

# New modeling and control methods for flexible rotors with magnetic bearings toward passing through critical speeds caused by elastic modes

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**Abstract**—This paper proposes a new modeling and control method for flexible rotors using active magnetic bearings (AMB) toward passing through many critical speeds and realizing high-speed rotation. To achieve this, it is necessary to control vibration caused by some elastic modes under acting gyroscopic effect. A suggested model “Extended Reduced Order Physical Model” enables to express motion and vibration simultaneously. Then, we present a controller that combined PID and LQ controllers applied for the model.

**Index Terms** – rotor dynamics, multi-body dynamics, modelling method, vibration control, gyroscopic effect

## I. INTRODUCTION

Active magnetic bearings (AMB) systems have been applied to various machines such as grinding machines, vacuum pumps and energy storage flywheel system. Because it enables to support rotor without friction and has applicability for machines that demands rotor used in high-speed. However, according to rotate it in higher speed, we must consider to control more elastic modes of rotor and precession caused by gyroscopic effect. In this paper, a modeling technique for a flexible rotor-AMB system that can express vibration of elastic modes and gyroscopic effect is considered. And we propose a control method applied for the model.

The model is designed in order to obtain an exact multi-degree-of-freedom model of the flexible rotor-AMB system[1]. Utilizing the obtained model, we designed a state feedback control system which combines both PID and LQ controllers[2]. PID controller is used for stabilizing the originally unstable rotor- AMB system corresponded with control of the rigid mode, whereas LQ control with state feedback loops is adapted to control many elastic modes of vibration. One of the authors had applied these modeling and control method to a flexible rotor[3]. We have been developing bigger and more flexible rotor system. In this paper, The method is applied to the new rotor.

## II. CONTROL OBJECT AND ITS DYNAMIC CHARACTERISTICS

Fig.1 shows a schematic diagram of the flexible rotor used as the control object in this research. Mass of this rotor is about 10 kg. This rotor is levitated thrust axis by a PID controller independently designed. And the coupling among thrust axis and other axes is negligible. Therefore, this research considers only dynamics of the flexible rotor in radial direction later.

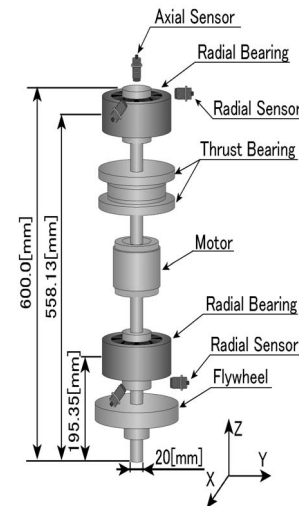


Fig.1 Schematic of flexible rotor-AMB system

Fig.2 shows the vibration mode shapes and their natural frequencies in free boundary condition obtained by FEM. In this study, we considered to control rigid modes, 1st elastic mode, and 2nd elastic mode.

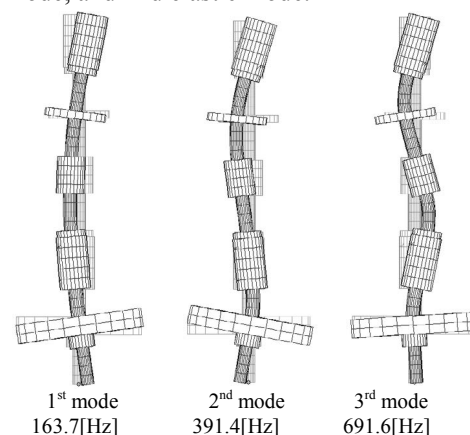


Fig.2 Modal shapes obtained by FEM

### III. EXTENDED REDUCED ORDER PHYSICAL MODELING METHOD

#### A. Design procedure of extended reduced order physical model

In order to simulate and control flexible structures, authors have already developed an effective modelling method called “Extended Reduced Order Physical Modeling Method (EROM)” which can be treated in multi-body dynamics[4]. This modelling method is very useful for dynamic simulations and controller design of flexible structures because it is possible to treat three-dimensional non-linear systems and to express a lumped parameter system.

