Study on Nonlinear Dynamics of Rotor System Equipped with AMB *

Gang Zhang Songsheng Li Ning Liu Gang Gao Zhifeng Zhao Gao Cheng

Research Institute of Bearing ,Shanghai University
UBC (Shanghai) Precision Bearing Manufacturing Co.,Ltd
Shanghai, 200072, China
gzhang@mail.shu.edu.cn;gzhanghd@sh163.net

Lie Yu

Theory of Lubrication and Bearing Institute
Xi'an Jiaotong University
Xi'an, 710049,China
tlbi@sun20.xjtu.edu.cn

***Abstract* –Taking an electric miller spindle equipped with AMB as the research object, this paper have established the non-linear dynamic model of five degrees of freedom for the rotor-AMB system, and carried on the numerical solution using the principle of multi-scale method and combined with the numerical analysis method. Besides, from the nonlinear viewpoint, the dynamic characteristics of rotor-AMB system have been studied; the bifurcation and chaos phenomenon of five degrees of freedom systems have been analyzed else. The results show, in the nonlinear scope, the rotor can appear the instability in certain situations, so there exists a question to judge the stability. The partial bifurcation curves generally enter from the sub-critical bifurcation region to the supercritical bifurcation region, then return to the sub-critical bifurcation region once more; The chaos phenomenon confirmed that, the rotor system can appear the chaos phenomenon under the certain condition, when striding across the critical speeds the chaos phenomenon is quite obvious.**

***Index Terms* –AMB(Active Magnetic Bearing) Rotor System Nonlinear Dynamics**

I INTRODUCTION

The rotor system equipped with AMB (active magnetic bearing) is essentially a typical nonlinear coupling electromechanical system[1], and the electromagnetic force produced by AMB is nonlinear function of displacement of the controlled object and the controlling electric current, therefore most of the actual rotor dynamics problem is nonlinear[2]. Indeed, for most of the engineering application, using the linearizable description is very precise; this is also the main reason why most engineering problems were still solved via linear analysis currently. Though, as the fast development of modern industries towards large power, high precision and high efficiency, using the non-linear dynamics to analyse the further dynamical question is needed more and more. The early research only can be used to dynamic control of system in small scope around the static operating point based on the hypothesis that the electromagnetic force is the linear function of displacement. Once the system receives external destabilization, particularly when the frequency and amplitude changes, the rotor system possibly appears large amplitude of vibration, causing the rotor bump heavily against the magnetic bearing to out of control. Therefore, dynamics research on the non-linear field is necessary, and the non-linear research will play increasingly important role in the modern industry. *Enrich* made a further research on the bifurcations and chaos phenomena existing in rotor system, and found that the characteristics of bifurcations and chaos in complex rotor system on the background of engineering had many similarities with the single freedom nonlinear system. Later, many scholars had discovered amount of bifurcations and chaos phenomena[2~9] in their researches, such as nonlinear force in oil-film bearing, nonlinear extrusion oil-film force, nonlinear crack rotor, nonlinear collisional friction, nonlinear rigid supporting and so on. But these researches are mostly based on liquid-sliding bearings. In the magnetic bearing research, at present the majority of the non-linear research generally is studied on single degree of freedom and two degrees of freedom of single bearing [10~13], but the paper about the nonlinear research on complex multi-freedom rotor-AMB system with higher-order and multi-dimension is extremely difficult to find. Though, there are many kinds of very complex forms of nonlinear force in rotor system to be studied, the form and phenomenon of bifurcation and chaos are basically consistent. These rotor systems generally enter or exit chaos through the period-double bifurcation, the Hopf bifurcation, the period-3 bifurcation and intermittent chaos and so on, these phenomena do not beyond the analysis results of non-linear Duffing equation of single degree of freedom[2]. This indicates there possibly exists something in common in the complex nonlinear system with periodic actuation, which also may perform through the simple system to describe, but it is pity that people has not discovered the commonness. The early analysis methods of nonlinear oscillation in rotor system mainly consist of the traditional harmonic analysis method, the small parameter method, the multi-scale method and so on, these methods are very complex for more than two degrees of freedom system analysis, and are unable to analyze the chaos response. Now, people are mainly making use of numerical integral method in combination with Floquet theory, Poincare mapping, center manifold

* This work is supported by National 863 Plan of Chinese Science and Technology Ministry (No. 2001AA423310).

theory to analyse the bifurcation and chaos of rotor system[5~7]. But for more than four degrees of freedom system, there is no better method except the numerical integral method. The key to further research on nonlinear dynamics is developing the nonlinear dynamic theory of higher dimension system, setting up the nonlinear dynamical model in conformity with the fact, enhancing the experimental research on nonlinear rotor dynamics, applying the analytical results to solve practical questions, and proposing a method to control the nonlinear vibration of system.

