array}{cc} \phi_i & \phi_{ni} \\ \phi_{inm} & \phi_{ninm} \end{array} \right]}_{\Phi} \left\{ \begin{array}{c} q_i \\ q_{ni} \end{array} \right\}, \quad \text{with } 1 \leq i \leq r, \text{ and, } r+1 \leq ni \leq n \quad (3)$$

q_i represents the modal quantities vector corresponding to the modes, whose frequencies are within the Target Frequency Bandwidth (TFB) in free motion, and q_{ni} represents the others. Φ is the classical modal matrix of the system, and is subdivised in four parts: ϕ_i and ϕ_{ni} both correspond to the measured dof. ϕ_i refers to the modes, whose frequencies are within the TFB, while ϕ_{ni} correspond to the other modes. Similarly, ϕ_{inm} and ϕ_{ninm} both correspond to the unmeasured dof. ϕ_{inm} refers to the modes, whose frequencies are within the TFB, while ϕ_{ninm} corresponds to the other modes.

Equation (3) can also be written as:

$$\left\{ \begin{array}{rcl} x_m & = & \underbrace{\phi_i q_i}_{x_{mi}} + \underbrace{\phi_{ni} q_{ni}}_{x_{mni}} \\ x_{nm} & = & \underbrace{\phi_{inm} q_i}_{x_{nmi}} + \underbrace{\phi_{ninm} q_{ni}}_{x_{nmni}} \end{array} \right. \quad (4)$$

During the identification of the CRM, the structure is put into motion by an impact type disturbance or initial conditions. When using a low-pass numerical filter, whose cutting-frequency equals the upper-limit of the TFB, the absolute values of the amplitudes of the q_{ni} can be considered as much lower than those of the q_i.

Consequently, (4) becomes:

$$\left\{ \begin{array}{rcl} |q_{ni}| & \ll & |q_{ni}| \\ x_{nmi} & = & \phi_{inm} \phi_i^{-1} x_{mi} \end{array} \right. \quad (5)$$

Equation (5) introduced in the first r equations of (2) gives the equation of the CRM:

$$\begin{array}{rcl} F_m & = & \underbrace{\left[M_m + M_{nm} \phi_{inm} \phi_i^{-1} \right]}_{M_{CRM}} \ddot{x}_{mi} \\ & + & \underbrace{\left[C_m + C_{nm} \phi_{inm} \phi_i^{-1} \right]}_{C_{CRM}} \dot{x}_{mi} \\ & + & \underbrace{\left[K_m + K_{nm} \phi_{inm} \phi_i^{-1} \right]}_{K_{CRM}} x_{mi} \end{array} \quad (6)$$

The equations of motion of the CRM represent the dynamic of the structure referred to the dof that can be measured and dynamically bounded to the modes that are within the TFB.

B. Modal observer and complex modes

In operating conditions, the structure is excited on all its modes. As the modal controller is tuned on the modes of the TFB, a modal filter is necessary to avoid spillover. Analogic filters are avoided, as it would induce time shifts in the sensors signals for such low frequency range, and thus, greatly lessen the controller efficiency. Therefore, an observer based on the CRM is used as a low pass filter with no time delays. Its parameters are chosen as a compromise between the convergence rapidity and the higher frequency signals filtered out. As the influence of the higher frequency signals of the structure is unknown, the observer parameters are tuned in simulation, using the signals issued from the CRM. An important and uniform numerical noise was added to the observer to take them into account during tuning.

The CRM can be expressed in a state-space formulation as:

$$\left\{ \begin{array}{c} \dot{x}_{mi} \\ \ddot{x}_{mi} \end{array} \right\} = A_{CRM} \left\{ \begin{array}{c} x_{mi} \\ x_{mi} \end{array} \right\} + B_{CRM} F_m \quad (7)$$

With A_{CRM} and B_{CRM} respectively the evolution matrix and the action matrix of the state-space formulation of the CRM.

$$A_{CRM} = \left[\begin{array}{cc} 0 & I \\ -M_{CRM}^{-1} K_{CRM} & -M_{CRM}^{-1} C_{CRM} \end{array} \right] \quad \text{and,} \quad B_{CRM} = \left[\begin{array}{c} 0 \\ -M_{CRM}^{-1} \end{array} \right] \quad (8)$$

As the error of the observer can be defined as:

$$\varepsilon = \left\{ \begin{array}{c} x_{mi} \\ \dot{x}_{mi} \end{array} \right\} - \left\{ \begin{array}{c} \hat{x}_{mi} \\ \hat{\dot{x}}_{mi} \end{array} \right\} \quad (9)$$

The dynamic of the observer depends on the choice of the parameters of L. C_{obs} is the observation matrix of the CRM.

$$\dot{\varepsilon} = (A_{CRM} - L C_{obs}) \varepsilon \quad (10)$$

The mass, stiffness and damping matrices of the CRM are not symmetric. Therefore, it is necessary to use the orthogonality of the left and right eigenvectors to decouple the CRM equations [7].