In the model, flexible structure is composed of two kinds of rigid bodies and springs. One type of rigid body named as “reference rigid body” expresses motion, while the other type of rigid body named “rigid body element” express vibration.

The procedure for constructing the extended reduced order physical modelling method is as follows,

1. Analyze the mode shapes and calculate the modal masses.
2. Choose the number of modes to be controlled and determine the positions of the rigid body elements.
3. Identify the mass matrix of the rigid body elements.
4. Identify the mass matrix of the reference rigid body.
5. Identify the stiffness matrix of the rigid body elements.
6. Construct the equations of motion.

In this study, the chosen mode number is determined to 4, two elastic modes to two directions(x, y axis direction). Then, number of the rigid body element is selected to 9. It is decided from designing conditions of modal masses of rigid body element written as below.

1. Agreement of motion energy in vibration mode.
2. Keep orthogonality to other vibration modes
3. Conservation of momentum in vibration mode
4. Conservation of angular momentum in vibration mode

Representing these conditions by equation becomes written as below.

$$\begin{bmatrix} \Phi_i^T \Phi_i \\ \Phi_i^T \Phi_j \\ \Phi^T \mathbf{I}_R \mathbf{I}_X \\ \Phi^T \mathbf{I}_R \mathbf{I}_Y \\ \Phi^T \mathbf{I}_R \mathbf{I}_Z \\ (\mathbf{I}_X^T \tilde{\mathbf{r}}_{Gm} \Phi)^T \mathbf{I}_R + \Phi^T \mathbf{I}_\Theta \mathbf{I}_X \\ (\mathbf{I}_Y^T \tilde{\mathbf{r}}_{Gm} \Phi)^T \mathbf{I}_R + \Phi^T \mathbf{I}_\Theta \mathbf{I}_Y \\ (\mathbf{I}_Z^T \tilde{\mathbf{r}}_{Gm} \Phi)^T \mathbf{I}_R + \Phi^T \mathbf{I}_\Theta \mathbf{I}_Z \end{bmatrix} \begin{bmatrix} \xi & \mathbf{0}_{3n \times 3n} \\ \mathbf{0}_{3n \times 3n} & \mathbf{I}_{3n \times 3n} \end{bmatrix} \begin{bmatrix} \mathbf{M}_{OB} \\ \mathbf{J}'_B \end{bmatrix} = \begin{bmatrix} \mu_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Here,

$$\mathbf{M}_{OB} = [\mathbf{M}_{OB1} \quad \dots \quad \mathbf{M}_{OBm}]^T, \mathbf{J}'_B = [\mathbf{J}'_{B1} \quad \dots \quad \mathbf{J}'_{Bm}]^T,$$

$$\xi = \text{diag}[\mathbf{I}_{31} \quad \dots \quad \mathbf{I}_{31}], \mathbf{I}_{31} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Phi$  : Modal matrix obtained from vibration mode shapes  
 $\mathbf{M}_{OB}$  : Mass of rigid body elements,  $\mathbf{J}_B$  : Moment of inertia of rigid body elements, n: Number of reference mode

The mass matrix of the reference rigid body  $\mathbf{M}_{OA}$  and the moment of inertia  $\mathbf{J}'_A$  are calculated to be equal to real mass  $\mathbf{M}_{real}$  and real moment of inertia  $\mathbf{J}'_{real}$  of the system in total. This is the reason the model can express motion and vibration simultaneously.

$$\mathbf{M}_{OA} = \mathbf{M}_{real} - \eta_m^T \mathbf{M}_{OB} \eta_m \quad (2)$$

$$\mathbf{J}'_A = \mathbf{J}'_{real} - (\eta_m^T \mathbf{J}'_{\eta_m} + \eta_m^T \tilde{\mathbf{r}}_{Gm}^T \mathbf{M}_{OB} \tilde{\mathbf{r}}_{Gm} \eta_m) \quad (3)$$

$$\eta_m = [\mathbf{I} \quad \dots \quad \mathbf{I}]^T \quad (m \times 3)$$

Matrix  $\eta_m$  has many number of the  $3 \times 3$  unit matrix  $\mathbf{I}$  as design points, arranged lengthways.