Concretely speaking, compared to classical linear dynamic theory, the main tasks of nonlinear dynamics are: (1)the system equilibrium points and as well as the judgment of stability, and the structural variation of steady-state solution when the parameters change (i.e. the solution bifurcation); (2) the result of the system in the long-run development under definite initial condition or the destabilization, namely the global study of nonlinear solutions, which belongs to the chaos dynamics field. In general, nonlinear dynamics of magnetic suspension can be divided into two parts, the study on autonomous system and non-autonomous system problems.

On basis of single degree of freedom nonlinear mathematical model, this paper establishes a nonlinear dynamical model of five degrees of freedom rotor system Equipped with AMB, and applies the fundamental theory of multi-scale method to deal with five degrees of freedom nonlinear differential equation. Also, the solution to average equation is got using numerical analysis method, and the bifurcations and chaos phenomena of five degrees of freedom system are analysed. Finally, partial process and result of experiment are given and verify the correctness of the above-mentioned theories.

II. DYNAMIC MODEL OF FIVE DEGREES OF FREEDOM ROTOR SYSTEM WITH AMB

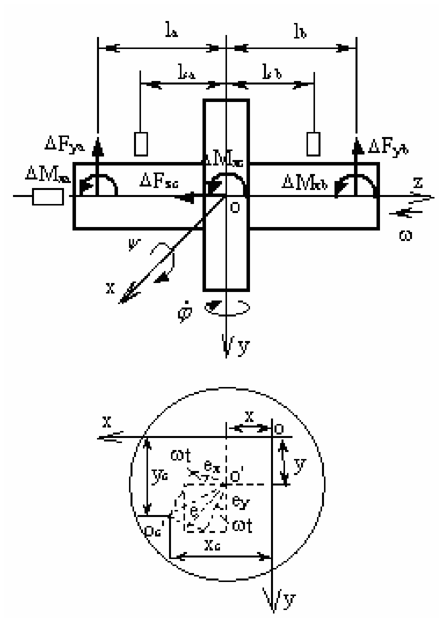


Fig.1 The dynamic loads and movements state of the rotor

Masses of literature indicate that, in the magnetic levitation bearing systems, the present non-linearized theory researches generally only go into two degrees of freedom systems. Fig.1 is stress state of motion for rigid rotor supported by five degrees of freedom of AMBs.

Considering the influence of electromagnetic force and unbalance force periodically, taking the single degree of freedom non-linear mathematical model as the foundation, the non-linear dynamic dimensionless equation of the five degrees of freedom rotor-AMB system is [15]:

$$M_b X^{//} + C_b X^{/} + K_b X + F_b = 0 \tag{1}$$

$$M_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$C_b = \begin{bmatrix} 2\frac{u_{xa}}{\omega_0} & 0 & 0 & 0 & 0 \\ 0 & 2\frac{u_{xb}}{\omega_0} & 0 & 0 & 0 \\ 0 & 0 & 2\frac{u_{ya}}{\omega_0} & 0 & 0 \\ 0 & 0 & 0 & 2\frac{u_{yb}}{\omega_0} & 0 \\ 0 & 0 & 0 & 0 & 2\frac{u_{z}}{\omega_0} \end{bmatrix} \tag{3}$$

$$K_b = \begin{bmatrix} (\frac{\omega_{xa}}{\omega_0})^2 & 0 & 0 & 0 & 0 \\ 0 & (\frac{\omega_{xb}}{\omega_0})^2 & 0 & 0 & 0 \\ 0 & 0 & (\frac{\omega_{ya}}{\omega_0})^2 & 0 & 0 \\ 0 & 0 & 0 & (\frac{\omega_{yb}}{\omega_0})^2 & 0 \\ 0 & 0 & 0 & 0 & (\frac{\omega_{z}}{\omega_0})^2 \end{bmatrix} \tag{4}$$