The eigenvalue problem can be formulated by assuming a solution for the state vector as:

$$x = exp(\lambda t)v_r \tag{11}$$

where λ is the eigenvalue, and v_r is the rigth eigenvector. The eigenvalue problem needs to be solved for both the right and left eigenvectors, that can be expressed as follow:

$$\begin{array}{c} A_{CRM}v_r = \lambda v_r \\ \text{and} \\ A_{CRM}^T v_l = \lambda v_l \end{array} \tag{12}$$

where v_l is the left eigenvector. The right and left eigenvectors can be normalized as:

$$\begin{array}{c} v_{ls}^T v_{rt} = 2\delta st, \text{ and, } v_{ls}^T A_{CRM} v_{rt} = 2\lambda_t \delta_{st} \\ s,t = 1,2,\ldots,2r \end{array} \tag{13}$$

The eigenvectors and eigenvalues can be written as:

$$\begin{array}{c} v_{rs} = a_s \pm ib_s, v_{ls} = c_s \pm id_s; s = 1,2,\ldots,r \\ \lambda_s = \sigma_s \pm i\omega_s; s = 1,2,\ldots,r \end{array} \tag{14}$$

The modal matrices can be expressed in real components as:

$$\begin{array}{c} R = [a_1, b_1, a_2, b_2, \ldots, a_r, b_r] \\ L = [c_1, d_1, c_2, d_2, \ldots, c_r, d_r]; \end{array} \tag{15}$$

with the relationships,

$$\begin{array}{c} R^T L = I \\ \text{and,} \\ R^T A_{CRM} L = \Lambda \end{array} \tag{16}$$

with,

$$\Lambda = \begin{bmatrix} \begin{bmatrix} \sigma_1 & \omega_1 \\ -\omega_1 & \sigma_1 \end{bmatrix} & \cdots & 0_{2\times2} \\ \vdots & \ddots & \vdots \\ 0_{2\times2} & \cdots & \begin{bmatrix} \sigma_r & \omega_r \\ -\omega_r & \sigma_r \end{bmatrix} \end{bmatrix} \tag{17}$$

Equation (7) is decoupled as:

$$\dot{q} = \Lambda q + L^T B_{CRM} F_m \tag{18}$$

III. Inverse model of the AMB

Active Magnetic Bearings equations are well known throughout litterature. Ampere's theorem is widely used to express the electromagnetic force of the AMB as a function of its material properties. However, those properties are not always known with great precision. Usually, the AMB parameters (input intensity, air gap distance to the bearing) are set so that the rotor mass is in operating position. Equivalent stiffness respect to the distance and the intensity are then estimated [8]. Thus, the electromagnetic behavior of the AMB is linearized around its operating position.
In our case, an inverse model of the AMB is necessary to express the efforts issued from the modal controller to the efforts applied to the structure. The relation between the intensity, the air gap and the electromagnetic force is:

$$F_{AMB} = I^2(\alpha/(a+\beta)^2) \tag{19}$$

with F_{AMB}, I and a respectively the electromagnetic force, the input intensity and the air gap. α and β are the AMB parameters issued from its material properties. The electromagnetic forces are strongly nonlinear according to the input current and the air gap. The function of the inverse model of the AMB is to make this relation linear for the controller. Experimentally, the AMBs are driven by a current amplifier fed by a voltage issued from a dSpace (Matlab ®) card. Several voltages are applied to the current amplifier and the response of the system is compared to the CRM's. A specialized neural network is used for each AMB to identify their material parameters (Fig. 1). These parameters are then extracted from the neural network weights. Then, they were used to construct the inverse model of the AMBs. The inverse model of the AMBs is then implemented in the control loop before the current amplifier (Fig. 2). The current amplifier gain is called $g_{amplifier}$. The equation of the inverse model of the AMB for the magnet i is written as:

$$V_{control(i)} = (a_i + \beta_i)/(g_{amplifier}\sqrt{\alpha_i})\sqrt{F_{AMB(i)}} \tag{20}$$

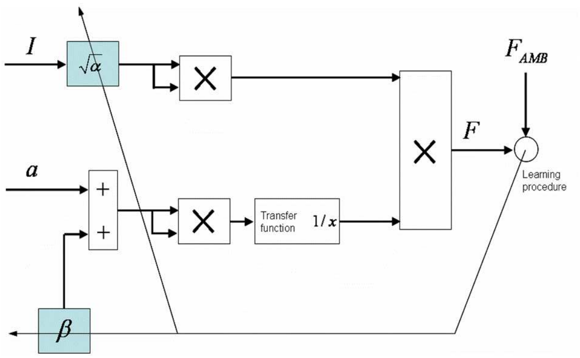


Fig. 1. Specialized neural network for parameters identication.