The stiffness matrix  $\mathbf{k}$  is obtained from the following equation.

$$\mathbf{k} = \Phi^{-T} \omega^2 \mu \Phi^{-1} \quad (4)$$

where  $\omega$  is a matrix including natural frequency of each modes

Compared with modal masses of rigid body element that is a part of expressing vibration modes, mass of reference rigid body is designed as total mass of the model has same value from real mass of structure.

Fig.3 shows a model of our rotor obtained by applying the EROM concept.

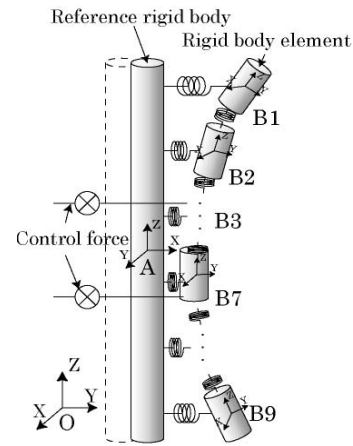


Fig. 3 Extended Reduced Order Physical Model

Next, I explain about relation between the model and real rotor using this figure. Rigid body labeled as A is reference rigid body. Rigid bodies labeled as B1~B9 are rigid body elements. B2, B6 are points of the actuators in the radial direction. B1, B7 are the sensing point in the radial displacement. And B5 is center of gravity of the

rotor. B9 means end of the rotor. Each rigid body have respectively 6 degrees of freedom (6DOF). Each rigid body elements and reference rigid body are connected with springs each other.

Parameters of the rigid body elements and the reference rigid body are determined by the procedure mentioned above and presented in Table 1.

Table 1 Parameters of rigid body elements and reference rigid body

Element	Distance from center [m]	Mass [kg]	Moment of Inertia[kgm <sup>2</sup> ]	
			X-axis	Y-axis
B1	0.3767	-0.05489	-0.1188	-0.1188
B2	0.3349	0.9217	0.05467	0.05467
B3	0.23721	0.5786	1.2003	1.2003
B4	0.1116	0.8567	0.9110	0.9110
B5	0	-0.1644	-0.3966	-0.3966
B6	-0.02791	-0.4220	1.0466	1.0466
B7	-0.05581	-0.1936	1.6247	1.6247
B8	-0.09767	1.1033	-0.1911	-0.1911
B9	-0.2233	0.07381	-0.3170	-0.3170
A	0	8.3154	-3.6214	-3.6214

### B. Equations of motion

In this section, we conduct equation of motion using the method of multi-body dynamics “the constraint addition method”[5]. Firstly, the equations of motion of the reference rigid body A without a restraint in 6 DOF are given like this.

$$\mathbf{M}_{OA} \dot{\mathbf{V}}_{OA} = \mathbf{F}_{OA} \quad (5)$$

$$\mathbf{J}'_{OA} \dot{\boldsymbol{\Omega}}'_{OA} + \tilde{\boldsymbol{\Omega}}'_{OA} \mathbf{J}'_{OA} \boldsymbol{\Omega}'_{OA} = \mathbf{N}'_{OA} \quad (6)$$

$\dot{\mathbf{V}}_{OA}$  : Velocity of reference rigid body,

$\mathbf{F}_{OA}$  : External force of reference rigid body

$\boldsymbol{\Omega}'_{OA}$  : Angular speed of reference rigid body,

$\mathbf{N}'_{OA}$  : Torque of reference rigid body

Similarly, the equations of motion of the rigid body elements (B1~B9) without restraint (6 DOF) are given like this

$$\mathbf{M}_{OB} \dot{\mathbf{V}}_{OB} = \mathbf{F}_{OB} \quad (7)$$

$$\mathbf{J}'_{OB} \dot{\boldsymbol{\Omega}}'_{OB} + \tilde{\boldsymbol{\Omega}}'_{OB} \mathbf{J}'_{OB} \boldsymbol{\Omega}'_{OB} = \mathbf{N}'_{OB} \quad (8)$$

Each symbols mean same in (5) and (6). Suffix “B” means the value of rigid body element B1~B9.