$$F_b = -\begin{bmatrix} (\frac{A_1}{\omega_0}x_b^{/} + \frac{A_2}{\omega_0^2}x_b - \frac{J_z\alpha 2\pi\Omega}{J_y}y_b^{/} + \frac{J_z\alpha 2\pi\Omega}{J_y}y_a^{/}) + 4\pi^2\Omega^2(m_a e_a\sin(\omega_k\Omega t+\theta_a) + m_b e_b\sin(\omega_k\Omega t+\theta_b))/m \\ (\frac{A_3}{\omega_0}x_a^{/} + \frac{A_4}{\omega_0^2}x_a + \frac{J_z(1-\alpha)2\pi\Omega}{J_y}y_b^{/} - \frac{J_z(1-\alpha)2\pi\Omega}{J_y}y_a^{/}) + 4\pi^2\Omega^2(m_a e_a\sin(\omega_k\Omega t+\theta_a) + m_b e_b\sin(\omega_k\Omega t+\theta_b))/m \\ \frac{A_5}{\omega_0}y_b^{/} + \frac{A_6}{\omega_0^2}y_b + \frac{c_{0r}A_7}{\omega_0}y_b y_b^{/} + \frac{c_{0r}A_8}{\omega_0^2}y_a^2 + \frac{c_{0r}A_9}{\omega_0^2}y_b^2 + \frac{c_{0r}A_{10}}{\omega_0}y_a y_a^{/} + \frac{J_z\alpha 2\pi\Omega}{J_x}x_b^{/} - \frac{J_z\alpha 2\pi\Omega}{J_x}x_a^{/} + \frac{A_{11}}{c_{0r}\omega_0^2} + 4\pi^2\Omega^2(m_a e_a\cos(\omega_k\Omega t+\theta_a) + m_b e_b\cos(\omega_k\Omega t+\theta_b))/m \\ \frac{A_{12}}{\omega_0}y_a^{/} + \frac{A_{13}}{\omega_0^2}y_a + \frac{c_{0r}A_{17}}{\omega_0}y_b y_b^{/} + \frac{c_{0r}A_{14}}{\omega_0^2}y_a^2 + \frac{c_{0r}A_{15}}{\omega_0^2}y_b^2 + \frac{c_{0r}A_{16}}{\omega_0}y_a y_a^{/} - \frac{J_z(1-\alpha)2\pi\Omega}{J_x}x_b^{/} + \frac{J_z(1-\alpha)2\pi\Omega}{J_x}x_a^{/} + \frac{A_{18}}{c_{0r}\omega_0^2} + 4\pi^2\Omega^2(m_a e_a\cos(\omega_k\Omega t+\theta_a) + m_b e_b\cos(\omega_k\Omega t+\theta_b))/m \\ 0 \end{bmatrix} \tag{5}$$

$$X^{//} = \left[x_a^{//}, x_b^{//}, y_a^{//}, y_b^{//}, z^{//}\right]^T$$
$$X^{/} = \left[x_a^{/}, x_b^{/}, y_a^{/}, y_b^{/}, z^{/}\right]^T \qquad (6)$$
$$X = \left[x_a, x_b, y_a, y_b, z\right]^T$$

It is the foundation of the multi-scale method analysis below, and gives the theoretical basis to the solution to bifurcation and chaos movements.

III EXAMPLE ANALYSES OF PARTIAL BIFURCATIONS AND THE FREQUENCY RESPONSE CURVE

Making use of the multi-scale method to carry on the perturbation analysis to dimensionless movement equation of the rotor-AMB system, the key research on rotor-AMB system is the non-linear dynamic response in the primary resonance, and its averaged equation is obtained. In this foundation, using MATLAB6.5 software to carry on the numerical simulation, the frequency-response equation and partial bifurcation figure are obtained.

In assigning the values of the parameters u、φ_0、ω and so on ,E_{xa}, E_{xb}, E_{ya}, E_{yb} are defined as the function of the coordinate parameter σ, and its function curve is frequency response curve. For the purpose of simplicity, here only the first equation is selected to give the explanation, simultaneously, in order to obtain the single modal frequency-response equation, some kind of stipulation to one of the E_{xa}, E_{xb} must be produced. According to the literature [16,17], supposing that $E_{xb} = (\frac{1}{\alpha} - 1)E_{xa}$, Taking it into the equation, it is derived:

$$[(\frac{u_{xa}}{\omega_0})^2 + (\frac{\sigma_{xa}}{2\Omega})^2 - \frac{A_1^2}{4\omega_0^{\ 2}} * (\frac{1}{\alpha} - 1)^2 - \frac{A_2^2 E_{xb}^2}{4\Omega^2 \omega_0^{\ 4}} * (\frac{1}{\alpha} - 1)^2]E_{xa}^2 - 4(-\frac{J_z \alpha 2\pi\Omega}{2J_y} B_{yb} + \frac{J_z \alpha 2\pi\Omega}{2J_y} B_{ya}) * (\frac{A_1}{2\omega_0}\cos\varphi_{xb} + \frac{A_2}{2\Omega\omega_0^2}\sin\varphi_{xb}) * (\frac{1}{\alpha} - 1)E_{xa} - 4(-\frac{J_z \alpha 2\pi\Omega}{2J_y} B_{yb} + \frac{J_z \alpha 2\pi\Omega}{2J_y} B_{ya})^2 \qquad (7)$$