IV. Experiment and results

A. Experimental bench

A clamped-free beam made of steel is used for the demonstration of the feasibility of the method. An AMB made with two rectangular 80 coils magnets is used. The mobile part of the ferromagnetic core is fixed on the beam in order to close the magnetic circuit. The first four modes were chosen to be controlled. Therefore, four displacement

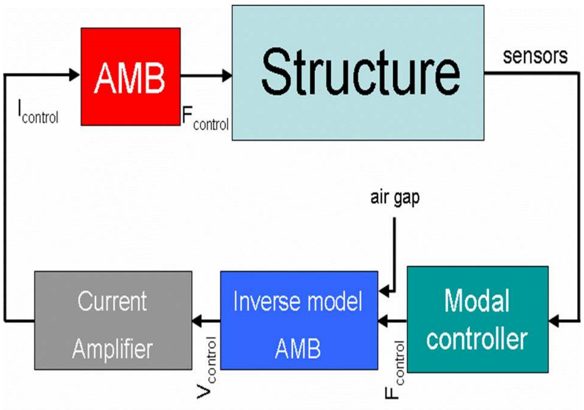
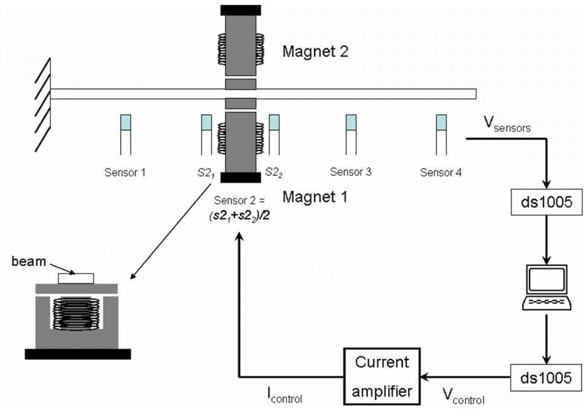
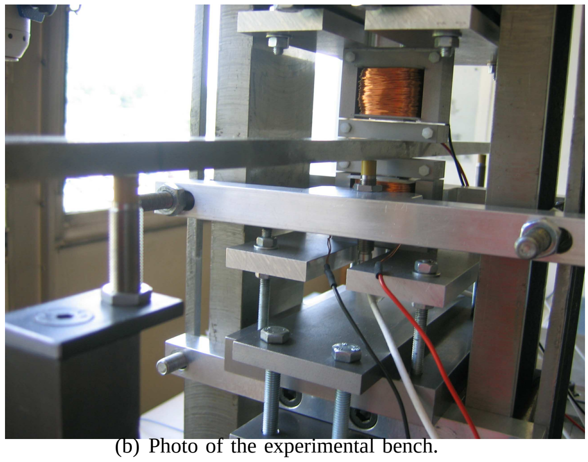


Fig. 2. Implementation of the inverse model of the AMB in the control loop.

sensors were necessary for the identification of the CRM of the beam. Only one actuator (the AMB) is used for control. In a modal control strategy, it is necessary that the actuator is collocated to one of the sensors. Thus, the displacement of the spot corresponding to the actuator was obtained by taking the average value of the sensors placed on either sides of the AMB. The calibration setup is composed of a standard PC connected to a PowerPC in which a dSpace hardware card (ds1005) is implemented. The controller is tuned on the Simulink® interface. The sampling period of the analysis is set to $T_s = 10^{-4}s$. The controlling signals are tensions that feed a current amplifier, which drives the magnets. The experimental bench is shown on Fig. 3.



(a) Scheme of the experimental bench.



(b) Photo of the experimental bench.

Fig. 3. Experimental bench.