Equations (5) ~ (8) are equations of motion of the rotor without restraint. The generalized velocity  $\mathbf{H}$  without restraint is given by

$$\mathbf{H} = [\mathbf{V}_{OA} \quad \boldsymbol{\Omega}'_{OA} \quad \mathbf{V}_{OB} \quad \boldsymbol{\Omega}'_{OB}]^T \quad (9)$$

The mass matrix include the moment of inertia  $\mathbf{M}^H$  and external force matrix  $\mathbf{F}^H$  are given by

$$\mathbf{M}^H = \begin{bmatrix} \mathbf{M}_{OA} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}'_{OA} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{OB} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}'_{OB} \end{bmatrix} \quad (10)$$

$$\mathbf{F}^H = [\mathbf{F}_{OA}^T \quad \mathbf{N}'_{OA}^T \quad \mathbf{F}_{OB}^T \quad \mathbf{N}'_{OB}^T]^T \quad (11)$$

Arranged equations (9) ~ (11) can be written by

$$\mathbf{M}^H \dot{\mathbf{H}} = \mathbf{F}^H \quad (12)$$

Secondly, when  $\mathbf{h}$  is represents the modal velocity, the equation of motion with restraints will be derived. The generalized velocity  $\mathbf{S}$  in the system with restraints is given by

$$\mathbf{S} = [\mathbf{V}_{OA}^T \quad \boldsymbol{\Omega}'_{OA}^T \quad \mathbf{h}^T]^T \quad (13)$$

Using the constraint addition method[4], the equation of motion of the rigid body model is derived. The general equations of the constraint addition method are as follows.

$$\mathbf{M}^S \dot{\mathbf{S}} = \mathbf{F}^S \quad (14)$$

$$\mathbf{H} = \mathbf{H}_S \mathbf{S} + \mathbf{H}_{\bar{S}} \quad (15)$$

$$\mathbf{M}^S = \mathbf{H}_S^T \mathbf{M}^H \mathbf{H}_S \quad (16)$$

$$\mathbf{F}^S = \mathbf{H}_S^T \left[ \mathbf{F}^H - \mathbf{M}^H \left( \frac{d\mathbf{H}_S}{dt} \mathbf{S} + \frac{d\mathbf{H}_{\bar{S}}}{dt} \right) \right] \quad (17)$$

where,

$\mathbf{M}^S$  : Generalized mass matrix constraints

$\mathbf{F}^S$  : Generalized force vector constraint

$\mathbf{M}^H$  : Mass matrix unconstraint

$\mathbf{H}$  : Generalized velocity unconstraint

$\mathbf{F}^H$  : Force vector unconstraint

### C. State space representation

The equation of motion derived in the foregoing section is non-linear. In order to design the linear control system, it is necessary to linearize the equation of motion and to create a linearized state space form. If small terms  $\delta\dot{\mathbf{S}}$  and  $\delta\mathbf{F}^S$  of  $\dot{\mathbf{S}}$  and  $\mathbf{F}^S$  are deleted, (14) is written as follows.

$$\mathbf{M}_0^S \delta\dot{\mathbf{S}} = \delta\mathbf{F}^S \quad (18)$$

where the matrix with subscript 0 means substitution of the equilibrium position. The relationship between generalized velocity  $\mathbf{S}$  and generalized coordinate  $\mathbf{Q}$  with a derivative with respect to time is

$$\dot{\mathbf{Q}} = \mathbf{L}\mathbf{S} \quad (19)$$

Equation (14) and (19) can be expressed in matrix form as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{S} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} (\mathbf{M}^S)^{-1} \mathbf{F}^S \\ \mathbf{L}\mathbf{S} \end{bmatrix} \quad (20)$$