In the above equation, if let

$$A_\alpha = (\frac{u_{xa}}{\omega_0})^2 + (\frac{\sigma_{xa}}{2\Omega})^2 - \frac{A_1^2}{4\omega_0^{\ 2}} * (\frac{1}{\alpha} - 1)^2 - \frac{A_2^2 E_{xb}^2}{4\Omega^2 \omega_0^{\ 4}} * (\frac{1}{\alpha} - 1)^2$$

$$B_\alpha = -4(-\frac{J_z \alpha 2\pi\Omega}{2J_y} B_{yb} + \frac{J_z \alpha 2\pi\Omega}{2J_y} B_{ya}) * (\frac{A_1}{2\omega_0}\cos\varphi_{xb} + \frac{A_2}{2\Omega\omega_0^2}\sin\varphi_{xb}) * (\frac{1}{\alpha} - 1)$$

$$C_\alpha = -4(-\frac{J_z \alpha 2\pi\Omega}{2J_y} B_{yb} + \frac{J_z \alpha 2\pi\Omega}{2J_y} B_{ya})^2$$

The condition of the real number solution appearing is

$$\Delta = B_\alpha^2 - 4A_\alpha C_\alpha \geq 0 \text{ and } A_\alpha \neq 0 \qquad (8)$$

When Δ=0, the system has the unique solution, known from the literature[4], this solution is the saddle point. Furthermore, by deducing the above equation, it is known that, under Δ>0 conditions, when the rotor speed ω_0 increased, the Δ value reduces. This is identical with the experimental phenomenon that when the rotor speed increases, the vibration of the rotor intensifies. So it explained, although we do not have computed the Jacobi matrix value of the system, it is still was allowed to distinguish the system stability by the Δ value. Therefore, in this situation, Δ has become another sigh to judge whether the system is stable or not.

It is usually very difficult to seek the condition satisfying Δ>0 from equation (8). Here we only discuss E_{xb}=0, a special condition, which physical meaning is in fig.1 the B end's rigidity of radial magnetic bearing is infinity, so that the *x* direction vibration displacement of the rotor is zero.

Suppose that u_{xa}=2、φ_{xb}=30°, according to the third critical speed is 23607rmp, the $E_{xa} \propto \sigma_x$ bifurcation response curve is as Fig.2 shows.

Fig.2 pointed out that, the smaller the σ_{xa} value is, closer the speed approaches the critical one 23607rmp obtained by computing. When σ_{xa}=0, there occurs resonance when the speed arriving the critical point and the E_{xa} value suddenly increases. Otherwise, the bigger the σ_{xa} value is, the more the rotor speed deviate the critical speed as well as the rotor vibrates gently, the E_{xa} value reduces along with it. Seen from the tendency of the bifurcation response curve, the curve after striding across the sub-critical bifurcation area and the supercritical bifurcation area returns to the sub-critical bifurcation once more, the co-dimension is 2.

Fig.3 is basically consistent with Fig.2. What is different is Fig.3 is a reflection of bifurcation response curve in the fourth critical speeds ω=26328rpm.

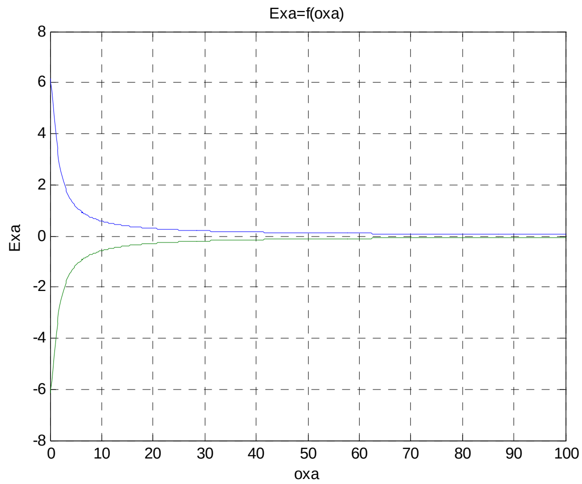


Fig.2 Responses of bifurcation when ω=23607rpm

The analysis of Fig.3 is like that of Fig.2. As the same principle, the other critical speeds can be carried on the similar partial bifurcation analysis. Finally, indicated, the single modal frequency-response curve is basically consistent with the Fig.2 and Fig.3. This explained, according to, the critical speed in this to calculate can truly have the resonance effect in critical point, the more the critical point deviate, the smaller the vibration is to be, this tallies with the experiment phenomenon, which conforms with the recognized vibration theory, therefore the linear mechanics and the nonlinear mechanics analysis obtained very good verifying here.