B. Results

The evolution matrix A_{CRM} is identified after the structure is excited with an impact hammer. The signals issued from the displacement sensors are used for the identification. As the structure responds on more than four modes, a low-pass frequency analogic filter is used to verify (5). The evolution matrix is identified according to (21).

$$\begin{cases} \begin{Bmatrix} x_{mi}(k+1) \\ \dot{x}_{mi}(k+1) \end{Bmatrix} = A_{d-CRM} \begin{Bmatrix} x_{mi}(k) \\ \dot{x}_{mi}(k) \end{Bmatrix} \\ A_{d-CRM} = expm(A_{CRM}T_s) \end{cases} \quad (21)$$

Similarly, the action matrix is identified when the structure is excited by a harmonic excitation. The excitation frequency is chosen within the TFB, and the identification is done only when the structure motion is steady. Equation (22) was used for the identification.

$$\begin{cases} \begin{Bmatrix} x_{mi}(k+1) \\ \dot{x}_{mi}(k+1) \end{Bmatrix} - A_{d-CRM} \begin{Bmatrix} x_{mi}(k) \\ \dot{x}_{mi}(k) \end{Bmatrix} = \Delta(X(k)) \\ \Delta(X(k)) = B_{d-CRM} F_m(k) \\ B_{d-CRM} = A_{CRM}^{-1}(A_{d-CRM} - I_{2r\times 2r})B_{CRM} \end{cases} \quad (22)$$

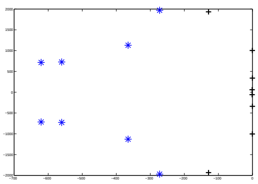
The CRM was used for the tuning of the observer gains in simulation. The observer poles are compared to those of the CRM on Fig. 4a. They are a compromise between the convergence rapidity and noise energy transferred. The effect of the CRM-based-observer are also visible on Fig. 4 (b and c). It filters out the influence of the modes that are not included in the TFB. Therefore, the modal controller will be fed by only the quantities of the modes corresponding to the CRM.

Similarly, the controller is tuned with the use of the CRM in simulation. The modal forces are calculated as a linear feedback on the modal quantities, which are converted in real forces that will be applied on the structure (23). An integral feedback was also applied to insure the positioning of spot corresponding on sensor 2. The CRM-based observer and the modal controller are then used for the control of the structure. Responses between controlled and uncontrolled structure submitted to an impact are shown on Fig. 5.

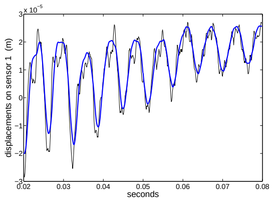
$$\begin{cases} F_{mod} = -K_{mod} q \\ F_{control} = (B_{CRM}^T B_{CRM}) B_{CRM}^T L^{-T} F_{mod} \end{cases} \quad (23)$$

V. Concluding Remarks

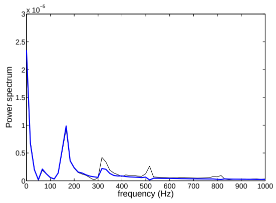
The work exposed in this paper describes a new approach for the modal control of structures with the use



(a) CRM poles (+) and observer poles (∗).



(b) Time reconstruction of the observer (bold line) compared to the structure response (thin line).



(c) Frequency spectrum of the reconstruction of the observer (bold line) compared to the frequency spectrum of the structure response (thin line).

Fig. 4. CRM-based observer and its influence.

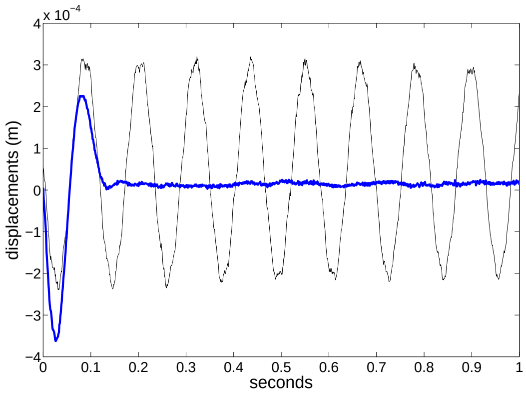


Fig. 5. Comparison between the controlled (bold line) and uncontrolled structure (thin line) submitted to an impact excitation.

of highly non linear actuators. The use of a condensed reduced model provides great advantages for modal control. Its identification is easy, as no finite element model is necessary to obtain a condensed reduced model precise enough for control. The number of modes that can be controlled equals the number of the available sensors. To make the effort generated by the AMB the image of the force computed by the controller, a inverse model is necessary. A neural network is used for this task, as it provides a great flexibility of procedure. In the last part of the work, it is shown that the method offers great efficiency for both the dynamic control and positioning control. The approach is thus validated. In forthcoming study, the method will be extended to a beam with free-free boundary conditions (with very low stiffness strings on both ends), and eventually will be applied to rotors in working conditions (in magnetic levitation).

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