This equation is a first-order ordinary differential equation. From the above equation, the state-space form

$$\dot{\mathbf{X}}_c = \mathbf{A}_c \mathbf{X}_c + \mathbf{B}_c \mathbf{U} \quad (21)$$

$$\mathbf{Y}_c = \mathbf{C}_c \mathbf{X}_c \quad (22)$$

can be obtained by linearization. When state valuable  $\mathbf{X}_c$  is

$$\mathbf{X}_c = [\mathbf{S}^T \quad \mathbf{Q}^T]^T \quad (23)$$

system matrix  $\mathbf{A}_c$  is the partial derivative of the right side of Eq.(20) with respect to state valuable  $\mathbf{X}_c$  [6]

$$\mathbf{A}_c = \frac{\partial}{\partial \mathbf{X}} \left[ \begin{array}{c} (\mathbf{M}^S)^{-1} \mathbf{F}^S \\ \mathbf{S} \end{array} \right] \Bigg|_{\substack{\mathbf{Q}=\mathbf{Q}_{eq} \\ \mathbf{S}=\mathbf{S}_{eq}}} \quad (24)$$

Also, the control matrix  $\mathbf{B}_c$  is the partial derivative of the right hand side of Eq.(20) with respect to input  $\mathbf{U}$ .

$$\mathbf{B}_c = \frac{\partial}{\partial \mathbf{U}} \left[ \begin{array}{c} (\mathbf{M}^S)^{-1} \mathbf{F}^S \\ \mathbf{S} \end{array} \right] \Bigg|_{\substack{\mathbf{Q}=\mathbf{Q}_{eq} \\ \mathbf{S}=\mathbf{S}_{eq}}} \quad (25)$$

$$\mathbf{C}_c = \frac{\partial}{\partial \mathbf{X}} \left[ \begin{array}{c} \mathbf{R}_{OB2} \\ \mathbf{R}_{OB7} \end{array} \right] \Bigg|_{\substack{\mathbf{Q}=\mathbf{Q}_{eq} \\ \mathbf{S}=\mathbf{S}_{eq}}} \quad (26)$$

$\mathbf{Q}_{eq}$  and  $\mathbf{S}_{eq}$  are the equilibrium positions of generalized coordinates and generalized velocitys, respectively.

#### 4. NUMERICAL ANALYSIS OF ROTOR DYNAMICS

##### A. Frequency response

Simulated frequency responses using the space state equation (21) are shown in Fig.4 and 5. The resonance peaks appeared near 35 Hz and 85 Hz are of two rigid modes (parallel mode and conical mode), and 160 Hz and 390 Hz are 1<sup>st</sup> and 2<sup>nd</sup> elastic modes. The gyroscopic effect is clearly appeared in Fig.5 though it did not appear in Fig.4. Fig.5 shows frequency response of the model rotational speed at 30 Hz. It is recognized that the gyroscopic effect act to divide into two natural frequencies in each modes. The identical natural vibrations are coupled in the X-Y direction, when the gyroscopic effect is acted. Therefore, the natural frequencies move to the higher frequency and the lower frequency separately. As mentioned above, this modeling method has expressed gyroscopic effect.

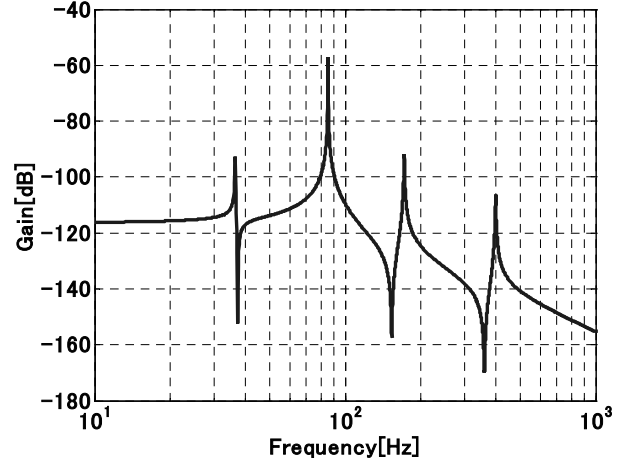


Fig.4 Frequency response of the model at rotational speed 0[Hz]