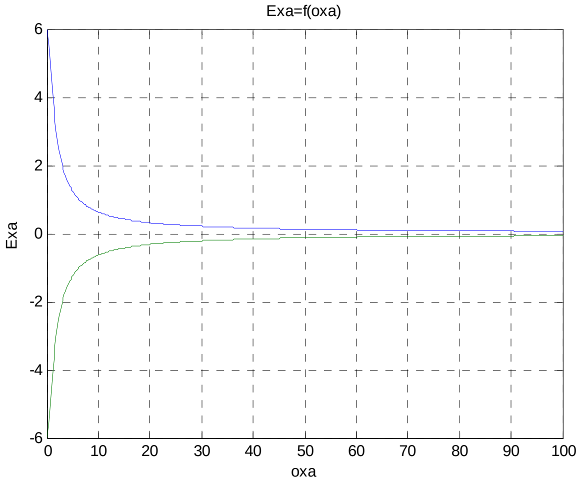


Fig.3 Responses of bifurcation when ω=26328rpm

The diagram of bifurcation curves indicated that, in five degrees of freedom rotor-AMB system, the bifurcations' types only have two kinds of the sub-critical bifurcation and the supercritical bifurcation, the bifurcation first strides from the sub-critical area across the supercritical area then returns to the sub-critical area.

IV CHAOS MOVEMENTS AND ITS NUMERICAL SIMULATION

A. The Basic Concepts about Chaos Mechanics

In nearly 40 years, a significant progress of the non-linear dynamics theory is discovering the chaos phenomenon. In the certain system, there may appear the similar stochastic phenomenon, this kind of rate process is extremely sensitive to the change of the initial value, therefore it is impossible to forecast the nonlinear system, and this kind of situation is the so-called chaos phenomenon.

The chaos phenomenon can only appear in the non-linear dynamic system. The linear model cannot have the chaos, because the linear system movement all has the rule, but in the nonlinear system, the chaos movement and the state of equilibrium, the periodic motion or almost periodic motion is completely different, it is one kind of complex movement that limited in some region or the track never come in the same.

As a result of the non-regularity and highly the complexity in the chaos movement, the majority research all uses the numerical method:

1) Oscillograms

The equations' solution $x=x(x_0,\lambda,t)$ has defined as a solution curve, which decided by the initial value x_0 and the parameter λ , t plays the parameter role. If the equation solution is regarded as the function of t, it expresses the time course of system movement, which is called the oscillogram. The main goal to analyse chaos movement is the behaviour in the state of steady motion. Because the chaos movement has the characteristic of partial instability and the whole stability, taking any initial value will obtain the nearly quite same behaviour in the long time steady motion condition.

2). Phase diagram

Phase diagram is the trajectory diagram. Regarding dynamic system expressed in n dimensional constant differential equation, the phase diagram is the projection of the system solution curve in vector space; this projection curve is namely the phase track.

3). Power spectrum

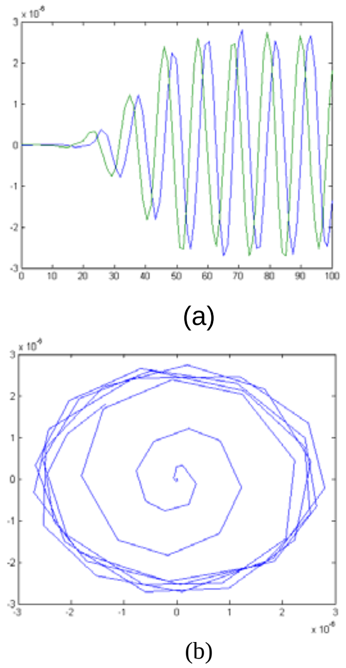
Because the chaos movement is non-periodic and complex, its power spectrum is different with that of the periodic motion or the almost periodic motion, which is separate spectral line, but is the continual spectrum.。

B Numerical Simulation Examples

Applying the numerical analysis method as mentioned to forecast the chaos movement of the rotor-AMB system; the analysis to the constant differential equation above is carried on using Matlab6.5 software. Considering the standard type of the averaged equation, using fourth or fifth-order Runge-Kutta method, the numerical computation is carried on to this constant differential equation and the chaos mechanics behavior is distinguished by using two methods of the system phase diagram and the oscillogram.

Fig.4 and Fig.5 are the oscillogram and phase diagram obtained by computer simulations, using the ode45 function in MATLAB 6.5 software 。

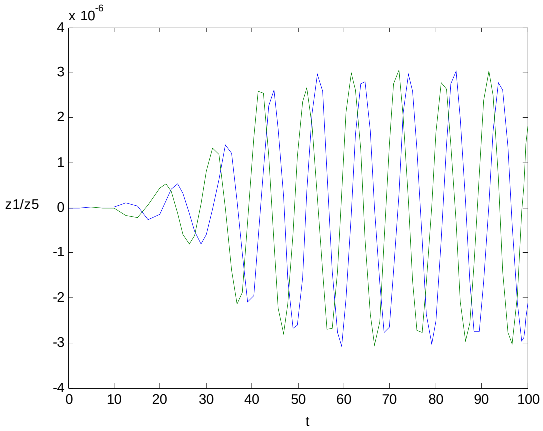
Fig.4 (a) is the oscillogram when ω=23607rpm, which reflects the changes of z_1 and z_5 along with the time t; fig. (b) is the phase diagram made up of z_1 and z_5. The oscillogram shows, the vibration amplitude of the system cannot grow infinitely along with time, but is to be nearly invariable after arriving some value.



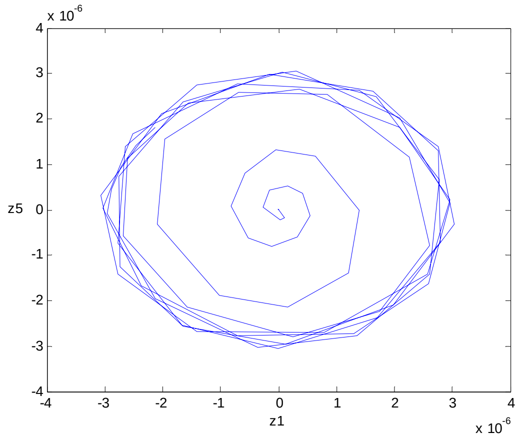
(a)

(b)

Fig.4 The oscillogram (a) and phase diagram (b) when ω=23607rpm



(a)



(b)

Fig.5 The oscillogram (a) and phase diagram (b) when ω=26328rpm

The phase diagram indicates that, under this group of parameters, the system can appear the chaos movement characteristics that the part is instability and the whole is stability. Using some conclusions in Literatures [1, 10, 17] for reference, the phase diagram is regarded as the trajectory of the axis center in the A end of rotor, which is a kind of movement limited in some region or its orbit never repeat and its state is extremely complicated. This shows, although the axis center diverges, this kind of diverge is not unlimited. When the axle center diverges to certain extent, it no longer continues to spread, but maintains this kind of condition and nearly is stable to move.

Fig.5 is the oscillogram and phase diagram when ω=26328rpm, which is similar to Fig.4 except it is more disorder and the chaos phenomenon is more obvious. Although, to some extent, the center orbits don't continue to spread, this kind of condition runs more unstably.

In fact, the analysis above can be found in the actual rotor movement. Because 23607rmp is the third order critical speed and 26328rmp is the fourth order critical speed. Through the experimental tests, the chaos phenomena have been seen.

Ⅴ EXPERIMENTS AND TESTS

In this experimental process, part of photographed pictures as follows: Fig.6 shows situation of the electric miller spindle equipped with AMB, Fig.7 is the outward appearance of system controller and the internal structure.