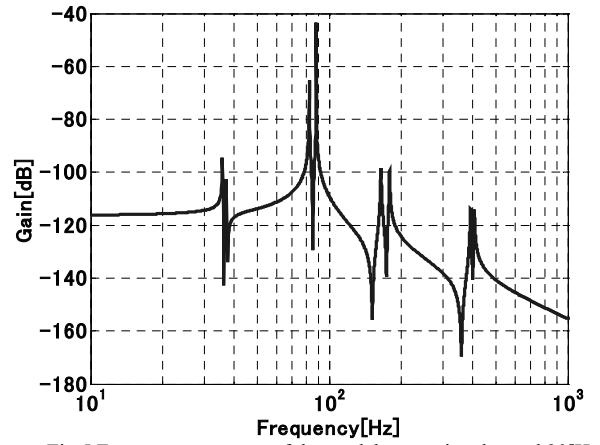


Fig.5 Frequency response of the model at rotational speed 30[Hz]

##### B. Whirl chart

In order to demonstrate the usefulness of extended reduced order physical model that can treat the gyroscopic effect of the flexible rotor, variation of natural frequencies depended on rotational speed is examined by calculation. Fig. 6 shows the relationship between rotational speed and natural frequency of the rotor used in this study. Natural frequencies of two elastic modes located at 160 Hz and 390 Hz under 0 rpm are separated into two frequencies as the rotational speed increases.

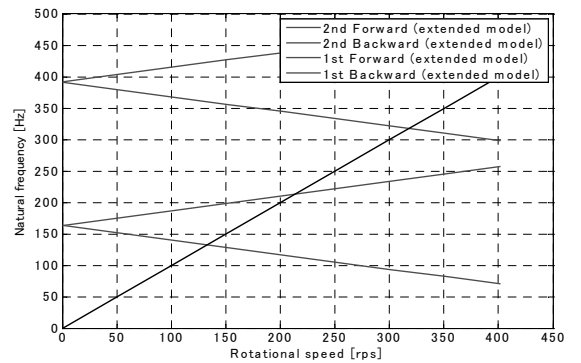


Fig.6 Natural frequency changed by gyroscopic effect

## 5. CONTORL SYSTEM DESIGN

### A. PID Control

Since the rotor-AMB system is essentially an unstable, PID control is used to stabilizes the system. The two rigid modes (parallel and conical) are also controlled. Fig.7 shows the associated block diagram.

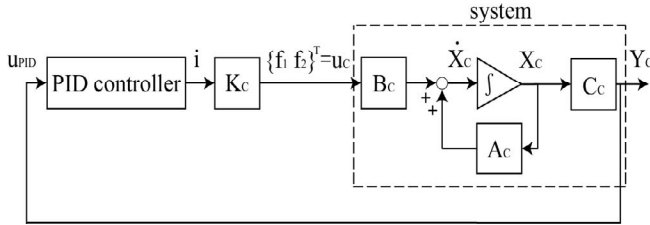


Fig.7 Block diagram

The transfer function of PID controller is designed as follows,

$$\frac{i}{u_{PID}} = \frac{1}{Ls + R} \left( \frac{\alpha_1 T_{d1}s + 1}{T_{d1}s + 1} \right) \left( \frac{\alpha_2 T_{d2}s + 1}{T_{d2}s + 1} \right) \quad (27)$$

Therefore, it can be written in the state space representation as below,

$$\dot{X}_h = A_h X_h + B_h u_h \quad (28)$$

$$Y_h = C_h X_h \quad (29)$$

Here,

$$X_h = \{\ddot{e} \quad \dot{e} \quad e\}^T, \quad u_h = u_{PID}, \quad Y = i$$

$$A_h = \begin{bmatrix} \frac{RT_{d1}T_{d2} + L(T_{d1} + T_{d2})}{LT_{d1}T_{d2}} & \frac{R(T_{d1} + T_{d2}) + L}{LT_{d1}T_{d2}} & -\frac{R}{LT_{d1}T_{d2}} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_h = \begin{bmatrix} 1 \\ \frac{LT_{d1}T_{d2}}{L} \\ 0 \\ 0 \end{bmatrix}$$

$$C_h = [\alpha_1 \alpha_2 T_{d1} T_{d2} \quad \alpha_1 T_{d1} + \alpha_2 T_{d2} \quad 1]$$