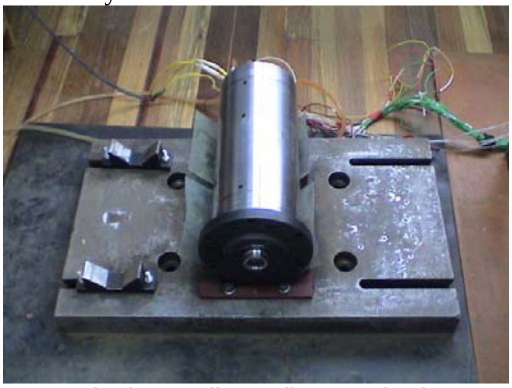


Fig.6 The electric miller spindle equipped with AMB

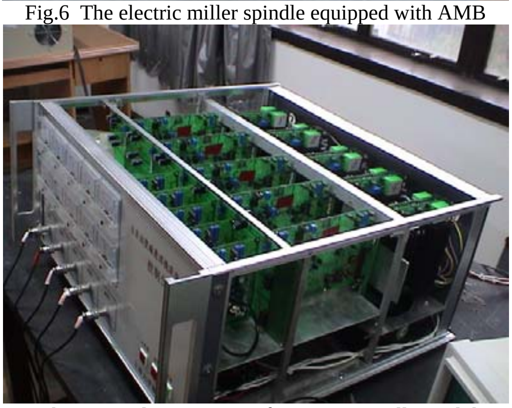


Fig.7 The outward appearance of system controller and the internal structure.

In the process, the vibration is big when the rotor runs between 20000rmp~30000rmp， especially in 27000rmp about, it is more obvious。

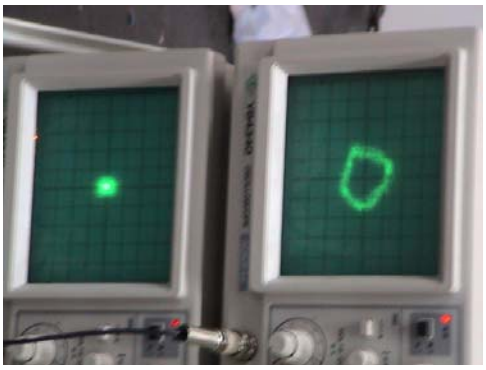


Fig.8 The Lissajous figure when ω= 24000rmp

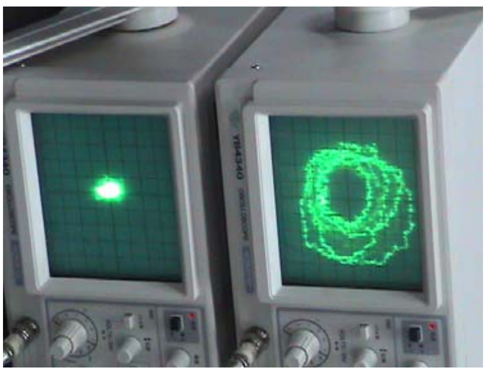


Fig.9 The Lissajous figure when ω=27000rmp

The above Fig.8 and Fig.9 demonstrate the trajectory diagram of movement, when the rotor system uses some group of PID parameters. Obviously, the rotor vibrates fiercer in 27000rmp than in 24000rmp. In two figures, the left graph demonstrated axle center path in the front bearing place, the right graph demonstrated the axle center path of the rear bearing. Fig.9 shows the vibration in rear bearing place even has the tendency to diverge, and the vibration is extremely fierce and the chaos movement characteristics that the part is instability and the whole is stability are very obvious.

The nonlinear analysis result tallies with the experimental process. The non-linear research indicated the vibration amplitude maintain invariable when it increase to some peak value to, the center orbit will be a close cycle. Before the rotor stride across the critical speeds, the vibration will intensify and the amplitude will increase; after it, the rotor will vibrate gently and the amplitude will reduce. Also, the axle center path will from the unceasing proliferation to gradually purse up, and will form so-called "limit cycle". In the experiment, taking the amplitude in 24000rmp about as the example, the rotor vibrates gradually intensifies before 24000rmp, the amplitude achieved the peak value in 24000rmp about, which approaches to the third order critical speed 23607rmp, so that resonates produce. After 24000rmp, the rotor vibration is gentle gradually; the axle center path also gradually reduces. In Fig.9, the rotor speed 27000rmp approaches to the fourth order critical speed 26328rmp and the resonance produces except it is more disorder and the chaos phenomenon is more obvious than Fig.8. Although, to some extent, the center orbits don't continue to spread, this kind of condition runs more unstably.

VI CONCLUTIONS AND PROSPECTS

This article have established the non-linear dynamic model of five degrees of freedom rotor-AMB system, and carried on processing five degrees of freedom nonlinear differential equation with the multi-scale method. Besides, the numerical solution to the averaged equation with the numerical analysis method has been done, and five degrees of freedom systems bifurcation and the chaos phenomenon has been analysed. The research shows, the rotor can appear the un-stabilization in certain situations, so there exists a question to judge the stability; The Hopf bifurcation explained the bifurcation type of the rotor system generally enters from the sub-critical bifurcation to the supercritical bifurcation, then returns to the sub-critical bifurcation area once more. The chaos phenomenon confirmed that, the rotor system may appear the chaos phenomenon under the certain condition, when stride across the critical speeds the chaos phenomenon is quite obvious, before the critical speed the amplitude of the vibration gradually to increase, after striding across the critical speeds the amplitude gradually to reduce, the trajectory diagram of the axis center which indicated with the chaos phenomena is consistent.