The designed parameter and frequency response with PID controller are shown as follows.

$$\alpha_1 = 5.775, \quad \alpha_2 = 3.224$$

$$T_{d1} = 1.531 \times 10^{-4}, \quad T_{d2} = 6.170 \times 10^{-4}$$

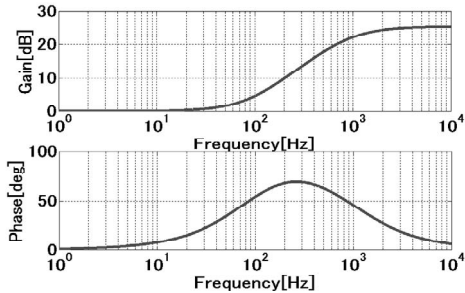


Fig.8 Frequency response of PID controller

### B. Filtered LQ control system combined with PID

Fig.9 shows a block diagram of the system combined PID controller with LQ controller. The feedback gain vectors,  $K_{PID}$  is tuning and  $K_{LQ}$  is obtained by applying LQ control law to the augmented system.

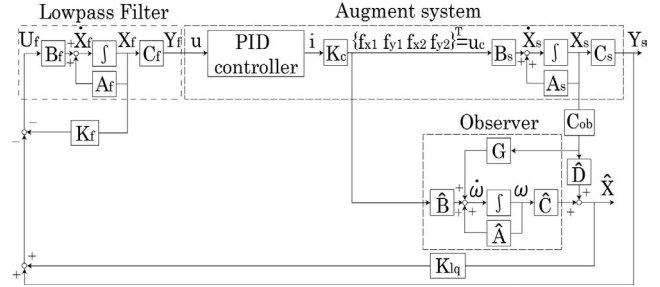


Fig.9 Block diagram of augmented system

Derived non-linear equation (14) is done in the linearized equation, and act out a state equation. The state equation and output equation combined with PID control system into this equation are shown as follows.

$$\dot{X} = AX + BU \quad (30)$$

$$Y = CX \quad (31)$$

Here,

$$A = \begin{bmatrix} A_f & \mathbf{0}_{8 \times 12} & \mathbf{0}_{8 \times 16} \\ B_h C_f & A_h & \mathbf{0}_{12 \times 16} \\ \mathbf{0}_{16 \times 8} & B_s C_h & A_s \end{bmatrix}, \quad B = \begin{bmatrix} B_f \\ \mathbf{0}_{12 \times 4} \\ \mathbf{0}_{16 \times 4} \end{bmatrix}, \quad C = [\mathbf{0}_{4 \times 8} \quad \mathbf{0}_{4 \times 12} \quad C_s]$$

$$X = \{X_f \quad X_h \quad X_c\}^T$$

$$U = -(I_{4 \times 4} C + K)X$$

Here, the state variable is defined as follows.

$$X_f = \{\dot{x}_{f1} \quad x_{f1} \quad \dot{y}_{f1} \quad y_{f1} \quad \dot{x}_{f2} \quad x_{f2} \quad \dot{y}_{f2} \quad y_{f2}\}^T$$

$$X_h = \{\ddot{e}_{x1} \quad \dot{e}_{x1} \quad e_{x1} \quad \ddot{e}_{y1} \quad \dot{e}_{y1} \quad e_{y1} \\ \ddot{e}_{x2} \quad \dot{e}_{x2} \quad e_{x2} \quad \ddot{e}_{y2} \quad \dot{e}_{y2} \quad e_{y2}\}^T$$

$$K = [K_f \quad K_{lq}]$$

Thus, the state equation of the augment system is written as follows.

$$\dot{X}_c = A_c X_c + B_c U_c \quad (32)$$

$$Y_c = C_c X_c \quad (33)$$

Here,

$$A_c = \begin{bmatrix} A_H & \mathbf{0} \\ B_s C_H & A_s \end{bmatrix}, \quad B_c = [B_H \quad \mathbf{0}]^T,$$

$$C_c = [\mathbf{0} \quad C_s]$$

## 5. COMPUTER SIMULATION

In this paper, simulation results show in frequency responses using only PID controller and controller combining PID with LQ, as shown in Fig.10,11 and 12. These frequency responses are calculated in case the rotor is at critical speeds obtained by Fig.6, and oscillated at

rigid body element B2 and observed at B1. It is clearly shown that the rigid modes are well suppressed when PID controller is only used, however, the elastic modes are not suppressed. This is because PID controller is used merely for controlling the rigid modes in this study. Contrarily, the controller combined PID with LQ is well acted to control the 1<sup>st</sup> elastic mode and also 2<sup>nd</sup> elastic mode. Especially, separated two natural frequencies appeared by gyroscopic effect as the rotational speed increases are well controlled. It is an effect of the used state feedback to bring robustness.

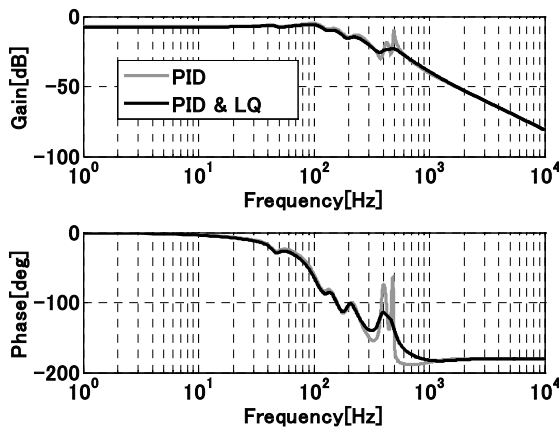


Fig.10 Simulation results of frequency response at 130(Hz)

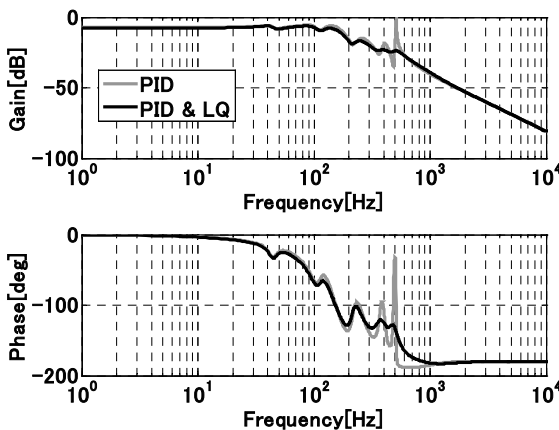


Fig.11 Simulation results of frequency response at 210(Hz)

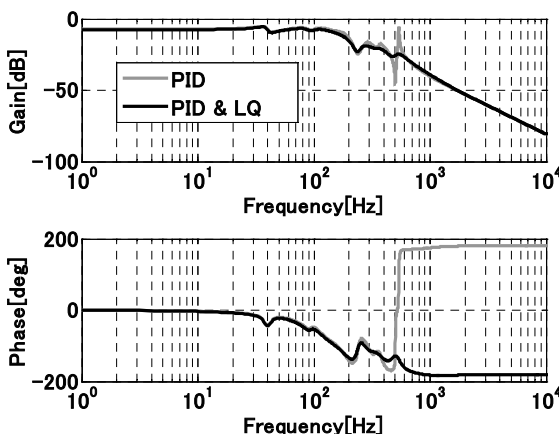


Fig.12 Simulation results of frequency response at 320(Hz)

## 6. CONCLUSIONS

In this paper, a controller design procedure for a flexible rotor-AMB system has been investigated. The proposed modeling technique was usefully applied to obtain the multi-degree of freedom model of the flexible rotor-AMB system exactly. Extended reduced order physical model of a flexible rotor-AMB system can simultaneously express the motion and vibration. The controller designed using controller combined PID with LQ control were effectively introduced to stabilize the flexible rotor and to control flexible bending vibration.

It is expected that the proposed modeling and controlling methods may invite a break through for high-speed rotations of flexible rotors to pass through more than the second critical speed. In the near future, we will try to prove utility of the model by experiment.

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