The experiment indicated that, the theoretical analysis is basically consistent with the experimental results, tallies well. The nonlinear analysis conclusions obtained by the theoretical calculations has been verified in experiments。

In the futural several years, UBC (Shanghai) Precision Bearing Manufacturing Co., Ltd will develop our industrial products in magnetic bearings with Research Institute of Bearing in Shanghai University. Now we have achieved two China patent and two United States patent (Fig.10).

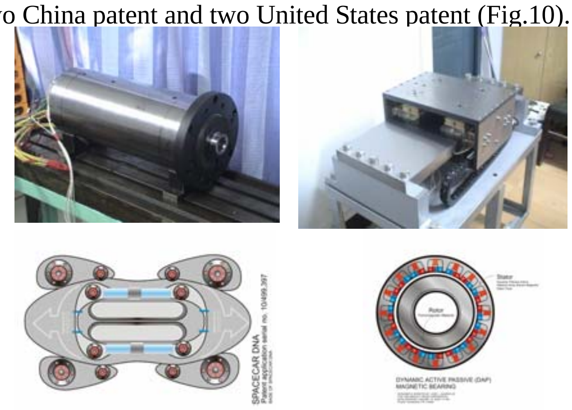


Fig 10 Patent of Magnetic Bearing

REFERENCES

[1] Lie Yu, Heng Liu. Dynamics of the Rotor-Bearing System Xi’an Xi’an Jiaotong University Press，2001

[2] Wei Zhang, Shaopu Yang, Jian Xu, Ming Ye, Zhiqiang Wu etc. Periodic Vibrations and Bifurcations in Nonlinear Systems, The Science Press, 2000

[3] Shaopu Yang, Yongjun Shen etc. The Bifurcations and Singularities in Nonlinear Systems, The Science Press, 2003

[4] Maoan Han, Shenshi Gu. The Theory and Method in Nonlinear Systems , The Science Press, 2001

[5] Bangchun Wen, Yinong Li, Qingkai Han. The Analytical Methods and Applications in Nonlinear Vibration Theory , Shen Yang , The Northeastern University Press , 2001

[6] Yushu Chen. The Bifurcations and Chaos Theory in Nonlinear Vibration Systems, Beijing, The High Education Press, 1993

[7] Yushu Chen. The Modern Analytical Methods in Nonlinear Dynamics, Beijing, The Science Press, 1992

[8] Xinjiang Zhang, Xinhua Wu, Songbo Xia. Study on Nonlinear Vibration of an Elastic Rotor-Bearing-Foundation System, Journal of Vibration Engineering，2001，14(2)：228-232

[9] Tiesheng Zheng, Stability and Bifurcation of High-order Local Nonlinear Rotor-bearing Dynamic System. [J] Acta Aeronautica Et Astronautica Sinica，1998，19(3)：284-292

[10] Haiyan Zhang, The Nonlinear Dynamics of Magnetic Bearing-Rotor System [The Master's Thesis] Beijing, Beijing University of Technology, 2002

[11] Adam Pilat. A Nonlinear AMB Model & Identification Methods, In Proceedings of the 9th Int. Symposium on Magnetic Bearings.Lexington Kentucky,USA, Aug. 3-6, 2004:99

[12] L.N.Virgin, T.F.Walsh and J.D.Knight, Nonlinear Beheavior of a Magnetic Bearing System [J], Transaction of the ASME, 1999,117: 582-588

[13] J.C.Ji and C.H.Hansen. Nonlinear Oscillations of a Rotor in Active Magnetic Bearings [J], Journal of Sound and Vibration, 2001,12(11): 1221-1224

[14] Yu Zhang, Yushu Chen, Qinsheng Bi. The Nonlinear Dynamics Research on Rotor-Bearing-Foundation System, Journal of Vibration Engineering, 1998，11(1)：24-30

[15] Gao Cheng, The Linear&Nonlinear Dynamics Research on a Milling Spindle Supported By Active Magnetic Bearings [The Master's Thesis] Shanghai, Shanghai University

[16] W.Zhang and Y.Tang. Global Dynamics of the Cable under Combined Parametrical and External Excitation [J], International Journal of Nonlinear Mechanics, 2000,37: 505-526

[17] A.H.Nayfeh, Applied Nonlinear Dynamics [W], New York, 1995